The *Ultimate* Formula Sheet for ACT Math

The ACT does not provide any formulas. Be prepared by making sure to have these ones memorized.

Fractions, Decimals, & Percentages: (for this section, r is the percent in <u>decimal</u> form)

$Fraction = \frac{part}{whole}$; $Percent = \frac{part}{100}$

Percent Increase or Decrease: $\frac{|old - new|}{old} \times 100\%$

Increase by a percent: multiply by $(1+r)$

Decrease by a percent: multiply by $(1-r)$

Simple Interest: $A = P(1+rt)$

Interest Compounded Annually: $A = P(1+r)^t$

Interest Compounded n times per year: $A = P\left(1+\frac{r}{n}\right)^{nt}$

Rates, Ratios, & Proportions:

General form of a conversion factor: $\left(\frac{ending\_units}{starting\_units}\right)$

Example: $10 feet\left(\frac{12 inches}{1 foot}\right) = 120 inches$

(Concentration of A x Volume of A)
+ (Concentration of B x Volume of B)
= Final concentration (Vol. of A + Vol. of B)

Distance = Rate x Time

Exponents, Roots, & Polynomials:

Multiplication Rule for Exponents: $a^b \cdot a^c = a^{b+c}$

Division Rule for Exponents: $\frac{a^b}{a^c} = a^{b-c}$

Power Rule for Exponents: $\left(a^b\right)^c = a^{bc}$

Negative Exponents: $a^{-b} = \frac{1}{a^b}$

Fractional Exponents: $a^{\frac{b}{c}} = \sqrt[c]{a^b}$ or $\left(\sqrt[c]{a}\right)^b$

$i = \sqrt{-1}$; $i^2 = -1$; $i^3 = -i$; $i^4 = 1$

$i^{4n} = 1$; $i^{4n+1} = i$; $i^{4n+2} = -1$; $i^{4n+3} = -i$

Complex Conjugates: $(a+bi)(a-bi)$

Parabolas:

<u>Standard Form</u>: $f(x) = ax^2 + bx + c$;

vertex= $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$;

y-intercept = c;

x-intercepts = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sum of solutions = $\frac{-b}{a}$

Discriminant = $b^2 - 4ac$; Pos=2 real roots

Zero= 1 real root; Neg=2 imaginary roots

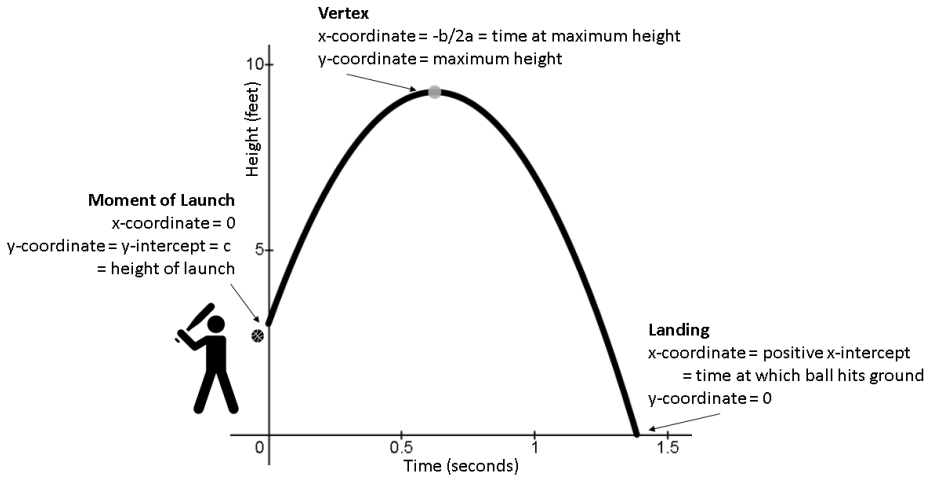
<u>Factored Form</u>: $f(x) = a(x-m)(x-n)$;

x-intercepts are m and n;

x-coordinate of vertex = $\frac{m+n}{2}$

<u>Vertex Form</u>: $f(x) = a(x-h)^2 + k$;

vertex = (h,k)



Difference of Squares: $a^2 - b^2 = (a+b)(a-b)$

Sum of Cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ Difference of Cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Perfect Square Trinomial: $a^2 + 2ab + b^2 = (a+b)^2$ and $a^2 - 2ab + b^2 = (a-b)^2$

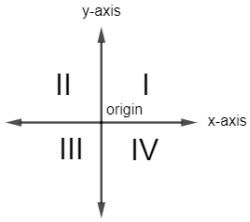
Completing the Square: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

Graphing Lines:

Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Slope of horizontal line = 0

Slope of vertical line = undefined



Standard Form: $Ax + By = C$

Slope-Intercept Form: $y = mx + b$

Point-Slope Form: $y - y_1 = m(x - x_1)$

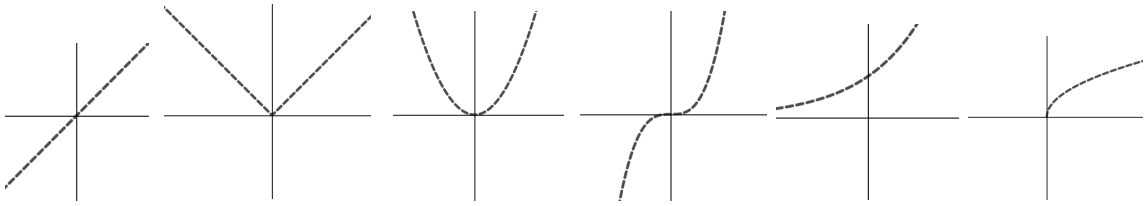
Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint Formula: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Parallel lines: equal slopes

$\perp$ Lines: slopes are opposite reciprocals

Parent Graphs & Transformations:



$y = x$ $y = |x|$ $y = x^2$ $y = x^3$ $y = a^x$ $y = \sqrt{x}$

Transformation	Visual effect
$f(x)+k$	Shift up by k units
$f(x)-k$	Shift down by k units
$f(x+h)$	Shift left by h units
$f(x-h)$	Shift right by h units
$-f(x)$	Reflect over the x axis (flip upside down)
$cf(x)$	Stretch vertically by a factor of c (becomes skinnier)
$\frac{1}{c}f(x)$	Shrink vertically by a factor of c (becomes fatter)

Data & Probability:

$$Average = \frac{sum\_of\_items}{number\_of\_items}$$

Median = center data point

Mode = most frequent data point

Range = maximum – minimum

$$Probability = \frac{desired\_outcomes}{possible\_outcomes}$$

Probability that independent events A and B will both happen: $P(A \cap B) = P(A) \times P(B)$

Probability that either A or B will happen: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Expected Value: $E(x) = \sum_{i=1}^{n} x_i \bullet P(x_i)$

Angles:

Vertical ∠' s are ≅

∠'s that form a linear pair are supplementary (add up to 180°)

∠' s that form a circle add up to 360°

When ∥ lines are cut by a transversal, all acute ∠' s are ≅ and all obtuse ∠' s are ≅

Triangles:

Area of a Triangle: $A = \frac{1}{2}bh$

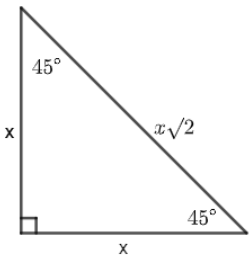
The three ∠' s of a Δ add up to 180°

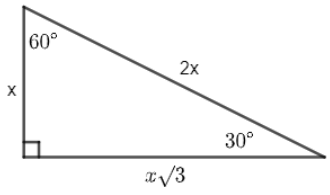
An exterior ∠ is equal to the sum of the two remote interior ∠' s

Pythagorean Theorem: $a^2 + b^2 = c^2$

Pythagorean Triples: 3-4-5 and 5-12-13

Special Right Triangles:

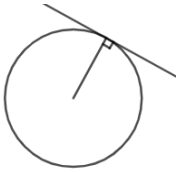




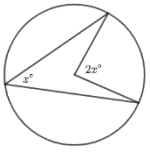
Circles:

Area of a Circle: $A = \pi r^2$

Circumference of a Circle: $C = 2\pi r$



A radius and tangent make a right ∠



A central ∠ is double the inscribed ∠

$\frac{x}{360}=\frac{arc}{circumference}$ and $\frac{x}{360}=\frac{sector}{area\_of\_circle}$ where x = central angle

Polygons: (for this section, n is the number of sides)

Area of a Rectangle: $A=lw$

Area of a trapezoid: $\frac{1}{2}(b_1+b_2)h$

Sum of the exterior angles: 360°

Sum of the interior angles: $180(n-2)$

One int. ∠ of a regular polygon: $\frac{180(n-2)}{n}$

# of diagonals: $\frac{n(n-3)}{2}$ (convex only)

Properties of Parallelograms:

1. Opp sides are ∥ and ≅
2. Opp ∠' s are ≅
3. Consec ∠'s are supplementary
4. Each diagonal forms a pair of ≅Δ's
5. Diagonals bisect each other

➔If they are ≅ it is a rectangle

➔If they are ⊥ it is a rhombus

6. $Area=base\times height$

Solids:

Volume of a Rectangular Prism (Box): $V=lwh$

Surface Area of a Box: $SA=2(lw+lh+wh)$

Volume of a Cylinder: $V=\pi r^2h$

Surface Area of a Cylinder: $SA=2\pi r^2+2\pi rh$

Volume of a Sphere: $V=\frac{4}{3}\pi r^3$

Volume of a Cone: $V=\frac{1}{3}\pi r^2h$

Volume of a Pyramid: $V=\frac{1}{3}lwh$

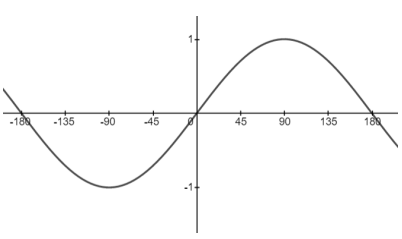
Trigonometry:

$\sin=\frac{opp}{hyp}$ $\cos=\frac{adj}{hyp}$ $\tan=\frac{opp}{adj}$ $\csc(x)=\frac{1}{\sin(x)}$ $\sec(x)=\frac{1}{\cos(x)}$ $\cot(x)=\frac{1}{\tan(x)}$

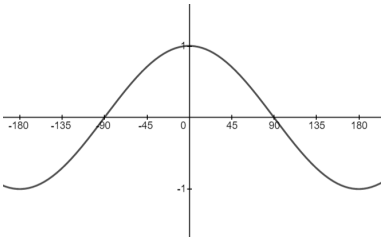
360°=2π radians $\tan x=\frac{\sin x}{\cos x}$ $\sin^2 x+\cos^2 x=1$ $\sin(x)=\cos(90-x)$

Law of Sines: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

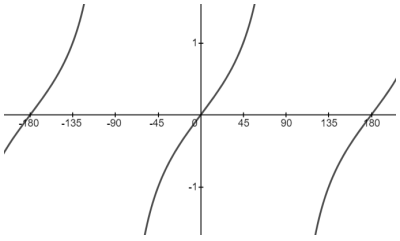
Law of Cosines: $a^2=b^2+c^2-2bc\bullet\cos(A)$



$y=\sin(x)$



$y=\cos(x)$



$y=\tan(x)$

If $y = A\sin(Bx - C) + D$ (also for cos, csc, and sec)

Amplitude: $|A|$ Period: $\frac{2\pi}{B}$ Phase Shift: $\frac{C}{B}$ Vertical Shift: D

If $y = A\tan(Bx - C) + D$ (also for cot)

Amplitude: none Period: $\frac{\pi}{B}$ Phase Shift: $\frac{C}{B}$ Vertical Shift: D

Sequences and Series: where a_1 = first term, n = number of terms, d = common difference, r = common ratio

Arithmetic sequence: $a_n = a_1 + (n-1)d$ Geometric sequence: $a_n = a_1 r^{n-1}$

Sum of an arithmetic series: $S_n = \frac{n}{2}(a_1 + a_n)$ Sum of a geometric series: $S_n = \frac{a_1(r^n - 1)}{r - 1}$

Logarithms:

If $\log_b a = x$, then $b^x = a$ $\log_b a = \frac{\log a}{\log b}$

Vector Addition: $\vec{a} + \vec{b} = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

Matrix Multiplication: Only possible when columns of first = rows of second

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \times \begin{pmatrix} E & F \\ G & H \end{pmatrix} = \begin{pmatrix} AE + BG & AF + BH \\ CE + DG & CF + DH \end{pmatrix}$$

Determinant of $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = AD - BC$

Conic Sections:

Circle: $(x-h)^2 + (y-k)^2 = r^2$, where (h,k) is the center and r is the radius

Ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ where (h,k) is the center, 2a is the horizontal axis, and 2b is the vertical axis

Horizontal Ellipse: $a^2 = b^2 + c^2$ Vertical Ellipse: $b^2 = a^2 + c^2$ where c is the distance from center to focus

Horizontal Hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ Vertical Hyperbola: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
