Lees asseblief die inligting op die volgende bladsy aandagtig deur!

Jou Handleiding

Hierdie is jou "handleiding", lees hom asseblief deeglik deur.

1. Hierdie PDF bestaan uit 10 vraestelle en 10 memos.

2. Die vraestelle en memos is gerangskik as Vraestel 1/Memo1/ Vraestel 2/Memo 2 ens.

3. Voor elke vraestel is ’n blad wat aandui dat jy met ’n nuwe vraestel en memo begin.

4. Die voorblaaie, instruksies en formule bladsye is verwyder om papier te spaar. Direk na hierdie bladsy is ’n enkele "instruksies" blad sowel as ’n "Formule blad".

5. Moet asseblief nie onnodig print nie. Probeer hiermee werk sonder om te print, dit sal ongelooflik wees vir die omgewing (en jou gatsak – ink en papier is duur).

6. Hierdie is vorige skool en departementele vraestelle wat verniet beskikbaar is op die internet. Dit beteken dat daar foute is in die memos maar dat dit reg is in die video's. Dit beteken ook jy kan hierdie pdf deel maar nie verkoop nie (jy het nie hierdie pdf by ons gekoop nie – maar die video's).

7. Jy gaan die meeste baat vind by hierdie program as jy die vraestelle uitwerk asof jy in ‘n eksamen sit (in die voorgeskrewe tyd en sonder hulp van jou handboek). Merk dit dan met die memos en kyk laastens die video's van die vrae wat jy nie verstaan nie.

8. Die Video's is beskikbaar op ons webblad: PolyMathic waar jy die betaling gemaak het. Gebruik die epos en Password wat jy gebruik het met "signup" om in te teken, gaan dan na "dashboard" en laastens klik jy op die "course".

9. Ek maak ook foute – daar is definitief foute wat deurglip. As jy dink iets is nie reg nie – kontak my! Jy het my nommer. Of klik op "questions and answers" op die kursus en laat weet my so.

10. Die belangrikste van alles kragtens jou subskripsie. Jy subskripsie hardloop van die dag wat jy gekoop het, tot die dag wat jy hom self kanselleer. Aan die einde van die jaar verwyder ek jou van die graad waarop jy tans is en plaas ek jou op die volgende graad. As jy kies om nie die subskripsie te stop deur die loop van jou skoolloopbaan nie moet

jy steeds onthou om hom te stop aan die einde van Gr12 anders gaan jy verewig aanhou betaal!

11. As enigiets nie werk soos dis moet nie (bv. ’n video wil nie speel nie) laat weet my op WhatsApp of direk op die kursus. Moet asb. nie ’n Facebook comment gaan los iewers nie – dis onmoontlik om by hulle almal uit te kom.

Instruksies en Inligting wat voor op ’n Vraestel verskyn.

Hierdie is min of meer hoe die instruksies vooraan elke vraestel lyk.

LEES DIE BLAD OP JOU AMPTELIKE VRAESTEL AANDAGTIG DEUR! Dit gaan waarskynlik effens verskil van die een.

Tyd: (dis hoe lank jy het om die vraestel te voltooi)

Punte: (uit hoeveel die vraestel tel)

1. Skryf jou naam en klas (bv. 11A) op die antwoordboek wat voorsien is.
2. Hierdie vraestel bestaan uit “x” vrae. Beantwoord ALLE vrae in die antwoordboek behalwe Vraag “y” wat op die grafiekpapier wat verskaf is beantwoord moet word. Vul jou naam in die aangeduide spasie bo-aan die grafiekpapier in.
3. Begin elke vraag op ‘n nuwe bladsy
4. Nommer die antwoorde PRESIES soos in die vraestel
5. Los ’n lyn oop tussen opeenvolgende vrae.
6. ‘n Nie-programmeerbare sakrekenaar mag gebruik word.
7. Jy mag toepaslike Wiskunde instrumente gebruik
8. Gebruik jou formuleblad!
9. Toon alle formules, vervangings en stappe
10. Rond alle antwoorde af tot “z” desimale plekke. (Gewoonlik 2 maar kan verskil).
11. Gee kort motiverings waar nodig
12. Skryf netjies en leesbaar

Formuleblad

Let asb op - hierdie is 'n Graad 12 Formule-blad. Gebruik slegs die formules wat jy nodig het vir die Graad waarin jy tans is.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m \quad \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

oppervlakte $\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

PolyMathic

Vraestel 1

Okt/Nov

Eksamen

PolyMathic

Graad 12 Eindeksamen
Totaal: 150
Tyd: 3ure

VRAAG 1

Die tabel hieronder gee die gemiddelde rand/dollar-wisselkoers en die gemiddelde maandelikse olieprys vir die jaar 2010.

	Jan.	**Feb.**	**Mrt.**	**Apr.**	**Mei**	**Jun.**	**Jul.**	**Aug.**	**Sep.**	**Okt.**	**Nov.**	**Des.**
Wisselkoers in R/$	7,5	7,7	7,2	7,4	7,7	7,7	7,6	7,3	7,1	7,0	6,9	6,8
Olieprys in $	69,9	68,0	72,9	70,3	66,3	67,1	67,9	68,3	71,3	73,6	76,0	81,0

1.1 Teken 'n spreidiagram op DIAGRAMVEL 1 om die wisselkoers (in R/$) teenoor die olieprys (in $) voor te stel. (3)

1.2 Beskryf die verband tussen die wisselkoers (in R/$) en die olieprys (in $). (2)

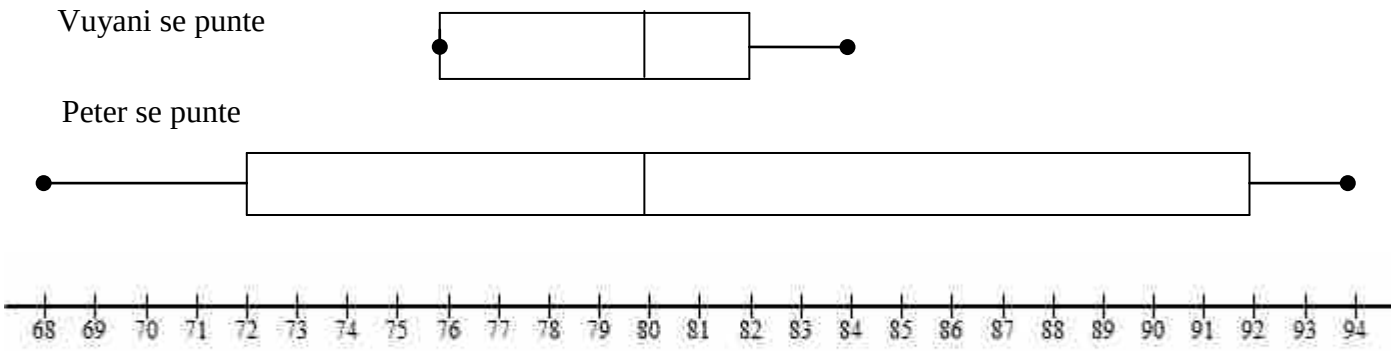
1.3 Bepaal die gemiddelde olieprys. (2)

1.4 Bepaal die standaardafwyking van die olieprys. (2)

1.5 Daar is gewoonlik kommer by die publiek as die olieprys hoër as twee standaardafwykings van die gemiddelde is. In watter maand(e) sou daar kommer by die publiek wees? (2)
[11]

VRAAG 2

Die mond-en-snor-diagramme hieronder stel Vuyani en Peter se punt vir hul Skoolgebaseerde Assesseringstake in 'n spesifieke vak deur die jaar voor.



2.1 Gee die variasiewydte van Peter se punte. (2)

2.2 Gee die minimum van Vuyani se punte. (1)

2.3 Lewer kommentaar oor wie jy dink 'n meer konsekwente prestasie deur die jaar gehad het. Motiveer jou antwoord deur na die waardes in die mond-en-snor-diagramme te verwys. (2)
[5]

VRAAG 3

Die gemiddelde persentasie van 150 leerders vir al hul vakke is in die kumulatiewefrekwensie-tabel hieronder opgesom.

PERSENTASIE INTERVAL	KUMULATIEWE FREKWENSIE
$x \leq 10$	5
$x \leq 20$	21
$x \leq 30$	50
$x \leq 40$	70
$x \leq 50$	88
$x \leq 60$	110
$x \leq 70$	135
$x \leq 80$	142
$x \leq 90$	147
$x \leq 100$	150

3.1 Teken die ogief (kumulatiewefrekwensie-grafiek) om die data hierbo voor te stel, op DIAGRAMVEL 2. (4)

3.2 Gebruik die ogief om die volgende te skat (benader):

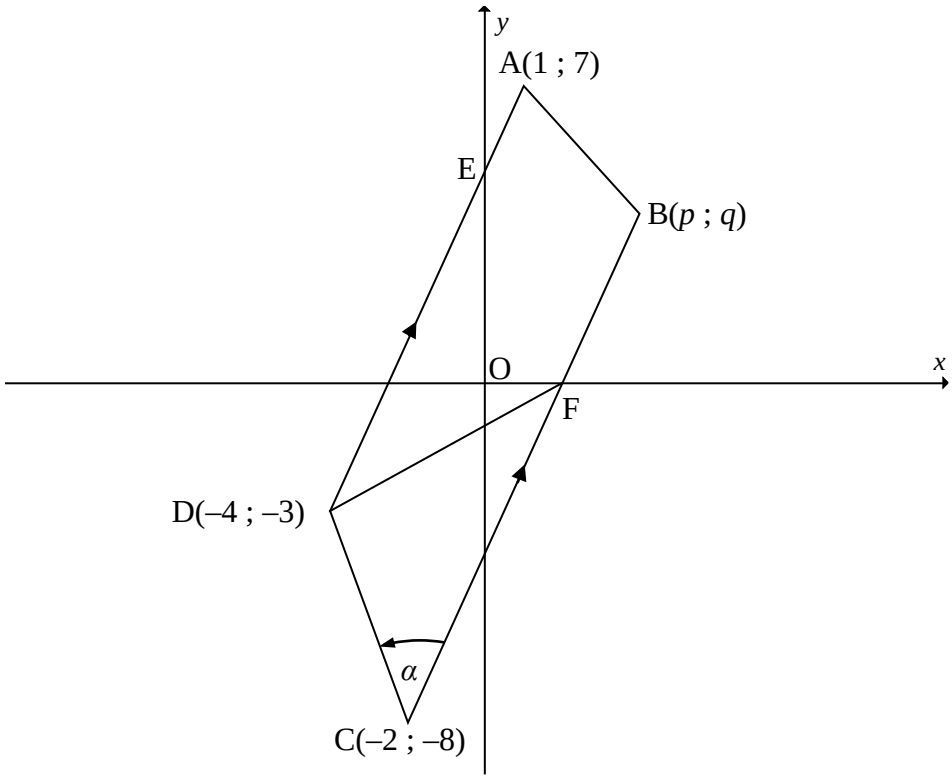
3.2.1 Die getal leerders wat minder as 85% behaal het (2)

3.2.2 Die interkwartielvariasiewydte (Toon ALLE berekeninge.) (3)

[9]

VRAAG 4

In die diagram hieronder is trapesium ABCD met AD // BC geteken. Die koördinate van die hoekpunte is A(1 ; 7); B(p ; q); C(–2 ; –8) en D(–4 ; –3). BC sny die x-as by F.
$D\hat{C}B = \alpha$.



4.1 Bereken die gradiënt van AD. (2)

4.2 Bepaal die vergelyking van BC in die vorm $y = mx + c$. (3)

4.3 Bepaal die koördinate van punt F. (2)

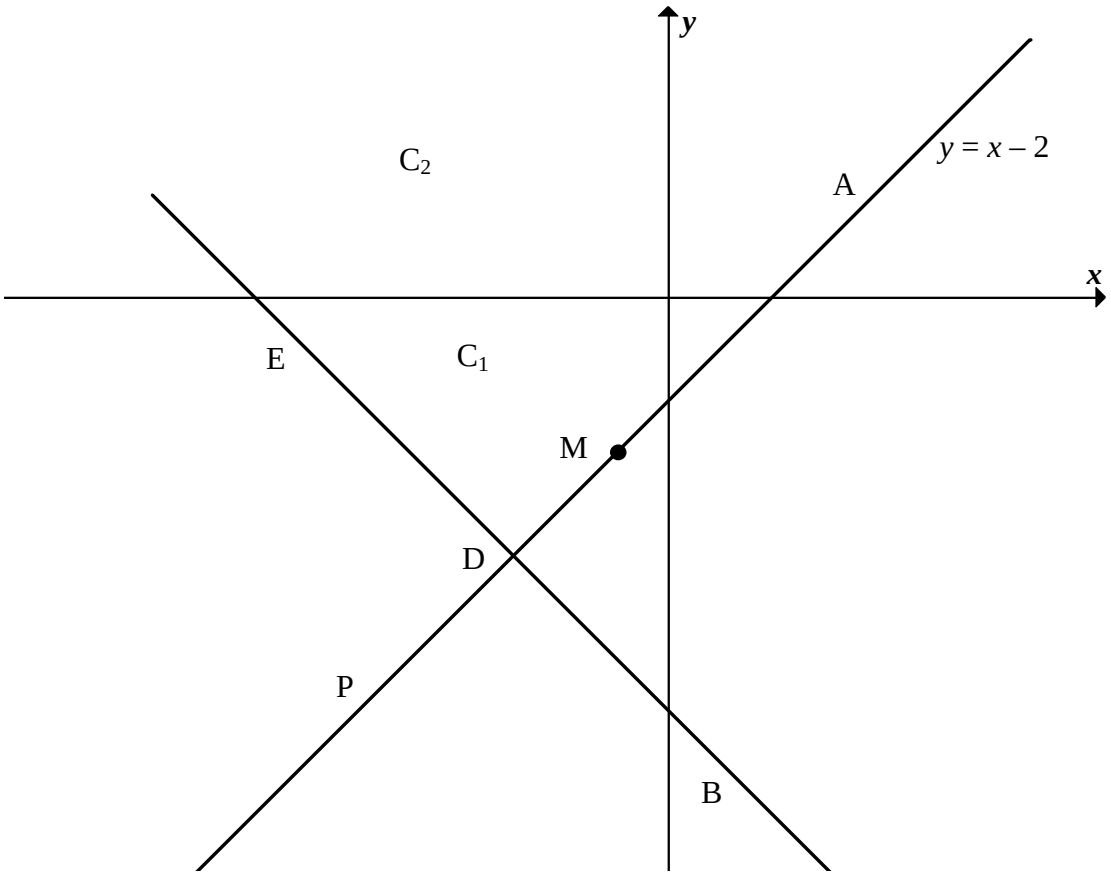
4.4 AB$^{/}$CD is 'n parallelogram met B$^{/}$ op BC. Bepaal die koördinate van B$^{/}$ deur 'n transformasie $(x\,;\,y) \rightarrow (x + a\,;\,y + b)$ wat A na B$^{/}$ stuur, te gebruik. (2)

4.5 Toon aan dat $\alpha = 48{,}37°$. (4)

4.6 Bereken die oppervlakte van ΔDCF. (6)
[19]

VRAAG 5

Sirkel C_1 en C_2 in die figuur hieronder het dieselfde middelpunt M. P is 'n punt op C_2. PM sny C_1 by D. Die raaklyn DB aan C_1 sny C_2 by B. Die vergelyking van sirkel C_1 word gegee deur $x^2 + 2x + y^2 + 6y + 2 = 0$ en die vergelyking van lyn PM is $y = x - 2$.



5.1 Bepaal die volgende:

5.1.1 Die koördinate van middelpunt M (3)

5.1.2 Die radius van sirkel C_1 (1)

5.2 Bepaal die koördinate van D, die punt waar lyn PM en sirkel C_1 sny. (5)

5.3 Indien gegee word dat DB $= 4\sqrt{2}$, bepaal MB, die radius van sirkel C_2. (3)

5.4 Skryf die vergelyking van sirkel C_2 neer in die vorm $(x-a)^2 + (y-b)^2 = r^2$. (2)

5.5 Is die punt F$\left(2\sqrt{5}\ ;\ 0\right)$ binne sirkel C_2? Ondersteun jou antwoord met berekeninge. (4)

[18]

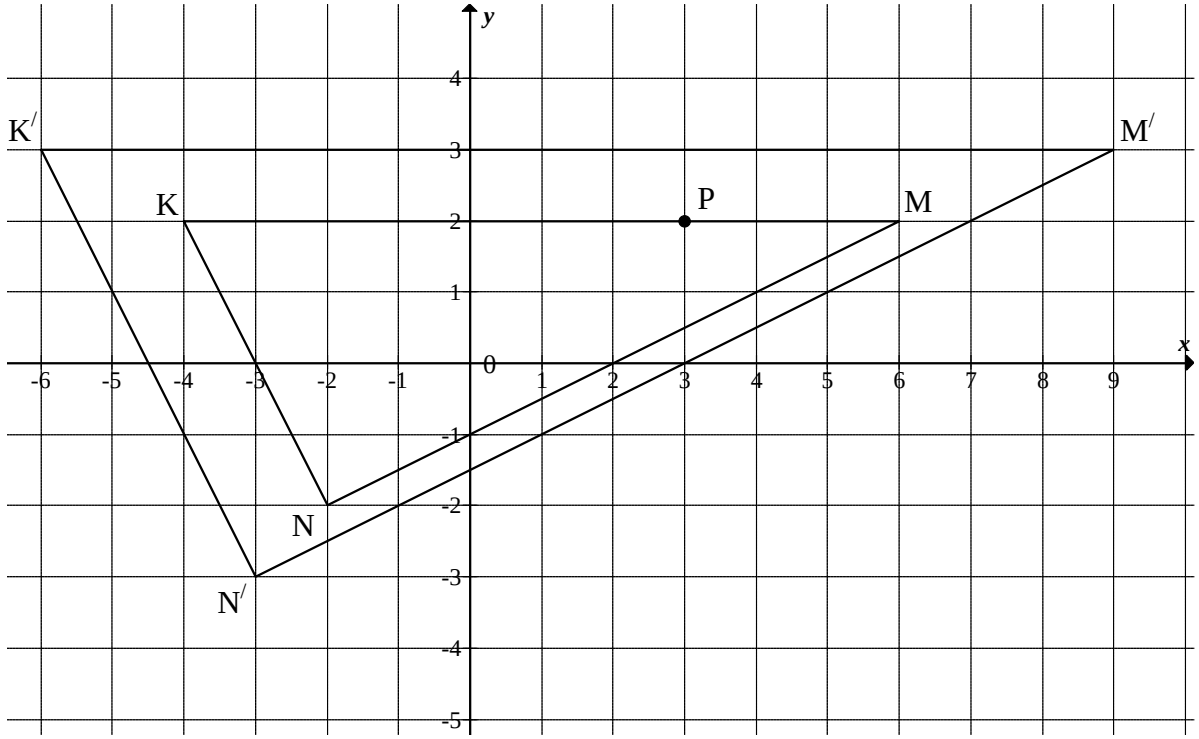
VRAAG 6

6.1 Skryf die koördinate van die beeld van punt A(–5 ; 3) neer nadat dit die volgende transformasies ondergaan het:

6.1.1 Translasie van 3 eenhede na onder en 4 eenhede na regs (2)

6.1.2 Refleksie om die *x*-as (2)

6.2 Beskou die volgende diagram:



6.2.1 In die diagram hierbo is driehoek KMN deur 'n sekere faktor vergroot om driehoek $K^{/}M^{/}N^{/}$ te vorm. Bepaal die faktor van vergroting. (2)

6.2.2 Gee die algemene reël vir die transformasie in VRAAG 6.2.1. (2)

6.2.3 Gebruik die antwoord op VRAAG 6.2.2 om die beeld $P^{/}$ van P(3 ; 2) te bepaal. (2)

6.2.4 M is die refleksie van K om die lyn met vergelyking $x = a$. Bepaal die waarde van die konstante a. (2)

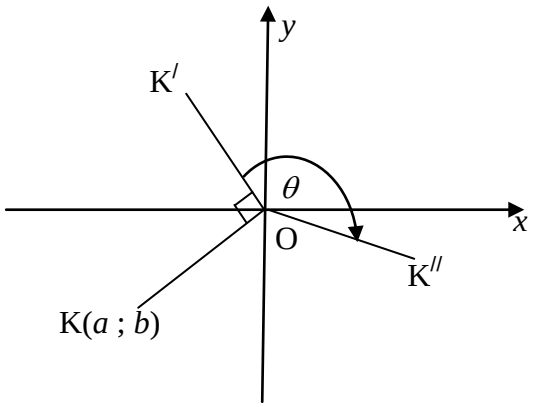
6.2.5 ΔKMN is 180° om die oorsprong geroteer om $\Delta K^{//}M^{//}N^{//}$ te vorm. Gee die koördinate van $K^{//}$. (2)

6.2.6 ΔKMN is 3 eenhede na regs en 1 eenheid na bo getransleer om $\Delta K^{///}M^{///}N^{///}$ te verkry. Skryf die verhouding van $\frac{K^{/}K^{///}}{K^{/}M^{///}}$ na die translasie neer. (3)

[17]

VRAAG 7

In die diagram hieronder is punt K(a ; b) kloksgewys deur 'n hoek van $90°$ om die oorsprong na $K^{/}$ geroteer en toe kloksgewys deur 'n hoek θ na $K^{//}$ geroteer.



7.1 Skryf die koördinate van punt $K^{/}$ in terme van a en b neer. (2)

7.2 Skryf die koördinate van $K^{//}$ in terme van a, b, $\sin\theta$ en $\cos\theta$ neer. Vereenvoudig indien nodig. (2)

7.3 T(–4 ; –2) word kloksgewys deur 'n hoek van (90° + θ) om die oorsprong geroteer om beeld $T^{/}$ te verkry. Bepaal, in die eenvoudigste vorm, die koördinate van $T^{/}$ in terme van θ. (2)

7.4 Bereken gevolglik of andersins die grootte van θ indien dit gegee word dat $T^{/}(2\sqrt{3}+1\ ;\ \sqrt{3}-2)$ en $90° < \theta < 180°$. (5)

[11]

VRAAG 8

8.1 Vereenvoudig so ver as moontlik: $1-\sin^2\theta+3-\cos^2\theta$ (2)

8.2 Vereenvoudig SONDER die gebruik van 'n sakrekenaar: $\sqrt{4^{\sin 150°}\times 2^{3\tan 225°}}$ (4)

8.3 Bewys dat $\dfrac{\cos^2 x\sin^2 x+\cos^4 x}{1-\sin x}=1+\sin x$ (4)

8.4 Bewys dat vir enige hoek θ, $\cos 3\theta=4\cos^3\theta-3\cos\theta$.
(Wenk: $3\theta=\theta+2\theta$) (4)

8.5 Indien $x=\cos 20°$, gebruik VRAAG 8.4 om aan te dui dat $8x^3-6x-1=0$. (2)
[16]

VRAAG 9

9.1 Vereenvoudig tot EEN trigonometriese funksie SONDER die gebruik van 'n sakrekenaar:

$$\frac{\cos 160°\tan 200°}{2\sin(-10°)}$$ (6)

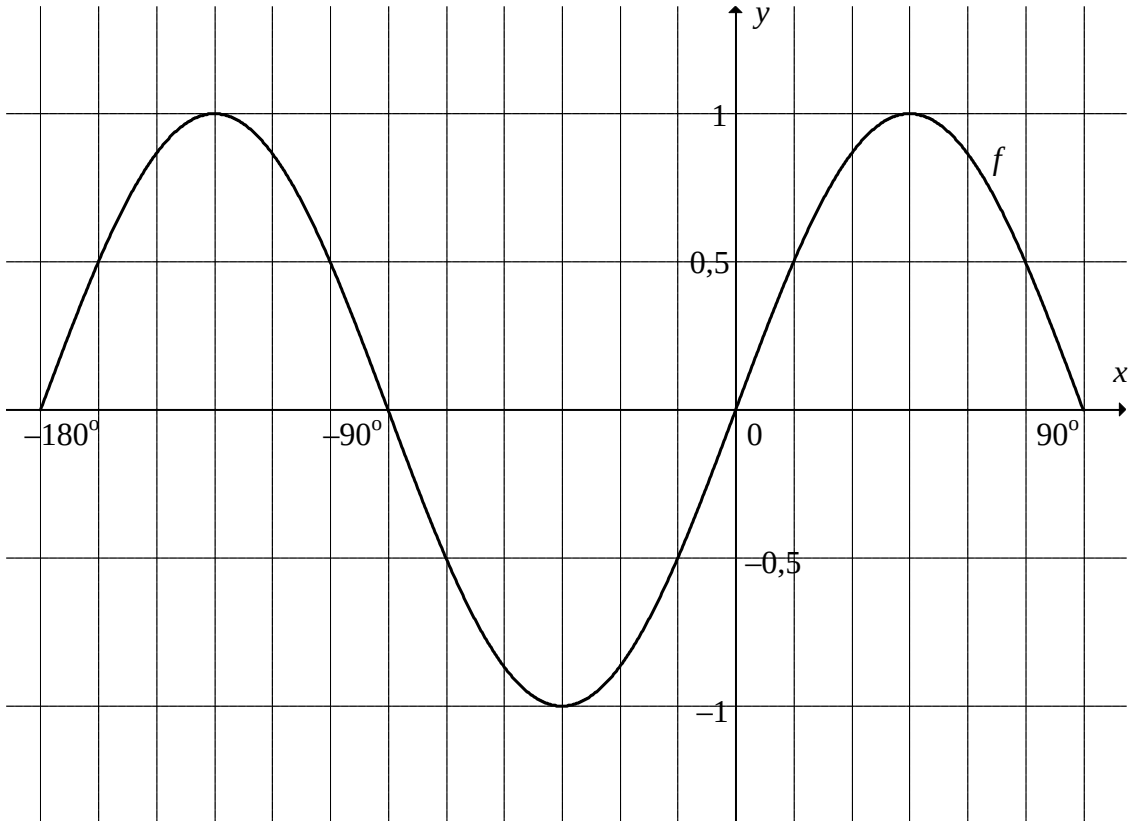
9.2 Beskou $\cos(x+45°)\cos(x-45°)$.

9.2.1 Dui aan dat $\cos(x+45°)\cos(x-45°)=\frac{1}{2}\cos 2x$. (4)

9.2.2 Bepaal gevolglik 'n waarde van x in die interval $0°\le x\le 180°$ waarvoor $\cos(x+45°)\cos(x-45°)$ 'n minimum is. (3)
[13]

VRAAG 10

Die grafiek van $f(x) = \sin 2x$ vir $-180° \leq x \leq 90°$ word in die skets hieronder aangedui.



10.1 Skryf die waardeversameling van f neer. (2)

10.2 Bepaal die periode van $f\left(\frac{3}{2}x\right)$. (2)

10.3 Teken die grafiek van $g(x) = \cos(x - 30°)$ vir $-180° \leq x \leq 90°$ op die assestelsel op DIAGRAMVEL 3. Merk ALLE x-afsnitte en draaipunte duidelik. (4)

10.4 Bepaal gevolglik of andersins die waardes van x in die interval $-180° \leq x \leq 90°$ waar $f(x) . g(x) < 0$. (4)

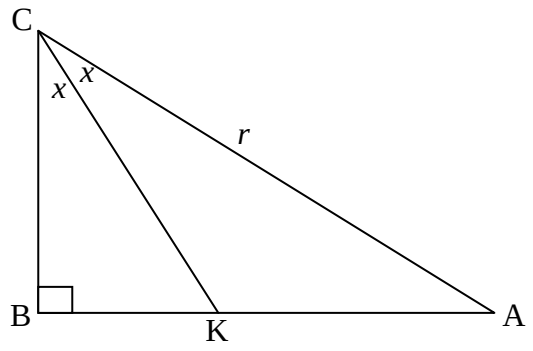
10.5 Beskryf die transformasie wat grafiek f moet ondergaan om $y = \sin(2x + 60°)$ te vorm. (2)

10.6 Bepaal die algemene oplossing van $\sin 2x = \cos(x - 30°)$. (6)

[20]

VRAAG 11

In die diagram hieronder is ABC 'n reghoekige driehoek. KC is die halveerder van $A\hat{C}B$. AC = r eenhede en $B\hat{C}K = x$.



11.1 Skryf AB in terme van x en r neer. (2)

11.2 Gee die grootte van $A\hat{K}C$ in terme van x. (1)

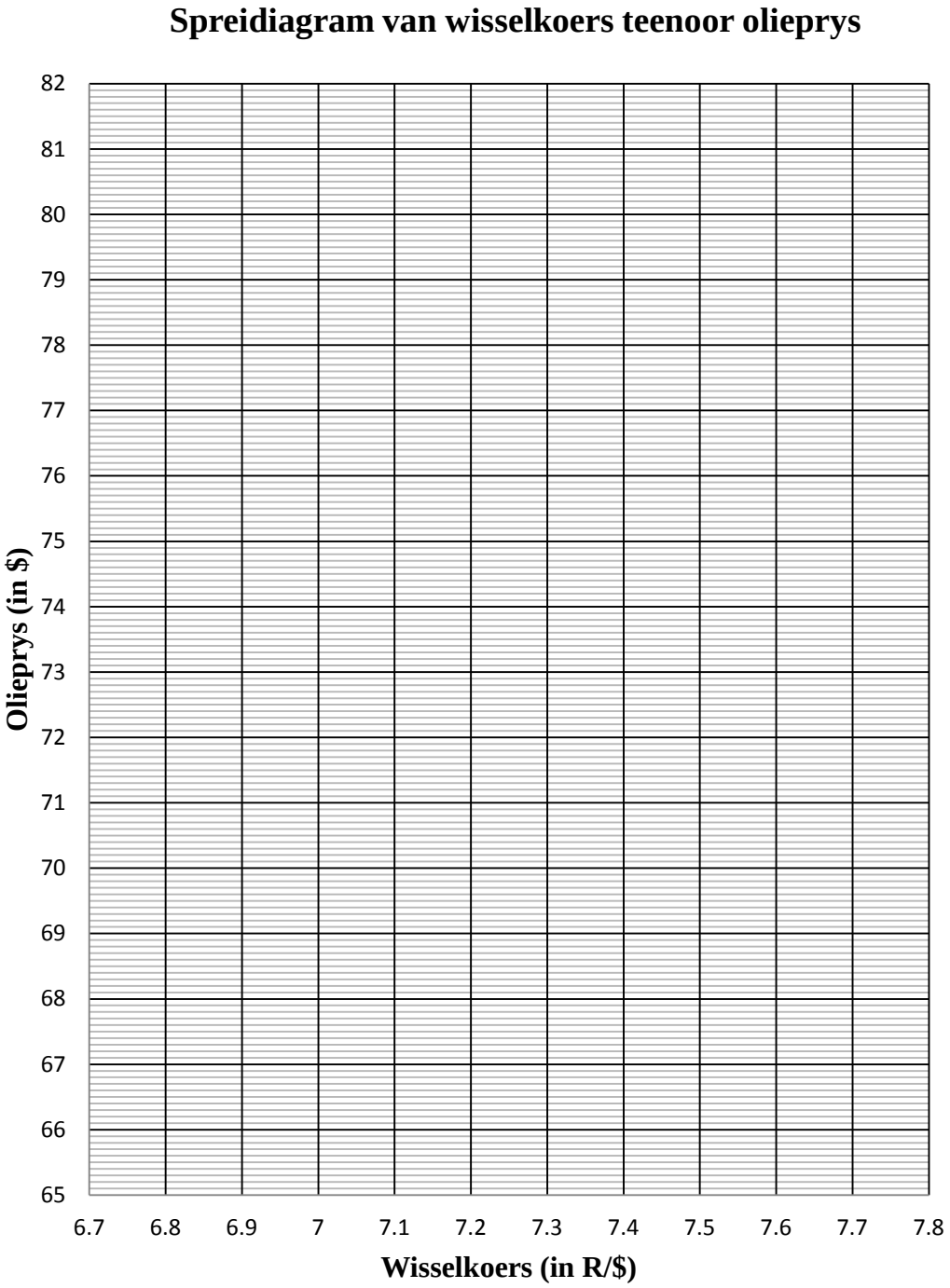
11.3 Indien dit gegee word dat $\frac{AK}{AB} = \frac{2}{3}$, bereken die waarde van x. (8)

[11]

TOTAAL: 150

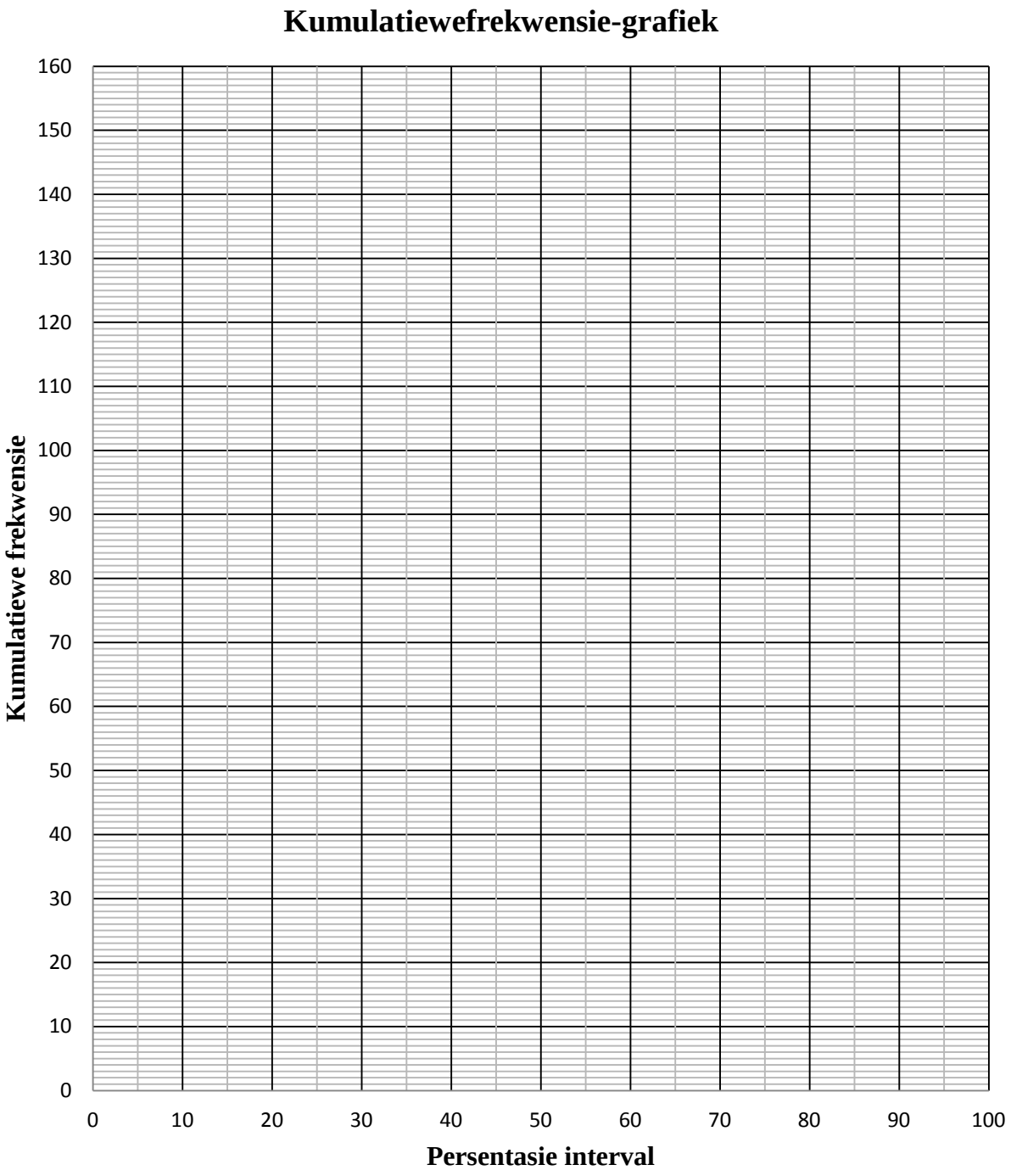
DIAGRAMVEL 1

VRAAG 1.1



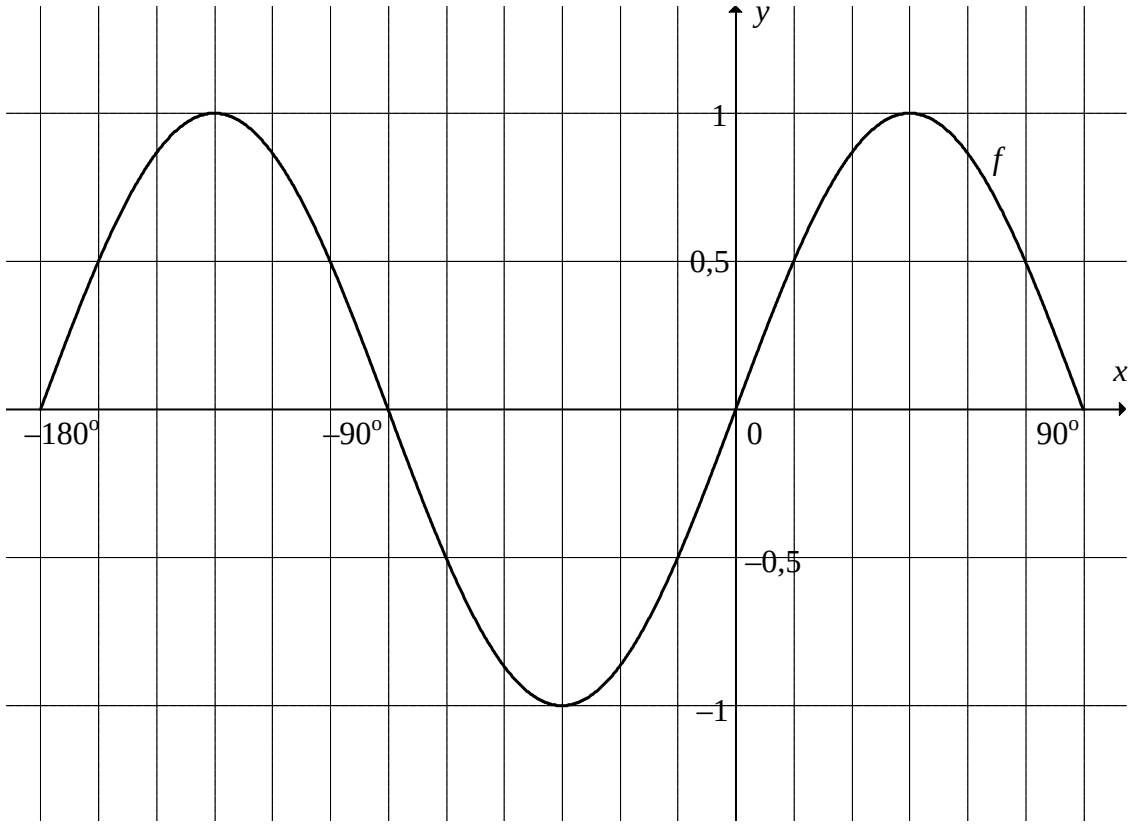
DIAGRAMVEL 2

VRAAG 3.1



DIAGRAMVEL 3

VRAAG 10.3

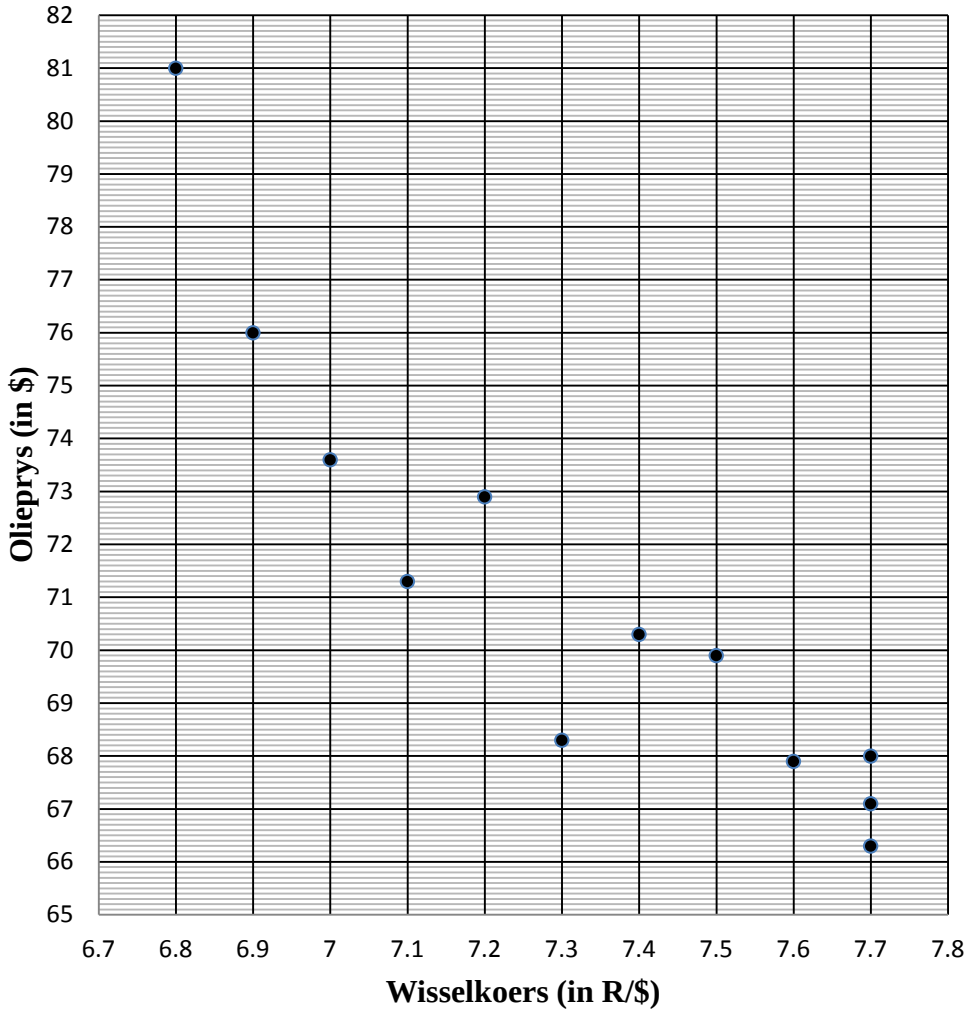


Memo

NOTA:

- Indien `n kandidaat `n vraag TWEE keer beantwoord het, merk slegs die EERSTE poging.
- Indien `n kandidaat `n poging om `n vraag te beantwoord gekanselleer het en die vraag nie weer gedoen het nie, merk die gekanselleerde poging.
- Konstante akkuraatheid is van toepassing in **ALLE** aspekte van die merk memorandum.

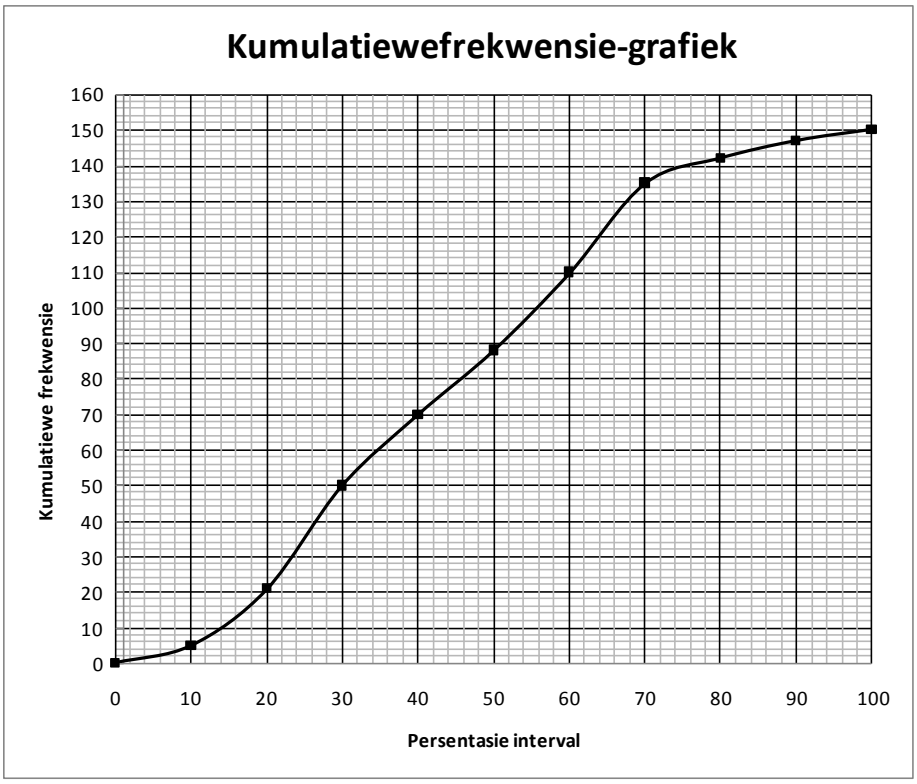
VRAAG 1

1.1	**Spreidiagram van wisselkoers versus olieprys**  	✓enige 4 punte korrek geplot ✓ enige 9 punte korrek geplot ✓ alle punte korrek geplot (3)
1.2	Soos die wisselkoers (R/$) vermeerder verminder die olieprys ($). **OF** Daar is `n negatiewe korrelasie tussen die wisselkoers en die olieprys.	✓✓ rede (2)
1.3	$\text{Gemiddelde} = \frac{852{,}6}{12}$ $= 71{,}05$	✓ 852,6 ✓ 71,05 (2)
1.4	Standaard afwyking is: $\sigma = 4{,}09$	✓✓ 4,09 (2)
1.5	2 standaard afwykings van die gemiddelde gemiddelde = 71,05+ 2(4,09) = 79,23 Die publiek sal besorg wees in Desember 2010	✓ 79,23 ✓ Des 2010 (2) **[11]**

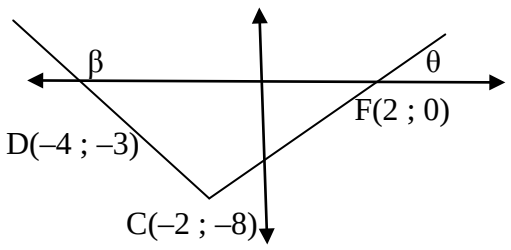
VRAAG 2

2.1	Variasiewydte van Peter se punte is 94 – 68 = 26	✓ 94 – 68 ✓ 26 (2)
2.2	Vuyani se minimum punte is 76	✓ 76 (1)
2.3	Vuyani was meer konstant gedurende die jaar want die variasiewydte van sy punte is nader om die mediaan waarde gerangskik **OF** die variasiewydte en die interkwartiel-wydte is kleiner as Peters.	✓ Vuyani ✓ rede (2) **[5]**

VRAAG 3

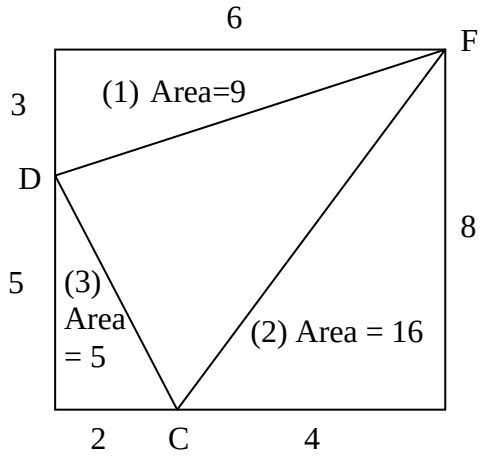
3.1		✓ plot punte by kummulatiewe frekwensie ✓ plot teen boonste limiet ✓ gegrond by (0 ; 0) ✓ gladde kurwe (4)
3.2.1	(85 ; ± 144) ± 144 leerders het punte onder 85% (Aanvaar: 144 to 146)	✓ (85 ; ± 144) ✓ ± 144 leerders (2)
3.2.2	Q_1 = 25 of 27 or 26 Q_3 = 61 of 62 or 64 Interkwartiel-wydte = 36 of 35 or 38	✓ onderste kwartiel ✓ boonste kwartiel ✓ IKW (3) **[9]**

VRAAG 4

4.1	$m_{AD} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{7-(-3)}{1-(-4)}$ $= 2$	✓ substitusie ✓ 2 (2)
4.2	AD//BC $m_{AD} = m_{BC} = 2$ $y - y_1 = m(x - x_1)$ $y - (-8) = 2(x - (-2)$ $\therefore y = 2x - 4$	✓ $m_{AD} = 2$ ✓ stel in formule ✓ $y = 2x - 4$ (3)
4.3	By F: $y = 0$ $0 = 2x - 4$ $x = 2$ F(2 ; 0)	✓ $y = 0$ ✓ $x = 2$ (2)
4.4	D is getransleer C volgens die reël: $D(x;y) \rightarrow C(x+2\ ;y-5)$ A moet ook getransleer word volgens die reël na B$^{/}$. $\therefore$A(1 ; 7) $\rightarrow$ B$^{/}$ (3 ; 2) **OF** $x_{B^{/}} = -2 + (1+4) = 3$ $y_{B^{/}} = -8 + (7+3) = 5$	✓ $x = 3$ ✓ $y = 2$ (2) ✓ $x = 3$ ✓ $y = 2$ (2)
4.5	$m_{BC} = 2$ $\tan\theta = 2$ $\theta = 63{,}43°$ $m_{DC} = \frac{-8-(-3)}{-2-(-4)} = -\frac{5}{2}$ $\tan\beta = -\frac{5}{2}$ $\beta = 180° - 68{,}20° = 111{,}80°$ $\alpha = 111{,}80° - 63.43° = 48{,}37°$  **OF**	✓ 63,43° ✓ $\tan\beta = -\frac{5}{2}$ ✓ 111,8° ✓ 48,37° (4)

	$DC = \sqrt{(-4+2)^2 + (-3+8)^2}$ $= \sqrt{29}$ $CF = \sqrt{(-2-2)^2 + (-8-0)^2}$ $= \sqrt{80}$ $DF = \sqrt{(2+4)^2 + (0+3)^2}$ $= \sqrt{45}$ $\cos\alpha = \dfrac{29+80-45}{2(\sqrt{29})(\sqrt{80})}$ $= 0{,}6643...$ $\alpha = 48{,}37°$ **OF** $DC = \sqrt{(-4+2)^2 + (-3+8)^2}$ $= \sqrt{29}$ $DB = \sqrt{(3+4)^2 + (2+3)^2}$ $= \sqrt{74}$ $BC = \sqrt{(3+2)^2 + (2+8)^2}$ $= \sqrt{125}$ $\cos\alpha = \dfrac{29+125-74}{2(\sqrt{29})(\sqrt{125})}$ $= 0{,}6643...$ $\alpha = 48{,}37°$	✓Subst in cos-formule ✓cos α onderwerp ✓0,6643... ✓48,37° (4) ✓Subst in cos-formule ✓cos α onderwerp ✓0,6643... ✓48,37° (4)
4.6	$DC = \sqrt{(-4+2)^2 + (-3+8)^2}$ $= \sqrt{29}$ $CF = \sqrt{(-2-2)^2 + (-8-0)^2}$ $= \sqrt{80}$ $Area\ \Delta DCF = \frac{1}{2}.DC.CF.\sin\alpha$ $= \frac{1}{2}(\sqrt{29})(\sqrt{80})\sin 48{,}37°$ $=18$ eenhede^2	✓ substitusie in formule ✓ $\sqrt{29}$ ✓ substitusie in formule ✓ $\sqrt{80}$ ✓ substitusie In die area reël ✓ 18 (6)

OF



Area ΔDCF = Area van reghoek – (1) – (2) – (3)

= 48 – 9 – 5 – 16

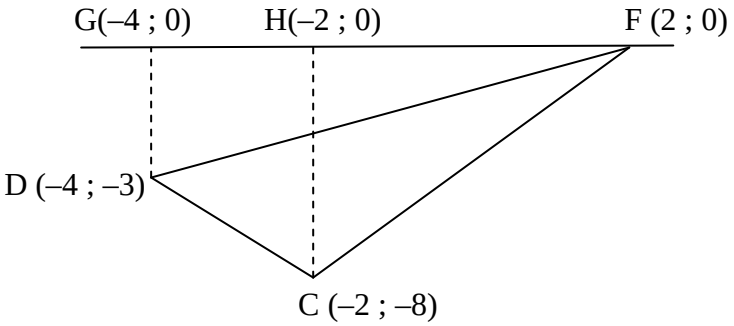
= 18 vierkante eenheid

✓ reghoek en area

✓ verband tussen areas
✓ (1) = 9
✓(2) = 16
✓(3) = 5
✓18 eenhede^2

(6)

OF



Area CDF = Area CHF + Area CDGH - Area DGF

$$= \frac{1}{2} \times 4 \times 8 + 2 \times \frac{1}{2}(3 \times 8) - \frac{1}{2} \times 6 \times 3$$

$$= 16 + 11 - 9$$

$$= 18 \text{ vierkante eenheid}$$

✓ teken loodregte lyne

✓ verband tussen areas
✓ 16
✓ 11
✓ 9
✓18vierkante eenheid

(6)

[19]

VRAAG 5

5.1.1	$x^2+y^2+2x+6y+2=0$ $x^2+2x+1+y^2+6y+9=-2+10$ $(x+1)^2+(y+3)^2=8$ $\mathrm{M}(-1\,;-3)$	✓ $(x+1)^2+(y+3)^2=8$ ✓ -1 ✓ -3 (3)
5.1.2	radius van sirkel $\mathrm{C}_1=\sqrt{8}$	✓ $\sqrt{8}$ (1)
5.2	$x^2+(x-2)^2+2x+6(x-2)+2=0$ $x^2+x^2-4x+4+2x+6x-12+2=0$ $2x^2+4x-6=0$ $x^2+2x-3=0$ $(x+3)(x-1)=0$ $x=-3$ *or* $x\neq 1$ $y=-3-2=-5$ $\therefore \mathrm{D}(-3;-5)$ **OF** $(x+1)^2+(y+3)^2=8$ *subst.* $y=x-2$ $(x+1)^2+(x-2+3)^2=8$ $(x+1)^2+(x+1)^2=8$ $x^2+2x-3=0$ $(x+3)(x-1)=0$ $x=-3$ *or* $x\neq 1$ $y=-3-2=-5$ **OF** $(x+1)^2+(y+3)^2=8$ *subst.* $y=x-2$ $(x+1)^2+(x-2+3)^2=8$ $(x+1)^2+(x+1)^2=8$ $(x+1)^2=4$ $x+1=\pm 2$ $x=-3$ *or* $x\neq 1$ $y=-3-2=-5$ **OF**	✓ substitusie ✓ standaard vorm ✓ faktore ✓ waarde van x ✓ waarde van y (5) ✓ substitusie ✓ standaard vorm ✓ faktore ✓ waarde van x ✓ waarde van y (5) ✓ substitusie ✓ vereenvoudiging ✓ vierkantswortel van albei kante ✓ waarde van x ✓ waarde van y (5)

	PM maak `n 45° hoek met die *x*-as. $\sqrt{8} = \sqrt{2^2 + 2^2}$ Dus: $x_D = x_M - 2 = -1 - 2 = -3$ $y_D = -3 - 2 = -5$	✓✓ $\sqrt{8} = \sqrt{2^2 + 2^2}$ ✓ waarde van *x* ✓ waarde van *y* (5)
5.3	$MD \perp DB$ (raaklyn ⊥ radius) $MB^2 = MD^2 + DB^2$ (Pythagoras) $= (\sqrt{8})^2 + (4\sqrt{2})^2$ $= 40$ MB is die radius van C_2 $MB = \sqrt{40}$	✓ raaklyn ⊥ radius ✓ substitusie in Pythagoras ✓ $\sqrt{40}$ (3)
5.4	$(x+1)^2 + (y+3)^2 = 40$	✓ LK ✓ RK (2)
5.5	Afstand van $(2\sqrt{5}\ ;\ 0)$ na middelpunt $= \sqrt{(2\sqrt{5}+1)^2 + (0+3)^2}$ $= 6{,}24$ $6{,}24 < 6{,}32\ (\sqrt{40})$ Afstand van $(2\sqrt{5}\ ;\ 0)$ na middelpunt < radius van sirkel. $(2\sqrt{5}\ ;\ 0)$ lê binne die sirkel.	✓substitusie in afstand formule ✓ 6,24 ✓6,24 < 6,32 ✓ afleiding (4) **[18]**

VRAAG 6

6.1.1	$A(-5\,;3)$ $A^{/}(-5+4\,;3-3)=(-1\,;0)$	✓ -1 ✓ 0 (2)
6.1.2	$A^{/}(-5\,;-3)$	✓ -5 ✓ -3 (2)
6.2.1	Skaal faktor van vergroting is $\frac{K^{/}M^{/}}{KM}=\frac{15}{10}=\frac{3}{2}$ **OF** $K(-4;2)\rightarrow K^{/}(-6;3)=K^{/}\left(\frac{3}{2}\times -4;\ \frac{3}{2}\times 2\right)$ Skaalfaktor is $\frac{3}{2}$	✓ $\frac{K^{/}M^{/}}{KM}$ ✓ $\frac{3}{2}$ ✓ $\left(\frac{3}{2}\times -4;\frac{3}{2}\times 2\right)$ ✓ $\frac{3}{2}$ (2)
6.2.2	$(x;y)\rightarrow\left(\frac{3}{2}x\ ;\ \frac{3}{2}y\right)$	✓ $\frac{3}{2}x$ ✓ $\frac{3}{2}y$ (2)
6.2.3	$P^{/}\left(\frac{3}{2}\times 3\ ;\ 2\times\frac{3}{2}\right)$ $=P^{/}\left(\frac{9}{2}\ ;\ 3\right)$	✓ $\frac{9}{2}$ ✓ 3 (2)
6.2.4	$a=1$	✓✓ $a=1$ (2)
6.2.5	$K^{//}(4;-2)$	✓ 4 ✓ -2 (2)

6.2.6	$K^{///}K^{/}=5$
	$K^{/}M^{///}=15$
	
	$\frac{K^{/}K^{///}}{K^{/}M^{///}}=\frac{5}{15}=\frac{1}{3}$
		✓ $K^{///}K^{/}=5$
		✓ $K^{/}M^{///}=15$
		
		✓ $\frac{1}{3}$
		(3)
		[17]

VRAAG 7

7.1	$K^{/}(b\ ;\ -a)$	✓ b ✓ $-a$ (2)

7.2	$K^{//}(b\cos\theta - a\sin\theta\ ;\ -a\cos\theta - b\sin\theta)$
	
	OF
	$K^{//}(a\cos(90+\theta) + b\sin(90°+\theta)\ ;\ b\cos(90°+\theta) - a\sin(90°+\theta)$
	$= K^{//}(-a\sin\theta + b\cos\theta\ ;\ -b\sin\theta - a\cos\theta)$
		✓ $b\cos\theta - a\sin\theta$
		✓ $-a\cos\theta - b\sin\theta$
		(2)

7.3	$T^{//}(-(-4)\sin\theta + (-2)\cos\theta\ ;\ -(-2)\sin\theta - (-4)\cos\theta)$
	$= T^{//}(4\sin\theta - 2\cos\theta\ ;\ 2\sin\theta + 4\cos\theta)$
	OF
	
	$T^{//}(-2\cos\theta - (-4)\sin\theta\ ;\ -(-4)\cos\theta - (-2)\sin\theta)$
	$= T^{//}(-2\cos\theta + 4\sin\theta\ ;\ 4\cos\theta + 2\sin\theta)$
		✓ $4\sin\theta - 2\cos\theta$
		✓ $2\sin\theta + 4\cos\theta$
		(2)
		
		✓ $4\sin\theta - 2\cos\theta$
		✓ $2\sin\theta + 4\cos\theta$
		(2)

7.4	$2\sqrt{3} + 1 = 4\sin\theta - 2\cos\theta$ .....(1) $\sqrt{3} - 2 = 2\sin\theta + 4\cos\theta$ .....(2) $(2)\times 2:\ \ 2\sqrt{3} - 4 = 4\sin\theta + 8\cos\theta$ ....(3) $(1)-(3):\ \ 5 = -10\cos\theta$ $-\frac{1}{2} = \cos\theta$ $\therefore\ \theta = 180° - 60° = 120°$ **OF** $2\sqrt{3} + 1 = 4\sin\theta - 2\cos\theta$ .....(1) $\sqrt{3} - 2 = 2\sin\theta + 4\cos\theta$ .....(2) $(1)\times 2:\ \ 4\sqrt{3} + 2 = 8\sin\theta - 4\cos\theta$ ....(3) $(2)+(3):\ \ 5\sqrt{3} = 10\sin\theta$ $\frac{\sqrt{3}}{2} = \sin\theta$ $\therefore\ \theta = 180° - 60° = 120°$ **OF**	✓ substitusie om vergelyking te vorm ✓ substitusie om vergelyking te vorm ✓ $5 = -10\cos\theta$ ✓ $-\frac{1}{2} = \cos\theta$ ✓ $120°$ (5) ✓ substitusie om vergelyking te vorm ✓ substitusie om vergelyking te vorm ✓ $5\sqrt{3} = 10\sin\theta$ ✓ $\frac{\sqrt{3}}{2} = \sin\theta$ ✓ $120°$ (5)

	$m_{OT} = \frac{1}{2} \Rightarrow \tan X\hat{O}T = \frac{1}{2}$ $X\hat{O}T = 206{,}565...°$ $m_{OT'} = \frac{\sqrt{3}-2}{2\sqrt{3}+1} \Rightarrow \tan X\hat{O}T^{//} = \frac{\sqrt{3}-2}{2\sqrt{3}+1} = -0{,}06....$ $X\hat{O}T = -3{,}434°$ $90° + \theta = 209{,}99...° \approx 210°$ $\theta = 120°$ **OF** $(TT')^2 = OT^2 + (OT')^2 - 2(OT)(OT').\cos(90° + \theta)$ $40 + 20\sqrt{3} = 40 - 40.\cos(90° + \theta)$ $\cos(90° + \theta) = -\frac{\sqrt{3}}{2}$ $90° + \theta = 150°$ $\theta = 60°$	✓ $\tan X\hat{O}T = \frac{1}{2}$ ✓ 206.565...° ✓ – 0,06... ✓ – 3.434° ✓ 120° (5) ✓$(TT')^2$ $= 40 + 20\sqrt{3}$ ✓substitusie in cos-reël ✓vereenvoudiging ✓150° ✓60° (5) **[11]**

VRAAG 8

8.1	$1-\sin^2\theta+3-\cos^2\theta$ $=4-(\sin^2\theta+\cos^2\theta)$ $=3$ **OF** $\cos^2\theta+3-\cos^2\theta$ $=3$	✓ vereenvoudiging ✓ 3 (2) ✓ substitusie met identiteit ✓ 3 (2)
8.2	$\sqrt{4^{\sin 150°}.2^{3\tan 225°}}$ $=\sqrt{4^{\sin 30°}.2^{3\tan 45°}}$ $=\sqrt{\left(2^2\right)^{\frac{1}{2}}.2^3}$ $=\sqrt{16}$ $=4$ **OF** $\sin 150°=\frac{1}{2}$ $\tan 225°=1$ $\sqrt{4^{\sin 150°}2^{3\tan 225°}}$ $=\sqrt{4^{\frac{1}{2}}2^3}$ $=\sqrt{2.2^3}$ $=\sqrt{16}$ $=4$	✓ herskryf met reduksie formules ✓ stel spesiale hoeke in ✓ vereenvoudiging ✓ 4 (4) ✓ $\sin 150°=\frac{1}{2}$ ✓ $\tan 225°=1$ ✓ $4^{\frac{1}{2}}=2$ ✓ 4 (4)
8.3	$LK=\frac{\cos^2 x(\sin^2 x+\cos^2 x)}{1-\sin x}$ $=\frac{\cos^2 x.(1)}{1-\sin x}$ $=\frac{(1-\sin^2 x)}{1-\sin x}$ $=\frac{(1+\sin x)(1-\sin x)}{1-\sin x}$ $=1+\sin x$ $=RK$	✓ faktorisering ✓ 1 ✓ $1-\sin^2 x$ ✓ faktore (4)

8.4	$\cos 3\theta$ $= \cos(2\theta + \theta)$ $= \cos 2\theta.\cos\theta - \sin 2\theta.\sin\theta$ $= (2\cos^2\theta - 1).\cos\theta - 2\sin\theta.\cos\theta.\sin\theta$ $= 2\cos^3\theta - \cos\theta - 2\sin^2\theta.\cos\theta$ $= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta).\cos\theta$ $= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$ $= 4\cos^3\theta - 3\cos\theta$	✓uitbreiding ✓ $2\cos^2\theta - 1$ ✓ $2\sin\theta.\cos\theta$ ✓$1-\cos^2\theta$ (4)
8.5	$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ $\cos 3(20°) = 4\cos^3(20°) - 3\cos(20°)$ $\frac{1}{2} = 4x^3 - 3x$ $8x^3 - 6x - 1 = 0$	✓$\theta = 20°$ ✓$\cos 60° = \frac{1}{2}$ (2) **[16]**

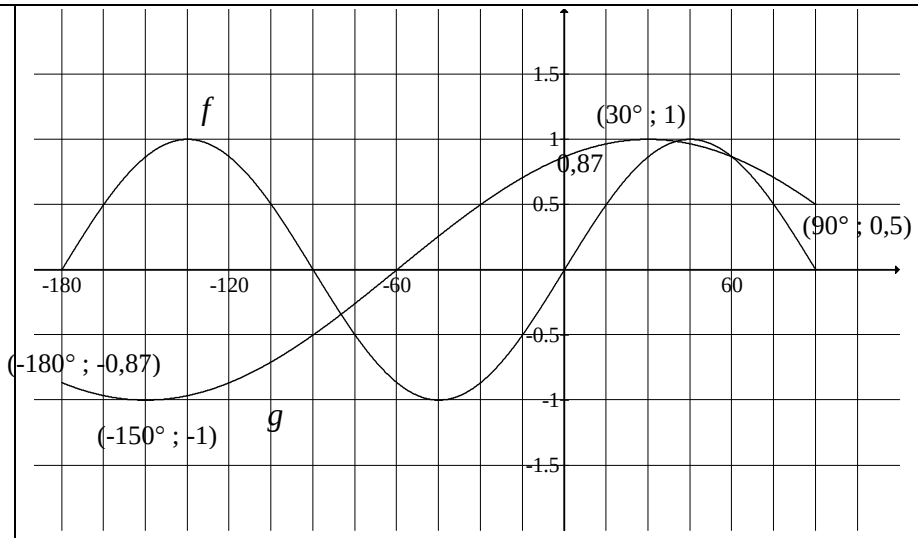
VRAAG 9

9.1	$\dfrac{\cos 160°.\tan 200°}{2\sin(-10°)}$ $= \dfrac{(-\cos 20°)(\tan 20°)}{2(-\sin 10°)}$ $= \dfrac{(-\cos 20°)\left(\dfrac{\sin 20°}{\cos 20°}\right)}{-2\sin 10°}$ $= \dfrac{2\sin 10°\cos 10°}{2\sin 10°}$ $= \cos 10°$	✓ $-\cos 20°$ ✓ $\tan 20°$ ✓ $-\sin 10°$ ✓ $\dfrac{\sin 20°}{\cos 20°}$ ✓ $2\sin 10°\cos 10°$ ✓ $\cos 10°$ (6)
9.2.1	$LK = \cos(x+45°).\cos(x-45°)$ $= (\cos x.\cos 45° - \sin x\sin 45°)(\cos x\cos 45° + \sin x\sin 45°)$ $= \cos^2 x.\cos^2 45° - \sin^2 x.\sin^2 45°$ $= \cos^2 x\left(\dfrac{\sqrt{2}}{2}\right)^2 - \sin^2\left(\dfrac{\sqrt{2}}{2}\right)^2$ or $\left[\cos^2 x\left(\dfrac{1}{\sqrt{2}}\right)^2 - \sin^2 x\left(\dfrac{1}{\sqrt{2}}\right)^2\right]$ $= \dfrac{1}{2}\cos^2 x - \dfrac{1}{2}\sin^2 x$ $= \dfrac{1}{2}(\cos^2 x - \sin^2 x)$ $= \dfrac{1}{2}\cos 2x$ **OF** $2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$ $\cos\alpha\cos\beta = \dfrac{1}{2}\left(\cos(\alpha+\beta) + \cos(\alpha-\beta)\right)$ Let $\alpha = x + 45°$ and $\beta = x - 45°$ $\therefore \cos(x+45°)\cos(x-45°)$ $= \dfrac{1}{2}\left(\cos((x+45°+x-45°) + \cos(x+45°-x+45°)\right)$ $= \dfrac{1}{2}\left(\cos 2x + \cos 90°\right)$ $= \dfrac{1}{2}\cos 2x$	✓ brei uit $\cos(x+45°)$ ✓ brei uit $\cos(x-45°)$ ✓ stel in spesiale hoeke ✓ vereenvoudiging (4) ✓✓ lei identiteit af ✓ substitusie ✓ vereenvoudiging (4)

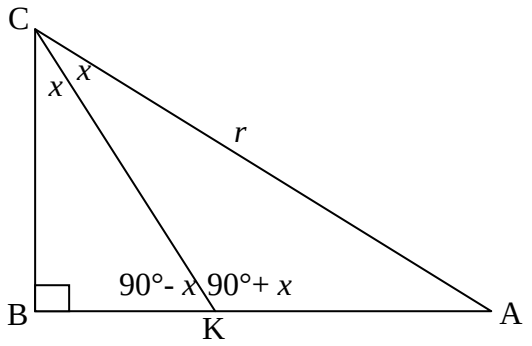
9.2.2	$\cos(x+45°)\cos(x-45°)$ het `n minimum waar $\frac{1}{2}\cos 2x$ `n minimum het. Die minimum waarde van $\cos 2x$ is -1 $\cos 2x = -1$ $2x = 180°$ $x = 90°$	✓ minimum waarde van -1 ✓$2x = 180°$ ✓ $x = 90°$ (3) **[13]**

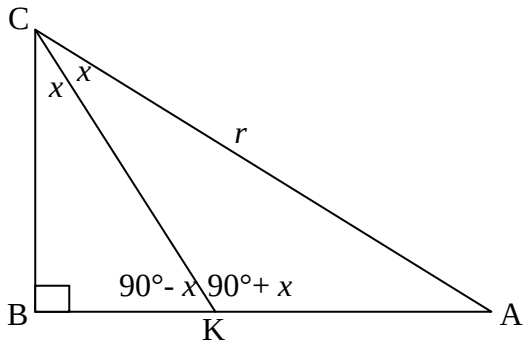
VRAAG 10

10.1	Waardeversameling $= [-1\ ;\ 1]$	✓✓ $[-1\ ;\ 1]$ (2)
10.2	$f\left(\frac{3}{2}x\right) = \sin 2\left(\frac{3}{2}x\right)$ $= \sin 3x$ $\therefore$ Periode $= \frac{360°}{3} = 120°$ **OF** $f\left(\frac{3}{2}x\right) = \sin 2\left(\frac{3}{2}x\right)$ $= \sin 3x$ $= \sin(3x + 360°)$ $= \sin 3(x + 120°)$ $\therefore$ Periode $= 120°$	✓ $\sin 3x$ ✓$120°$ (2) ✓ $\sin 3x$ ✓$120°$ (2)

10.3		✓ x afsnitte ✓✓ draaipunte ✓ vorm (4)
10.4	$(-180°;-90°)$ *of* $(-60°\ ;\ 0°)$ **OF** $-180° < x < -90°$ *of* $-60° < x < 0°$	✓ $> -180°$ ✓ $< -90°$ ✓ $> -60°$ ✓ $< 0°$ (4)
10.5	$y = \sin 2(x + 30°)$ $\therefore$ translasie van 30° na links	✓ translasie 30° ✓ na links (2)
10.6	$\sin 2x = \cos(x - 30°)$ $\sin 2x = \sin[90° - (x - 30°)]$ $= \sin(120° - x)$ $2x = 120° - x + 360°k; k \in Z$ $\quad$ $2x = 180° - (120° - x) + 360°k$ $3x = 120° + 360°k$ $\quad$ **of** $\quad 2x - x = 60° + 360°k$ $x = 40° + 120°k; k \in Z$ $\quad$ $x = 60° + 360°k; k \in Z$ **OF** $\sin 2x = \cos(x - 30°)$ $\cos(90° - 2x) = \cos(x - 30°)$ $90° - 2x = x - 30° + 360°k$ or $90° - 2x = 360° - (x - 30°) + 360°k$ $-3x = -120° + 360°k$ $\quad$ $-x = 300° + 360°k$ $x = 40° - 120°k; k \in Z$ $\quad$ $x = -300° - 360°k; k \in Z$ $\therefore x = 40° + 120°k$ of $x = 60° + 360°k$; $k \in Z$	✓ gebruik ko-funksie ✓ $2x = 120° - x + 360°k$ ✓ $x = 40° + 120°k$ ✓ $2x = 180° - (120° - x) + 360°k$ ✓ $x = 60° + 360°k$ ✓ $k \in Z$ (6) ✓ $\cos(90° - x) = \cos(x - 30°)$ ✓ $90° - 2x = x - 30° + 360°k$ ✓ $x = 40° - 120°k$ ✓ $90° - 2x = 360° - (x - 30°) + 360°k$ ✓ $x = -300° - 360°k$ ✓ $k \in Z$ (6) **[20]**

VRAAG 11

11.1	$\frac{AB}{r} = \sin 2x$ $AB = r\sin 2x$	✓ $\frac{AB}{r} = \sin 2x$ ✓ $AB = r\sin 2x$ (2)
11.2	$A\hat{K}C = 90° + x$	✓ $A\hat{K}C = 90° + x$ (1)
11.3	 *In* ΔAKC: $\frac{\sin A\hat{K}C}{AC} = \frac{\sin A\hat{C}K}{AK}$ $\frac{\sin(90° + x)}{r} = \frac{\sin x}{AK}$ $AK = \frac{r\sin x}{\sin(90° + x)} = \frac{r\sin x}{\cos x}$ $\frac{AK}{AB} = \frac{2}{3}$ $\frac{\left(\frac{r\sin x}{\cos x}\right)}{r\sin 2x} = \frac{2}{3}$ $\frac{\frac{\sin x}{\cos x}}{2\sin x\cos x} = \frac{2}{3}$ $\frac{\sin x}{\cos x} \times \frac{1}{2\sin x\cos x} = \frac{2}{3}$ $\frac{1}{2\cos^2 x} = \frac{2}{3}$ $4\cos^2 x = 3$ $\cos x = \frac{\sqrt{3}}{2}$ $x = 30°$	✓ sin reël ✓ substitusie ✓ maak AK onderwerp van die formule ✓ cos x ✓ 2sin$x$.cos$x$ ✓ $\frac{1}{2\cos^2 x}$ ✓ $\cos x = \frac{\sqrt{3}}{2}$ ✓ $x = 30°$ (8)
	OF	



Gebruik die sin-formule in ΔCBK en ΔCKA:

$\frac{\sin x}{BK} = \frac{\sin(90° - x)}{BC}$ en $\frac{\sin x}{KA} = \frac{\sin(90° + x)}{AC}$

$\therefore \frac{BK}{BC} = \frac{KA}{AC}$

$\therefore \frac{1}{BC} = \frac{2}{r}$

$\therefore BC = \frac{1}{2}r$

$\therefore \cos 2x = \frac{BC}{AC} = \frac{\frac{1}{2}r}{r} = \frac{1}{2}$

$\therefore 2x = 60°$

$\therefore x = 30°$

✓ $\frac{\sin x}{BK} = \frac{\sin(90° - x)}{BC}$

✓ $\frac{\sin x}{KA} = \frac{\sin(90° + x)}{AC}$

✓ $\frac{BK}{BC} = \frac{KA}{AC}$

✓ $\frac{1}{BC} = \frac{2}{r}$

✓ $BC = \frac{1}{2}r$

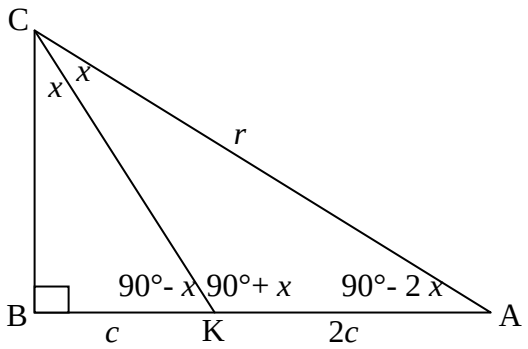
✓ $\cos 2x = \frac{1}{2}$

✓ $2x = 60°$

✓ $x = 30°$

(8)

OF



ΔCBK: KC = $\frac{c}{\sin x}$

ΔCKA: $\frac{\sin x}{2c} = \frac{\sin(90° - 2x)}{KC} = \frac{\sin(90° - 2x).\sin x}{c}$

$\therefore$ sin (90° - 2x) = $\frac{1}{2}$ = sin 30°

$\therefore$ 90° - 2x = 30°

x = 30°

✓ KC = $\frac{c}{\sin x}$

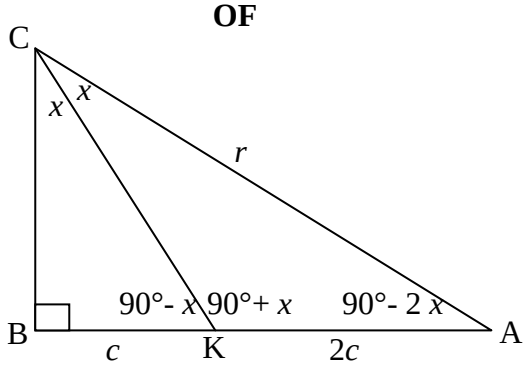
✓ $\frac{\sin x}{2c} = \frac{\sin(90° - 2x)}{KC}$

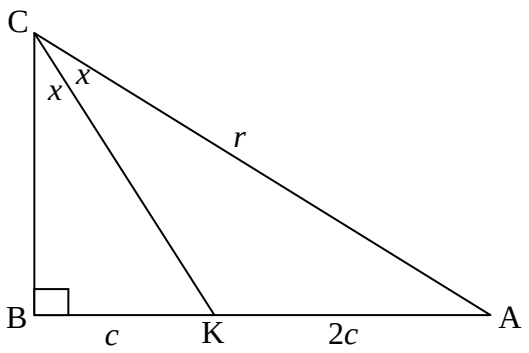
✓✓substitusie

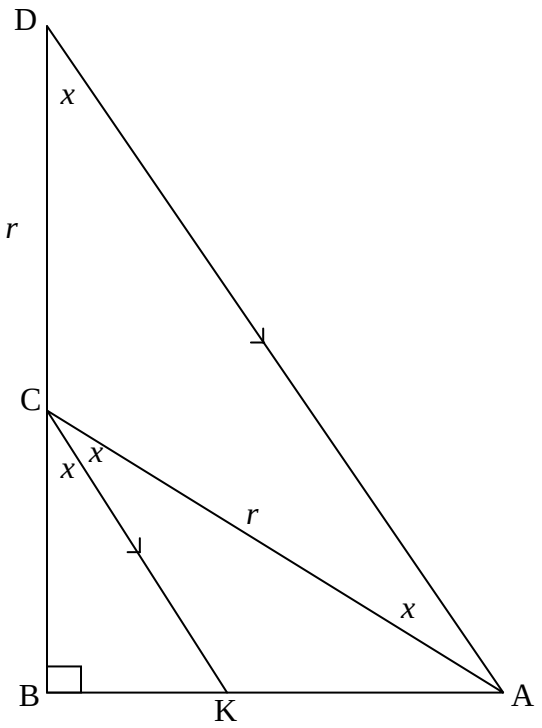
✓✓ sin (90° - 2x) = $\frac{1}{2}$

✓90° - 2x = 30°

✓ x = 30°

	OF  ΔCBK: $\sin 2x = \frac{3c}{r} = 2\sin x.\cos x$ $\therefore r\sin x = \frac{3c}{2\cos x}$ .....(1) ΔCKA: $\frac{2c}{\sin x} = \frac{r}{\cos x}$ $\therefore r\sin x = 2c\cos x$.....(2) Stel (1) en (2) gelyk: $2c.\cos x = \frac{3c}{2\cos x}$ $\therefore \cos^2 x = \frac{3}{4}$ $\therefore \cos x = \frac{\sqrt{3}}{2}$ $\therefore \quad x = 30°$ **OF**	(8) ✓ $\sin 2x = \frac{3c}{r}$ ✓ $2\sin x.\cos x$ ✓ $r\sin x = \frac{3c}{2\cos x}$ ✓ $\frac{2c}{\sin x} = \frac{r}{\cos x}$ ✓ $r\sin x = 2c\cos x$ ✓stel gelyk ✓ $\cos x = \frac{\sqrt{3}}{2}$ ✓30° (8)

	$\frac{AK}{KB} = \frac{2}{1} = 2$ $2 = \frac{\frac{1}{2}AK.BC}{\frac{1}{2}BK.BC}$ $= \frac{\text{area AKC}}{\text{area ABC}}$ $= \frac{\frac{1}{2}rCK\sin x}{\frac{1}{2}BC.CK\sin x}$ $= \frac{r}{BC}$ $\therefore \frac{BC}{r} = \frac{1}{2}$ $\therefore \cos 2x = \frac{1}{2}$ $\therefore 2x = 60°$ $\therefore x = 30°$	✓vermenigvuldig met $\frac{1}{2}BC$ ✓ area van driehoeke ✓ area formule in driehoeke ✓ $\frac{r}{BC} = 2$ ✓ $\frac{BC}{r} = \frac{1}{2}$ ✓ $\cos 2x = \frac{1}{2}$ ✓ $2x = 60°$ ✓ $x = 30°$ (8)
	OF  Deur gebruik te maak van die Interne Halveerder Stelling: $\frac{CB}{CA} = \frac{BK}{KA} = \frac{1}{2}$ $\cos 2x = \frac{1}{2}$ $2x = 60°$ $x = 30°$	✓✓ Deur die spesifieke stelling te noem ✓✓✓ $\frac{CB}{CA} = \frac{BK}{KA} = \frac{1}{2}$ ✓ $\cos 2x = \frac{1}{2}$ ✓ $2x = 60°$ ✓ $x = 30°$ (8)

	OF  Verleng BC na D en teken CK parallel aan DA. $C\hat{A}D = K\hat{C}A \quad and \quad B\hat{C}K = \hat{D}$ $\therefore DC = CA = r$ $\therefore \Delta BKC \,\|\|\| \, \Delta BAD$ $\therefore \frac{BK}{BA} = \frac{BC}{BD} = 3$ $\therefore BD = 3BC = BC + r$ $\therefore BC = \frac{1}{2}r$ $\therefore \cos 2x = \frac{\frac{1}{2}r}{r} = \frac{1}{2}$ $\therefore 2x = 60°$ $\therefore x = 30°$	✓ $DC = CA = r$ ✓ $\Delta BKC \,\|\|\| \, \Delta BAD$ ✓ $\frac{BK}{BA} = \frac{BC}{BD} = 3$ ✓BD = BC + r ✓ $BC = \frac{1}{2}r$ ✓ $\cos 2x = \frac{1}{2}$ ✓$2x$ = 60° ✓30° (8) **[11]**

TOTAAL: 150

PolyMathic

Vraestel 2

Okt/Nov

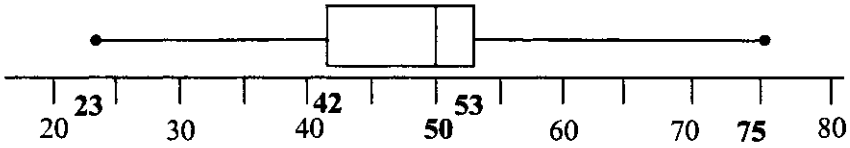
Eksamen

PolyMathic

Graad 12 Eindeksamen
Totaal: 150
Tyd: 3ure

VRAAG 1

Die vyfgetalopsomming van die hoogtes van bome drie maande nadat hulle geplant is, is (23 ; 42 ; 50 ; 53 ; 75). Hierdie inligting word in die mond-en-snordiagram hieronder aangetoon.



1.1 Bepaal die interkwartielvariasiewydte. (2)

1.2 Watter persentasie plante het 'n hoogte van meer as 53 cm? (2)

1.3 Tussen watter kwartiele het die hoogtes van die bome die kleinste variasie? Verduidelik. (2)

[6]

VRAAG 2

Die verband tussen bloedalkoholvlakke en die risiko om 'n motorongeluk te maak, word al jare lank bestudeer. Navorsing het die volgende resultate getoon:

BLOEDALKOHOLVLAK (%)	RELATIEWE RISIKO OM 'N MOTORONGELUK TE MAAK (%)
0,00	1,0
0,05	2,9
0,10	8,5
0,15	24,8
0,20	72,2
0,21	89,5

2.1 Teken 'n spreidiagram op DIAGRAMVEL 1 om die data voor te stel. (3)

2.2 Trek 'n lyn (of kromme) van beste passing op DIAGRAMVEL 1. (1)

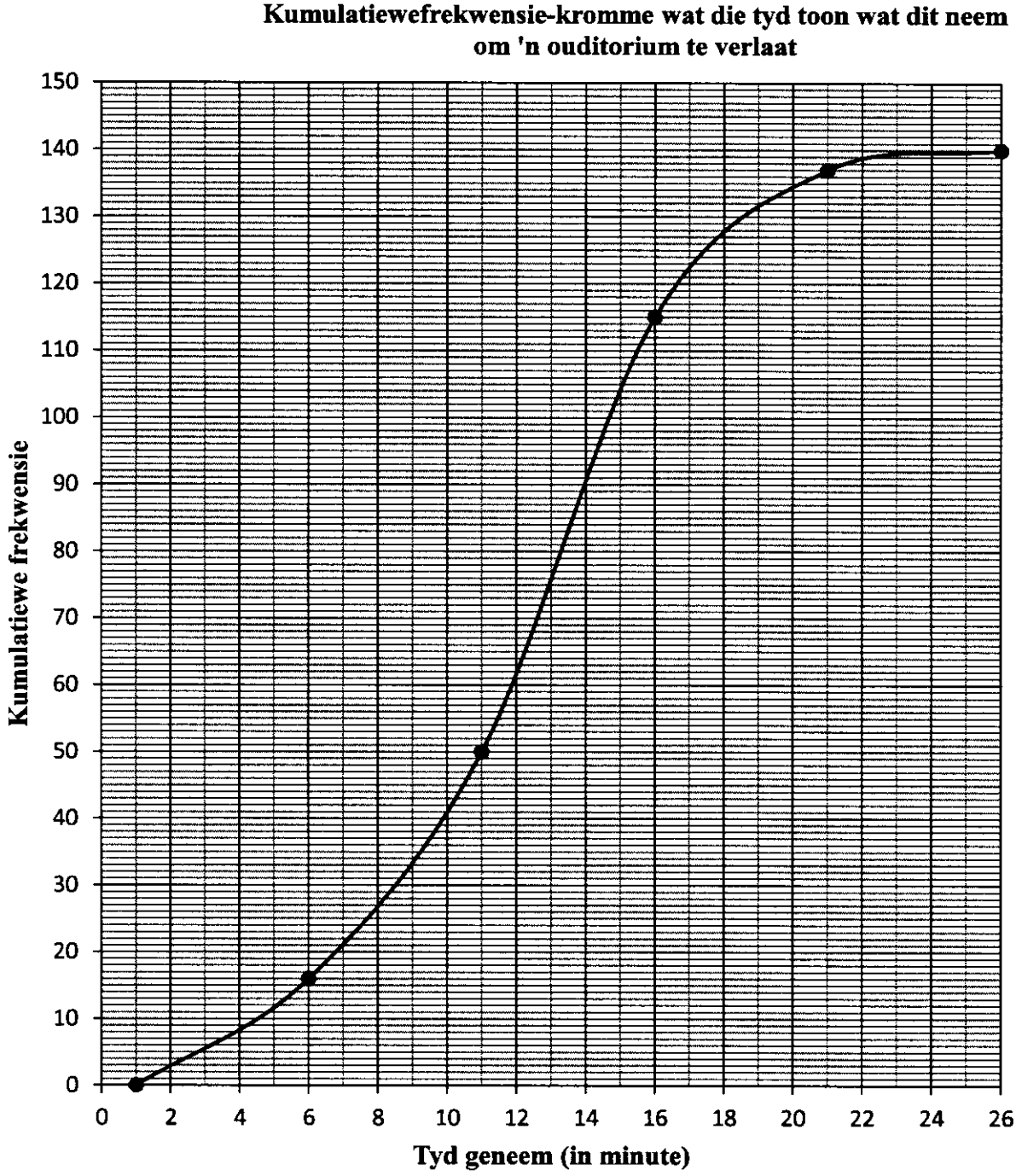
2.3 Beskryf die tendens van die data. (1)

2.4 Skat die waarskynlikheid om 'n motorongeluk te maak wanneer jou bloedalkoholvlak 0,18% is. (Die wettige perk van die bloedalkoholvlak is 0,05%.) (2)

[7]

VRAAG 3

Die kumulatiewefrekwensie-kromme (ogief) wat hieronder geteken is, toon die tyd (in minute) wat dit 140 konsertgangers neem om 'n ouditorium na 'n vertoning te verlaat.



3.1 Skat die getal mense wat meer as 15 minute geneem het om die ouditorium te verlaat. (2)

3.2 Skat die getal mense wat tussen 8 en 12 minute geneem het om die ouditorium te verlaat. (2)

3.3 Skryf die modale klas vir die data neer. (1)

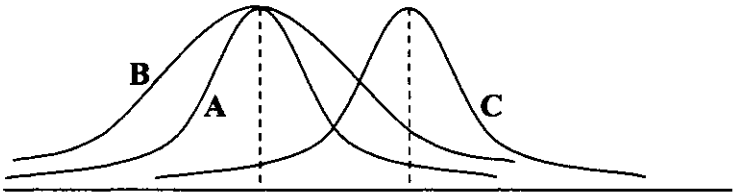
[5]

VRAAG 4

Die graad 10-klasse van drie skole het 'n kwartaaltoets geskryf. Al drie skole het dieselfde getal leerders in graad 10. Die uitslae van die toetse is in die tabel hieronder opgesom.

	SKOOL A	SKOOL B	SKOOL C
Gemiddeld	9,8	9,8	14,8
Standaardafwyking	2,3	3,1	2,3

Die verspreiding van die uitslae word in die diagram hieronder aangetoon.



4.1 In watter skool (A, B of C) is die meerderheid van die uitslae wyer rondom die gemiddeld versprei? Gee 'n rede vir jou antwoord. (2)

4.2 Wat is die verskil in die verspreiding rondom die onderskeie gemiddeldes van die punte in Skool A en Skool C? (1)

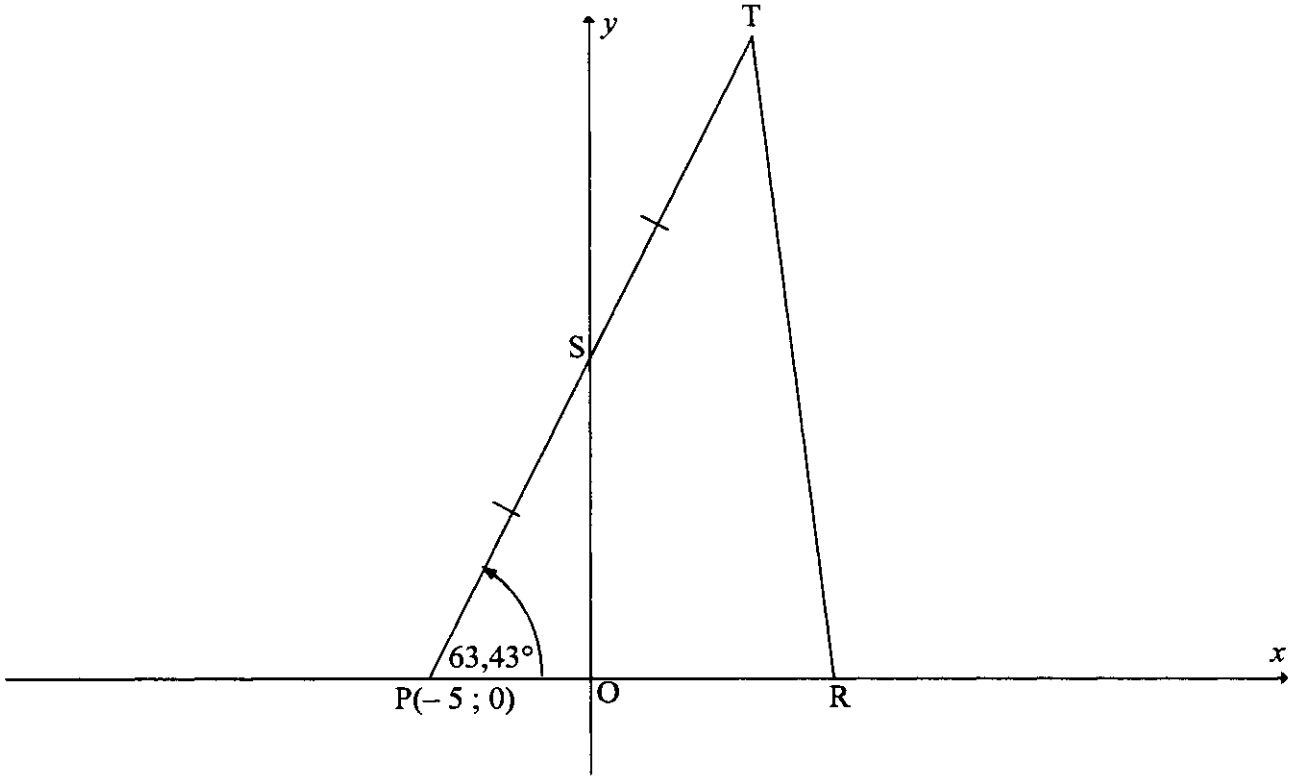
4.3 Verduidelik hoe die punte van Skool A aangepas moet word om met die punte van Skool C ooreen te stem. (2)

4.4 Indien elke punt in Skool C met 10% verlaag word, verduidelik hoe dit die gemiddeld en standaardafwyking van hierdie skool sal beïnvloed. (2)

[7]

VRAAG 5

In die diagram hieronder is P 'n punt (– 5 ; 0). Die inklinasie van lyn PT is 63,43°. S is die middelpunt en die y-afsnit van PT. R is 'n punt op die x-as sodanig dat PO : OR = 2 : 3.



5.1 Bepaal:

5.1.1 Die gradiënt van PT, korrek tot die naaste heelgetal (2)

5.1.2 Die vergelyking van PT in die vorm $y = mx + c$ (2)

5.1.3 Die afstand PS in wortelvorm (3)

5.1.4 Die koördinate van T (2)

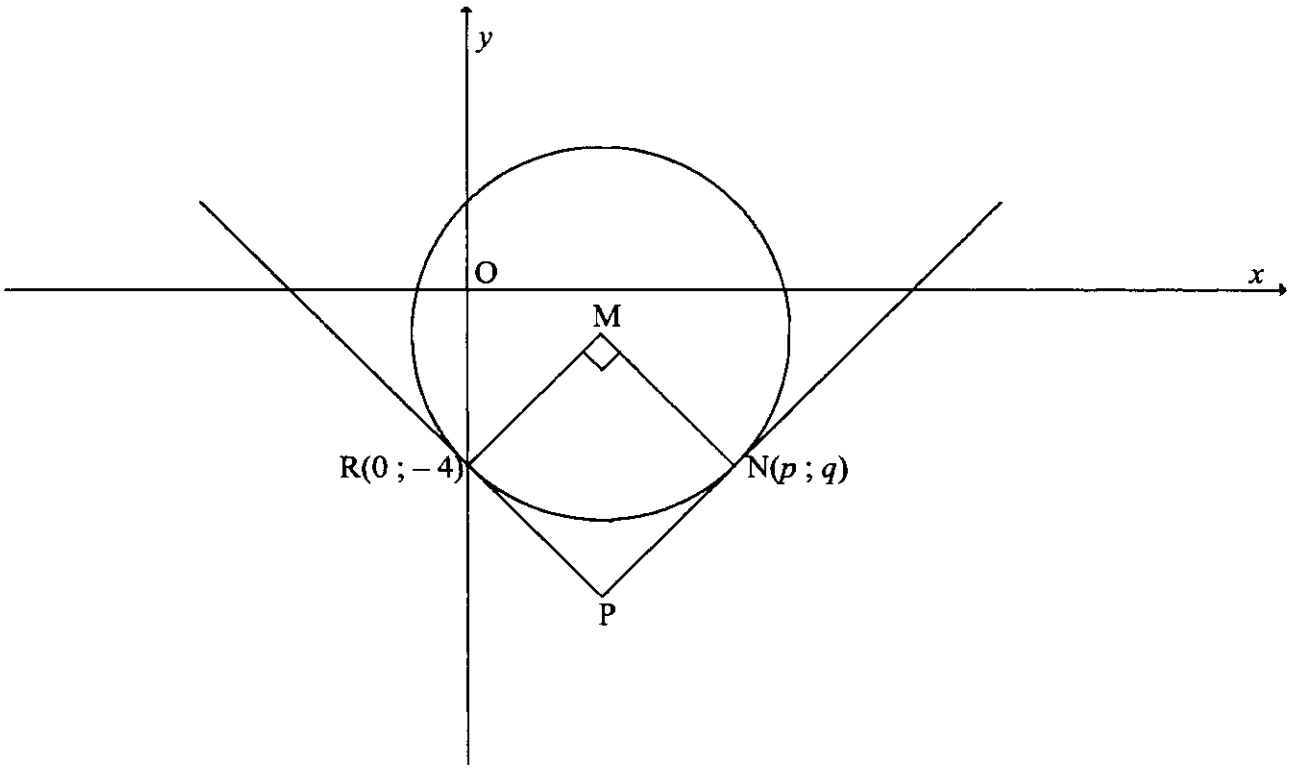
5.2 Bepaal die koördinate van R. (2)

5.3 Bereken die oppervlakte van ΔPTR. (4)

[15]

VRAAG 6

In die diagram hieronder is M die middelpunt van die sirkel met die vergelyking $x^2 + y^2 - 6x + 2y - 8 = 0$. Die sirkel gaan deur R(0 ; − 4) en N(p ; q). $R\hat{M}N = 90°$.
Die raaklyne aan die sirkel by R en N ontmoet by P.



6.1 Toon aan dat M die punt (3 ; –1) is. (4)

6.2 Bepaal die vergelyking van MR in die vorm $y = mx + c$. (3)

6.3 Toon aan dat $q = 2 - p$. (4)

6.4 Bepaal die waardes van p en q. (5)

6.5 Bepaal die vergelyking van die sirkel met middelpunt O wat deur punt N gaan. (2)

6.6 Bereken die oppervlakte van die sirkel met middelpunt M. (2)

6.7 Bereken die verhouding in sy eenvoudigste vorm: $\frac{NP}{MP}$ (4)

[24]

VRAAG 7

7.1 Bepaal die beeld van P(x ; y) as P deur 90° om die oorsprong in 'n kloksgewyse rigting geroteer en dan om die y-as gereflekteer word. (2)

7.2 Bepaal die beeld van P(x ; y) as P om die y-as gereflekteer en dan deur 90° om die oorsprong in 'n kloksgewyse rigting geroteer word. (2)

7.3 Mo en Ziya redeneer oor die beeld van P(x ; y) onder die volgende transformasies:

- Rotasie deur 90° om die oorsprong in 'n kloksgewyse rigting
- Refleksie om die y-as

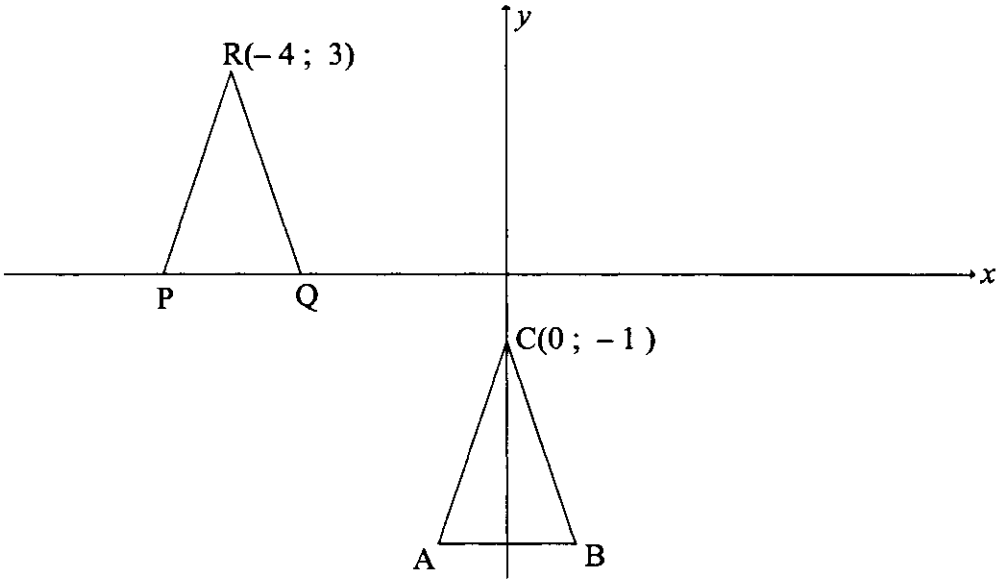
Mo beweer dat die volgorde waarin die transformasies uitgevoer word, die finale posisie van die beeld sal beïnvloed. Ziya redeneer dat die finale posisie van die beeld dieselfde sal wees, ongeag die volgorde waarin die transformasies uitgevoer word.

Watter een van die twee, Mo of Ziya, is in hierdie geval reg? Verduidelik. (2)

[6]

VRAAG 8

In die diagram is ABC 'n gelykbenige driehoek met hoekpunt C wat by (0 ; –1) lê. AB is ewewydig aan die x-as en AC = $\sqrt{10}$.



8.1 'n Rigiede (starre) transformasie word op ΔABC toegepas om ΔPQR te vorm, soos aangetoon. R(– 4 ; 3) is die beeld van C. Beskryf volledig, in woorde, die transformasie van ΔABC na ΔPQR. (2)

8.2 ΔPQR word om die lyn $y = x$ gereflekteer. Bepaal die koördinate van R$^{/}$, die beeld van R. (2)

8.3 ΔABC word deur die oorsprong vergroot om ΔA$^{/}$B$^{/}$C$^{/}$ te vorm, sodanig dat:

$$\frac{\text{oppervlakte van } \Delta A^{/}B^{/}C^{/}}{\text{oppervlakte van } \Delta ABC} = 16$$

8.3.1 Bepaal die skaalfaktor van die vergroting. (1)

8.3.2 Indien AC = $\sqrt{10}$ eenhede, skryf die lengte van A$^{/}$C$^{/}$ neer. (1)

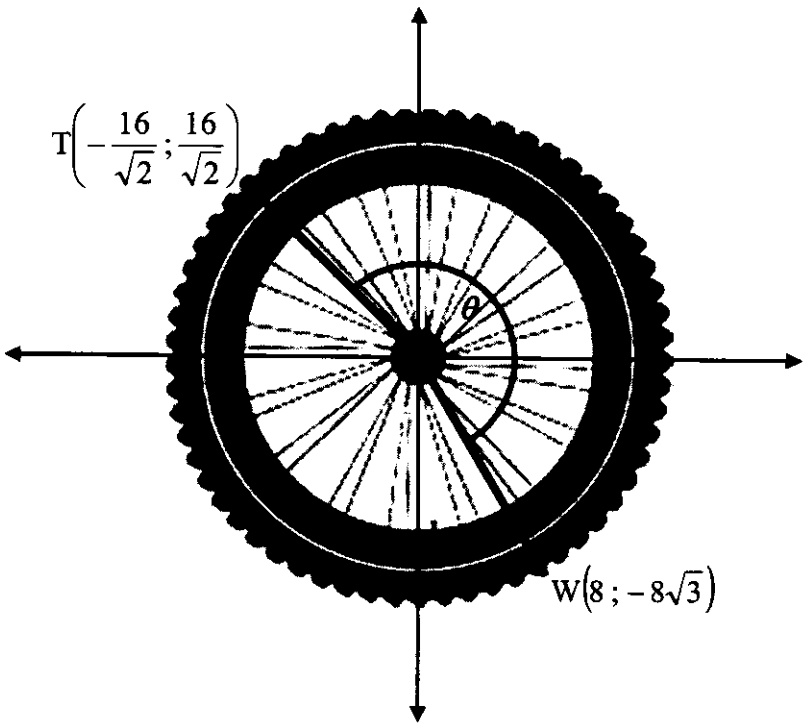
8.4 Nadat 'n rigiede (starre) transformasie op ΔABC toegepas is om ΔDEF te vorm, is F(0 ; 1) die beeld van C. As E die punt (s ; t) is, skryf 'n vergelyking in terme van s en t neer. (4)

[10]

VRAAG 9

'n Wiel word so geplaas dat sy middelpunt presies op die oorsprong van die Cartesiese vlak val. $T\left(-\frac{16}{\sqrt{2}};\frac{16}{\sqrt{2}}\right)$ is 'n punt op die buitenste rand van die wiel.

Wanneer die wiel in 'n kloksgewyse rigting om die oorsprong deur 'n hoek van θ gedraai word, val T direk op $W(8;-8\sqrt{3})$.



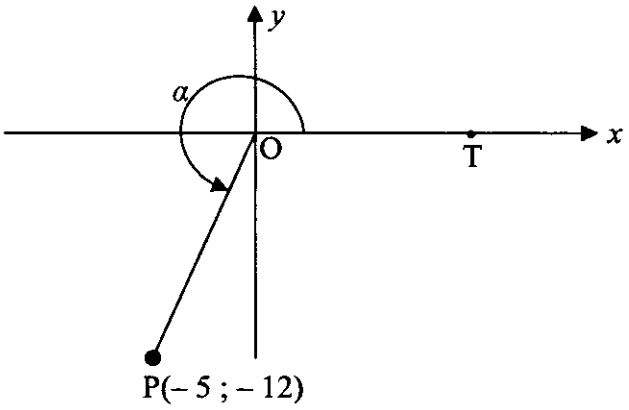
9.1 Toon aan dat $\theta = 195°$. (5)

9.2 As die wiel teen 'n konstante spoed in 'n kloksgewyse rigting geroteer word, neem dit 1,3 sekondes vir T om tot by W te beweeg. Bereken die spoed, in omwentelinge per minuut, waarteen die wiel roteer. (5)

[10]

VRAAG 10

In die diagram hieronder is refleks $T\hat{O}P = \alpha$ en P se koördinate is (– 5 ; – 12).



Bepaal die waarde van elk van die volgende trigonometriese verhoudings SONDER om 'n sakrekenaar te gebruik:

10.1 $\cos \alpha$ (3)

10.2 $\tan (180° - \alpha)$ (2)

10.3 $\sin (30° - \alpha)$ (3)

[8]

VRAAG 11

11.1 Bewys die volgende identiteit: $\dfrac{\cos^2(90° + \theta)}{\cos(-\theta) + \sin(90° - \theta)\cos\theta} = \dfrac{1}{\cos\theta} - 1$ (6)

11.2 Bepaal die algemene oplossing van: $\tan x \sin x + \cos x \tan x = 0$. (7)

11.3 Beskou die volgende uitdrukking: $2\sin^2 3x - \sin^2 x - \cos^2 x$

11.3.1 Vereenvoudig die uitdrukking tot 'n enkele trigonometriese verhouding van x. (3)

11.3.2 Skryf die maksimum waarde van die uitdrukking neer. (1)

11.4 Dit word gegee dat $p = \cos\alpha + \sin\alpha$ en $q = \cos\alpha - \sin\alpha$

11.4.1 Bepaal die volgende trigonometriese verhoudings in terme van p en/of q:

(a) $\cos 2\alpha$ (3)

(b) $\tan\alpha$ (4)

11.4.2 Vereenvoudig $\dfrac{p}{2q} - \dfrac{q}{2p}$ tot 'n enkele trigonometriese verhouding van α. (6)

[30]

VRAAG 12

12.1 Skets die grafieke van $f(x) = \tan x + 1$ en $g(x) = \cos 2x$ vir $x \in [-180° ; 180°]$ op dieselfde assestelsel wat op DIAGRAMVEL 2 gegee word. Toon duidelik alle afsnitte met die asse, draaipunte en asimptote aan. (6)

12.2 Skryf die periode van g neer. (1)

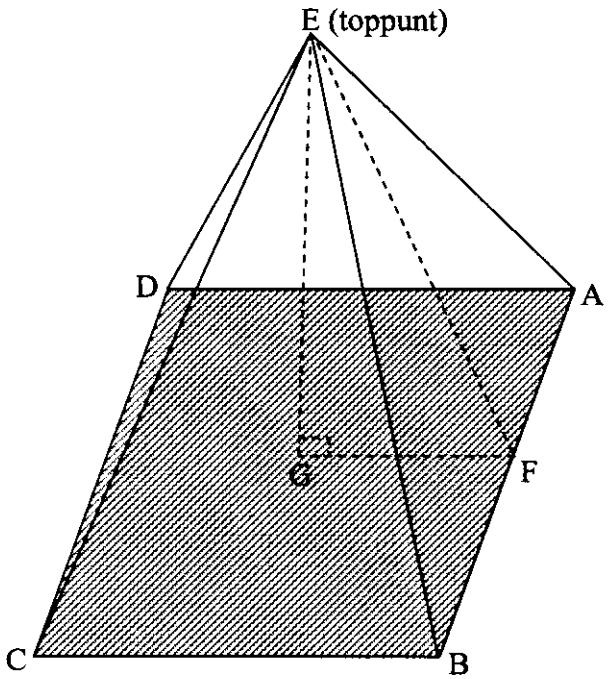
12.3 As $h(x) = -\cos 2(x + 10°)$, beskryf volledig, in woorde, die transformasie van g na h. (2)

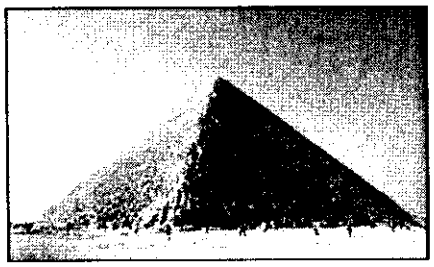
12.4 Vir watter waardes van x, waar $x > 0$, sal $f'(x)g(x) > 0$? (4)

[13]

VRAAG 13

Die Groot Piramide by Giza in Egipte is ongeveer 2 500 v.C. gebou. Die piramide het 'n vierkantige basis (ABCD) met sye 232,6 meter lank. Die afstand vanaf elke hoek van die basis na die toppunt (E) was oorspronklik 221,2 meter.





Groot Piramide by Giza in Egipte

13.1 Bereken die grootte van die hoek by die toppunt van 'n aansig van die piramide (byvoorbeeld $C\hat{E}B$). (3)

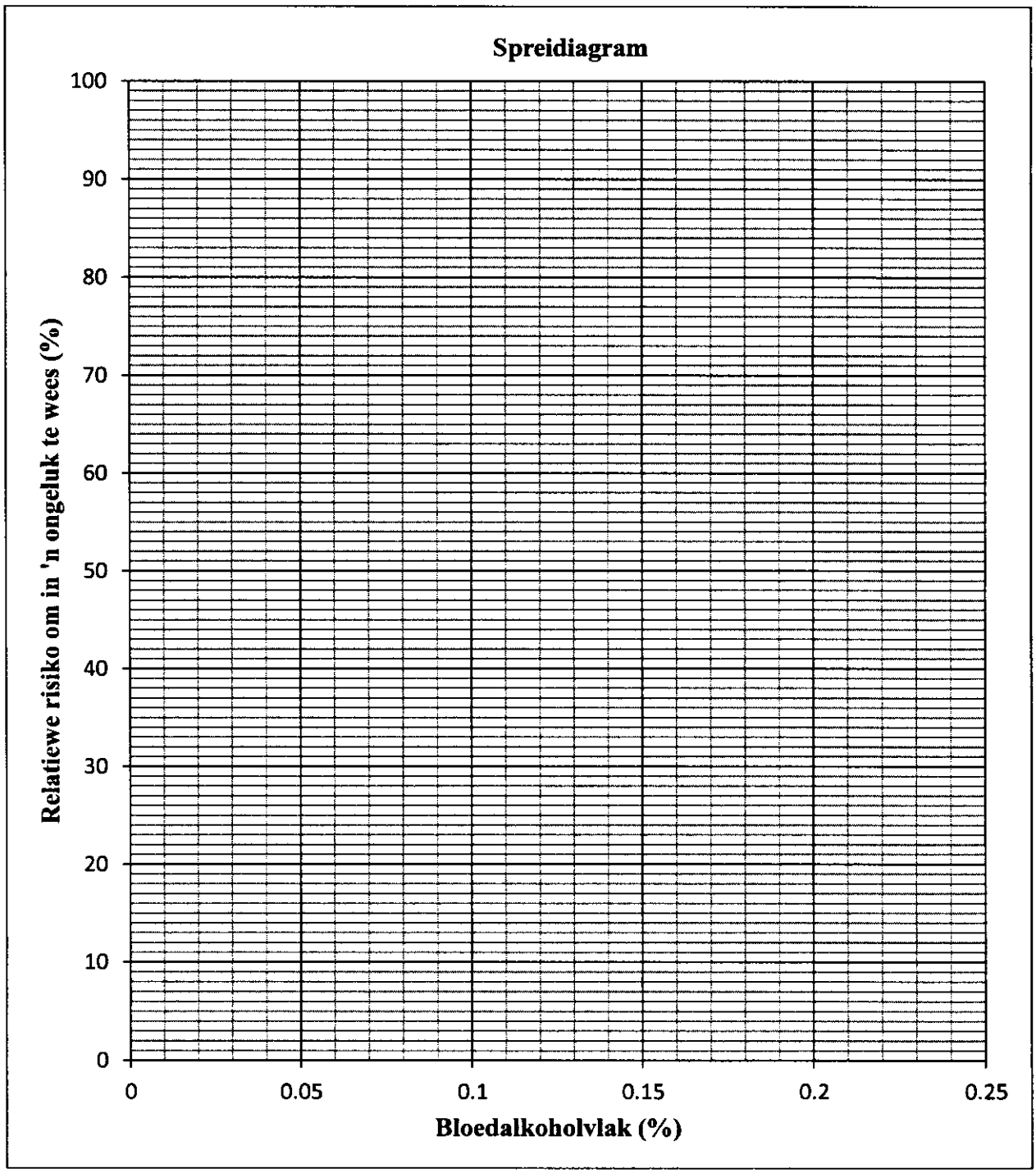
13.2 Bereken die hoek wat elke aansig met die basis vorm (byvoorbeeld $E\hat{F}G$, waar $EF \perp AB$ in ΔAEB). (6)

[9]

TOTAAL: 150

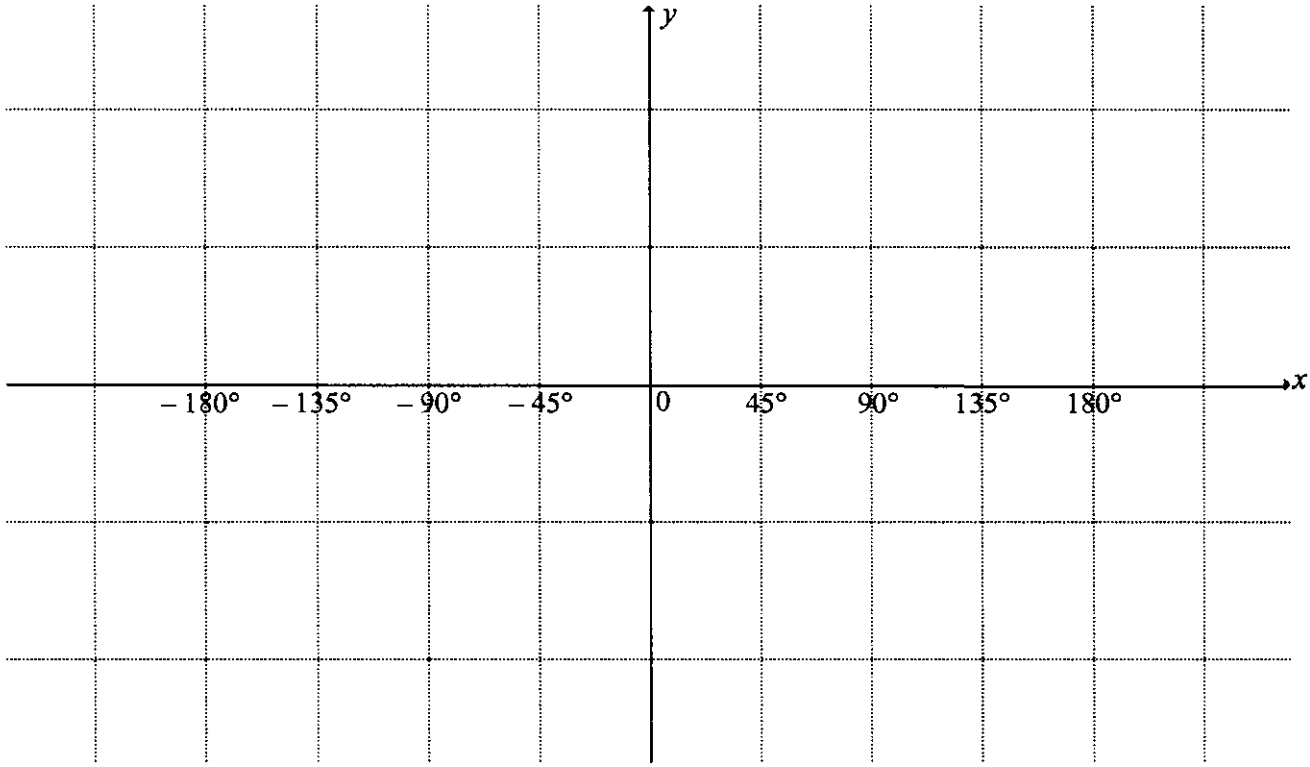
DIAGRAMVEL 1

VRAAG 2.1 en 2.2



DIAGRAMVEL 2

VRAAG 12

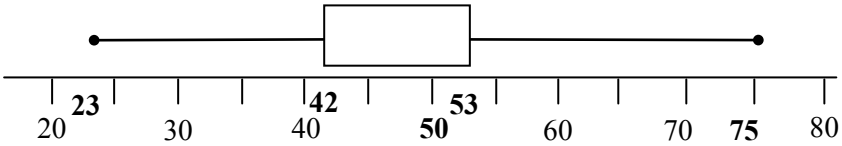


Memo

LET WEL:

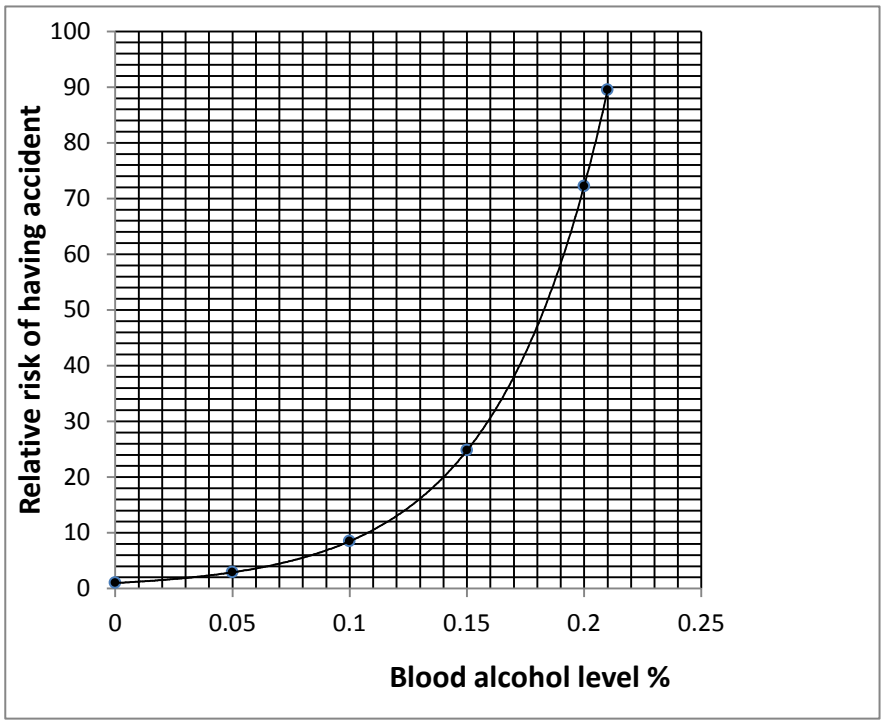
- *As 'n kandidaat 'n vraag TWEE keer beantwoord het, merk slegs die eerste poging.*
- *As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, merk die deurgehaalde antwoord.*
- *Volgehoue akkuraatheid is DEURGAANS in ALLE aspekte van die memorandum van toepassing.*
- *Aanvaarding van waardes/antwoorde om 'n problem op te los, is onaanvaarbaar.*

QUESTION/*VRAAG* 1



1.1	Interquartile range/*Interkwartielvariasiewydte* = 53 – 42 = 11 Answer only: Full marks No CA	✓critical values (42 ; 53) ✓11 (2)
1.2	25% of trees have a height in excess of 53 cm. *25% van bome het 'n hoogte van meer as 53 cm.*	✓✓25% (2)
1.3	Between Q_2(50) and Q_3 (53)/*Tussen Q_2 en Q_3* ***REASON / REDE*** The distance between these two quartiles is the smallest/*Die afstand tussen hierdie twee kwartiele is die kleinste.* **OR** The third quarter has smallest length / *Die derde kwart het die kortste lengte*	✓ Q_2 and Q_3 ✓ reason (2) **[6]**

QUESTION/*VRAAG* 2

2.1 and/en 2.2		6 points correctly plotted (3 marks) 4 points correctly plotted (2 marks) 2 points correctly plotted (1 mark) (3) ✓ exponential curve/ *eksponensiale kromme* (1)
2.3	The trend shows that as the blood alcohol levels increase, the risk of having an accident increases rapidly. *Die tendens(neiging) toon dat indien die bloed-alkoholvlakke toeneem, die risiko van 'n motorongeluk neem vinnig toe.*	✓reason (1)
2.4	Approximately 47% (Accept 44% - 51%)	✓✓ 47% (2) **[7]**

QUESTION/*VRAAG* 3

3.1	more than 15 minutes: 140 – 104 = 36 people Approximately 36 people (Accept 34 – 37) Answer only: Full marks	✓ 104 ✓ 36 (2)
3.2	At 8 minutes approximately 27 people and at 12 minutes approximately 62 people left the auditorium/*By 8 minute het ongeveer 27 mense en by 12 minute ongeveer 62 mense die ouditorium verlaat.* $\therefore 62 - 27 = 35$ Approximately 35 people left the auditorium between 8 and 12 minutes/*Ongeveer 35 mense het tussen 8 en 12 minute die ouditorium verlaat.*	✓ 27 and 62 ✓ 35 (Accept 33 – 36) (2)
3.3	Modal class/*modale klas*: $11 < x \leq 16$ **OR** $11 \leq x < 16$ Mark for critical values	✓ $11 < x \leq 16$ ✓ $11 \leq x < 16$ (1) **[5]**

QUESTION/*VRAAG* 4

	SCHOOL A	SCHOOL B	SCHOOL C
Mean	9,8	9,8	14,8
Standard deviation	2,3	3,1	2,3

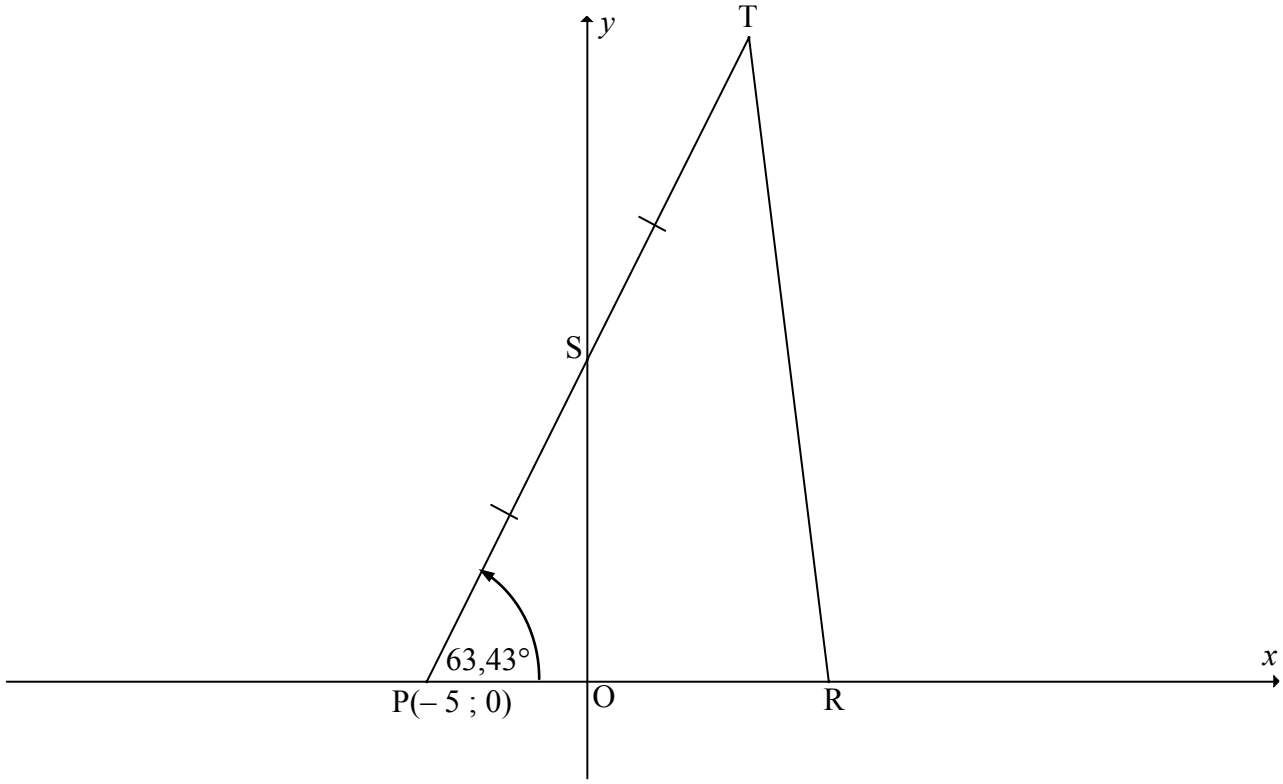
4.1	School B, because the standard deviation of B is the largest. *Skool B, want die standaardafwyking is die grootste.*	✓ School B ✓ reason (2)
4.2	There is no difference in the spread of the marks. *Daar is geen verskil in die verspreiding van punte nie.*	✓no difference / the same (1)
4.3	Add/increase each score in School A by 5 marks. *Vermeerder(tel by) elke punt in Skool A met 5 punte.*	✓ increase each mark *vermeerder elke punt* ✓ 5 marks (2)
4.4	The mean will decrease (by 10%) *Die gemiddelde sal verminder (met 10%).* The standard deviation will also decrease (by 10%) *Die standaardafwyking sal verminder (ook met 10%).*	✓ mean decreased /*gemiddeld verminder* ✓ SD decreased/ *SD verminder* (2) **[7]**

Explanation why values decrease by 10%:

$$mean = \frac{\sum 0{,}9x_i}{n} = 0{,}9\frac{\sum x_i}{n} = 0{,}9\bar{x}$$

$$SD = \sqrt{\frac{\sum (x_i - \bar{x})^2}{2}} = \sqrt{\frac{\sum (0{,}9x_i - 0{,}9\bar{x})^2}{n}} = \sqrt{\frac{0{,}9^2 \sum (x_i - \bar{x})^2}{n}} = 0{,}9\sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

QUESTION/*VRAAG* 5

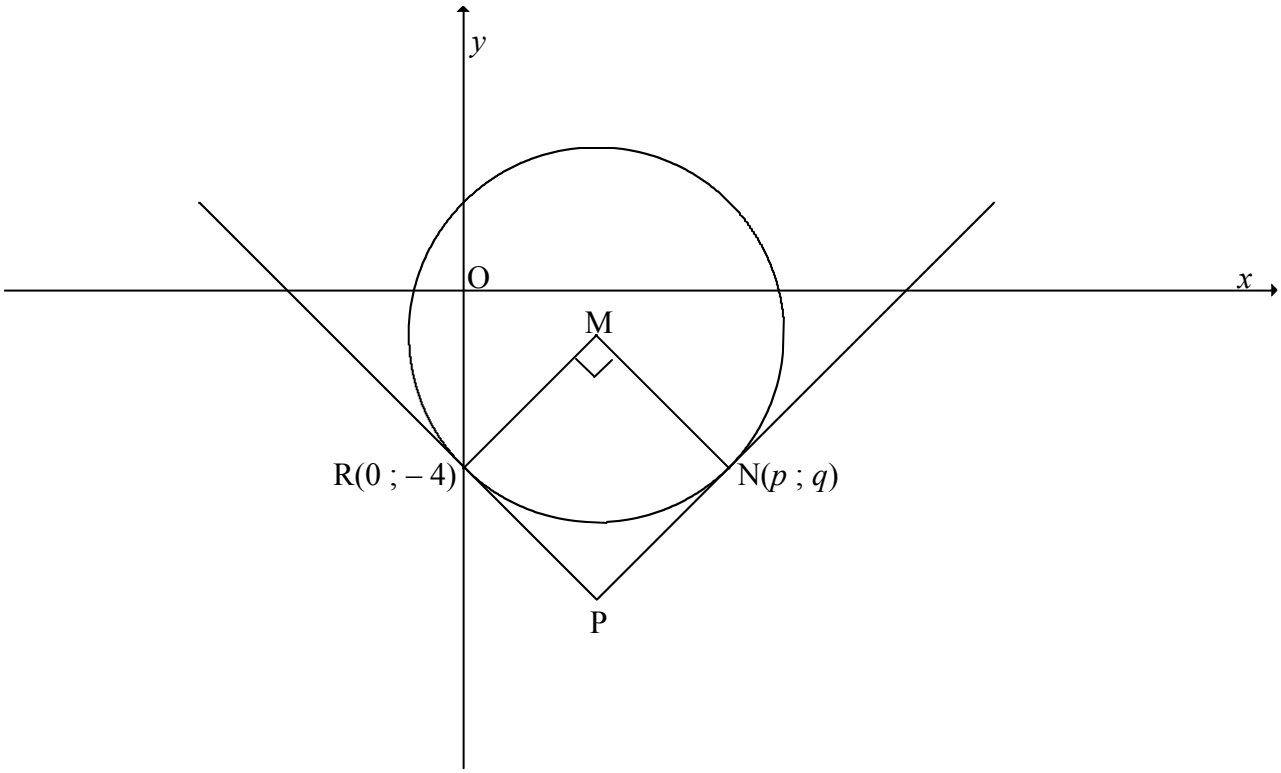


5.1.1	$m_{PT} = \tan\ 63{,}43°$ $= 2$	✓ tan 63,43° ✓ 2 (2) Answer only: full marks
5.1.2	Coordinates of P(– 5 ; 0) $y - y_1 = m(x - x_1)$ $y - 0 = 2(x + 5)$ $y = 2x + 10$ **OR** $y = mx + c$ $0 = (2)(-5) + c$ $c = 10$ $y = 2x + 10$ **OR** $m_{PT} = 2 = \tan 63{,}43°$ $\tan 63{,}43° = \frac{OS}{OP} = \frac{OS}{5} = 2$ $\therefore OS = 10$ $y = 2x + 10$	✓substitution of P(–5 ; 0) and $m = 2$ into equation ✓ equation (2) ✓substitution of P(–5 ; 0) and $m = 2$ into equation ✓ equation (2) ✓ $\frac{OS}{5} = 2$ ✓ equation (2)

5.1.3	$OS = 10$ units $PS^2 = (5)^2 + (10)^2$ $= 125$ $PS = \sqrt{125} = 5\sqrt{5}$ Accept PS = 11,18 **OR** P(–5 ; 0) ; OS = 10 units $PS^2 = (-5-0)^2 + (0-10)^2$ $= 25 + 100$ $= 125$ $PS = \sqrt{125} = 5\sqrt{5}$ Accept PS = 11,18 **OR** $\frac{PS}{5} = \frac{1}{\cos 63{,}43°}$ $\therefore PS = \frac{5}{\cos 63{,}43°}$ $PS = 11{,}18$ **OR** $\frac{PS}{10} = \frac{1}{\sin 63{,}43°}$ $\therefore PS = \frac{10}{\sin 63{,}43°}$ $PS = 11{,}18$	✓ OS = 10 ✓ substitution of correct distances into Pythagoras ✓ $\sqrt{125}$ (3) ✓ OS = 10 ✓ substitution of correct distances into Pythagoras ✓ $\sqrt{125}$ (3) ✓ ratio ✓ $PS = \frac{5}{\cos 63{,}43°}$ ✓11,18 (3) ✓ ratio ✓ $PS = \frac{10}{\sin 63{,}43°}$ ✓11,18 (3)
5.1.4	Let T be $(x ; y)$. Then $\frac{-5+x}{2} = 0$ and $\frac{0+y}{2} = 10$ $x = 5$ $y = 20$ T(5 ; 20) **OR** by inspection: T(5 ; 20)	✓ 5 ✓ 20 (2) ✓ 5 ✓ 20 (2)
5.2	$OR = \left(\frac{3}{2}\right)(5) = \frac{15}{2} = 7{,}5$ $R\left(\frac{15}{2}; 0\right)$ If only x-coordinate : 2 marks	✓$x = 7{,}5$ / $\frac{15}{2}$ ✓$y = 0$ (2)

5.3	Area $\Delta PTR = \frac{1}{2}(\text{base PR}) \times (\text{height})$ $= \frac{1}{2}(5 + \frac{15}{2}) \times 20$ $= 125$ square units **OR** Area $\Delta PTR = \frac{1}{2} PT.PR.\sin T\hat{P}R$ $= \frac{1}{2}\left(10\sqrt{5}\right)\left(\frac{25}{2}\right)\sin 63{,}43°$ $= 124{,}99$ square units	✓ area formula ✓ $5 + \frac{15}{2} = 12{,}5$ ✓ 20 ✓ 125 (4) ✓ area formula ✓ $10\sqrt{5}$ ✓ $\frac{25}{2}$ ✓ 124,99 (4) **[15]**

QUESTION/*VRAAG* 6



6.1	$x^2 + y^2 - 6x + 2y - 8 = 0$ $x^2 - 6x + 9 + y^2 + 2y + 1 = 8 + 9 + 1$ $(x-3)^2 + (y+1)^2 = 18$ $\therefore$ M(3 ; – 1) If only $(x-3)^2 + (y+1)^2 = r^2$ $(r^2 \neq 18)$, then 2 marks **OR** $x_M = -\frac{1}{2}(\text{coefficient of } x)$ $x_M = -\frac{1}{2}(-6)$ $x_M = 3$ $y_M = -\frac{1}{2}(\text{coefficient of } y)$ $y_M = -\frac{1}{2}(2)$ $y_M = -1$ $\therefore$ M(3 ; – 1)	✓ $x^2 - 6x + 9$ ✓ $y^2 + 2y + 1$ ✓ $(x-3)^2$ ✓ $(y+1)^2$ (4) ✓ $x_M = -\frac{1}{2}(-6)$ ✓ $x_M = 3$ ✓ $y_M = -\frac{1}{2}(2)$ ✓ $y_M = -1$ (4)

6.2	$m_{RM} = \frac{-1-(-4)}{3-0}$ $= 1$ y-intercept is – 4 $y = x - 4$	✓ substitution into gradient formula ✓ $m_{RM} = 1$ ✓ equation (3)
6.3	MR ⊥ RP (radius ⊥ tangent/*raaklyn*) $m_{MN} = m_{PR} = -1$ $\frac{q-(-1)}{p-3} = -1$ $-p+3 = q+1$ $q = 2-p$ **OR** MR ⊥ RP (radius ⊥ tangent/*raaklyn*) $m_{MN} = m_{PR} = -1$ $y-(-1) = -1(x-3)$ $y+1 = -x+3$ $y = -x+2$ $q = 2-p$	✓✓ $m_{MN} = -1$ ✓ substitution into gradient formula ✓ $-p+3 = q+1$ (4) ✓✓ $m_{MN} = -1$ ✓ $y = -x+2$ ✓ substitution into equation of line (4)
6.4	$(x-3)^2 + (y+1)^2 = 18$ $(p-3)^2 + (q+1)^2 = 18$ $(2-q-3)^2 + (q+1)^2 = 18$ $q^2 + 2q + 1 + q^2 + 2q + 1 - 18 = 0$ $2q^2 + 4q - 16 = 0$ $q^2 + 2q - 8 = 0$ $(q+4)(q-2) = 0$ $q = -4$ *or* $q \neq 2$ $p = 6$ **OR** MRPN is a square/*vierkant* (rectangle with/*reghoek met* MN = MR) $\therefore$ $M\hat{P}N = 45°$ But MR has a slope/gradient of 1, so RN || x-axis $\therefore$ $q = -4$ and $p = 2 - (-4) = 6$ **OR**	✓ method ✓✓ $q = -4$ ✓✓ $p = 6$ (5) ✓ method ✓✓ $q = -4$ ✓✓ $p = 6$ (5)

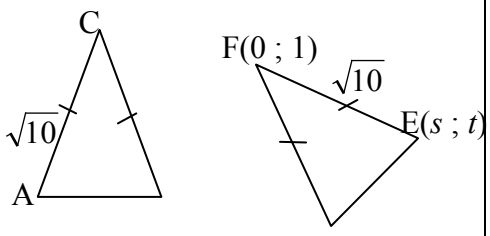
	$q = 2 - p$ $(p-3)^2 + (2-p+1)^2 = 18$ $(p-3)^2 = 9$ $\therefore \quad p - 3 = 3 \quad (p > 0)$ $p = 6$ $\therefore q = -4$ **OR** Using symmetry: $q = -4$ (since $y_M = y_R$) $-4 = 2 - p$ $p = 6$ OR $p = 2 \times 3$ (since $x_M = 2x_N$)	✓ method ✓✓ $p = 6$ ✓✓ $q = -4$ (5) ✓method ✓✓ q ✓✓ p (5)
6.5	$r^2 = (6)^2 + (-4)^2$ $= 36+16 = 52$ $x^2 + y^2 = 52$ **OR** $p^2 + q^2 = (6)^2 + (-4)^2$ $= 36+16 = 52$ $x^2 + y^2 = p^2 + q^2$ $= 52$	✓ substitution ✓ equation (2) ✓ substitution ✓ equation (2)
6.6	area of circle M $= \pi r^2$ $= \pi(\sqrt{18})^2$ $= 18\pi$ square *units* $= 56{,}55$ *square units*	✓ $r = \sqrt{18}$ ✓ area of circle (2)
6.7	MRPN is a square (all angles equals 90°, adj sides equal) $N\hat{M}P = 45°$ (diagonals of a square bisect the angles/*hoeklyne van vierkant halveer hoeke)* $\frac{NP}{MP} = \sin N\hat{M}P$ $= \sin 45°$ $= \frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$ **OR** MRPN is a square (all angles equals 90°, adj sides equal) $MP^2 = 18 + 18$ $= 36$ $MP = 6$ $\frac{NP}{MP} = \frac{\sqrt{18}}{6} = \frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$	✓ $N\hat{M}P = 45°$ ✓ ✓ $\frac{NP}{MP} = \sin N\hat{M}P$ ✓ $\frac{1}{\sqrt{2}}$ (4) ✓ $MN^2 = 18$ ✓$MP^2 = 36$ ✓6 ✓ $\frac{1}{\sqrt{2}}$ (4)

	OR By inspection: P(3 ; – 7) $\frac{NP}{MP} = \frac{\sqrt{(6-3)^2+(4-7)^2}}{\sqrt{(3-3)^2+(-7+1)^2}}$ $= \frac{\sqrt{18}}{6} = \frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$	✓ P(3 ; – 7) ✓$NP^2 = 18$ ✓MP = 6 ✓ $\frac{1}{\sqrt{2}}$ (4) **[24]**

QUESTION/*VRAAG* 7

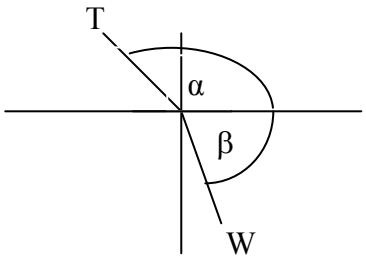
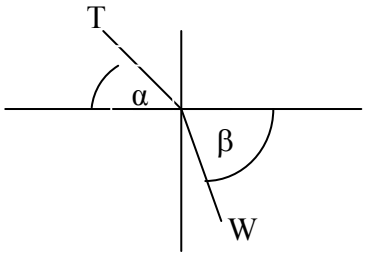
7.1	$(x\ ;\ y) \rightarrow (y\ ;\ -x) \rightarrow (-y\ ;\ -x)$	✓ $(y\ ;\ -x)$ ✓ $(-y\ ;\ -x)$ (2)
7.2	$(x\ ;y) \rightarrow (-x\ ;y) \rightarrow (y\ ;x)$	✓ $(-x\ ;\ y)$ ✓ $(y\ ;\ x)$ (2)
7.3	Mo's claim is correct/*Mo se bewering is korrek.* If order was unimportant then the image of P would be the same in both cases. This is not so./*As volgorde onbelangrik is, sal die beeld van P in beide gevalle dieselfde wees. Wat nie so is nie.* **OR** Choose any point $\neq (0\ ;\ 0)$ and show that their images both cases are not the same. For example: $(3\ ;\ 4) \rightarrow (4\ ;\ -3) \rightarrow (-4\ ;\ -3)$ $(3\ ;\ 4) \rightarrow (-3\ ;\ 4) \rightarrow (4\ ;\ 3)$ Mo is correct	✓ Mo ✓ reason (2) ✓ calculation of both images ✓ Mo (2) **[6]**

QUESTION/*VRAAG* 8

8.1	ΔABC is translated by 4 units to the left and 4 units up/ *ΔABC word getransleer met 4 eenhede na links en 4 eenhede opwaarts.* Accept $(x;y) \rightarrow (x-4;y+4)$	✓ translation/ *translasie* ✓ 4 left/*links* and 4 up/*opwaarts* (2)
8.2	$R^{/}$ $(3\ ;\ -4)$	✓ 3 ✓ -4 (2)
8.3.1	Area $\Delta A^{/}B^{/}C^{/}$ = 16 × Area of ΔABC Scale factor/*skaalfaktor* = 4	✓ 4 (1)
8.3.2	$AC = \sqrt{10}$ $A^{/}C^{/} = 4\sqrt{10}$	✓ $4\sqrt{10}$ (1)
8.4	EF = AC $\sqrt{(s-0)^2+(t-1)^2} = \sqrt{10}$ $\sqrt{s^2+(t-1)^2} = \sqrt{10}$ $s^2+(t-1)^2 = 10$ $s^2+t^2-2t+1-10=0$ $s^2+t^2-2t-9=0$ 	✓✓✓ recognising that EF = AC ✓ equation in terms of *s* and *t* (4) **[10]**

QUESTION/*VRAAG* 9

9.1	Anti-clockwise / Anti-kloksgewys: $-\frac{16}{\sqrt{2}}\cos(-\theta)-\frac{16}{\sqrt{2}}\sin(-\theta)=8$ .....(1) $\frac{16}{\sqrt{2}}\cos(-\theta)-\frac{16}{\sqrt{2}}\sin(-\theta)=-8\sqrt{3}$ .....(2) (1) +(2): $-\frac{32}{\sqrt{2}}\sin(-\theta) = 8-8\sqrt{3}$ $\sin(-\theta)=\frac{-8+8\sqrt{3}}{\frac{32}{\sqrt{2}}}$ $\sin\theta=\frac{-\sqrt{6}+\sqrt{2}}{4}=-0{,}258819...$ $\theta=180°+15°$ *or* $\theta=360°-15°$ $=195°$	✓ substitution into *x* image of rotation ✓ substitution into *y* image of rotation ✓ addition of equations ✓ value of sin θ ✓ 180° + 15° (5)
	OR	
	$-\frac{16}{\sqrt{2}}\cos(-\theta)-\frac{16}{\sqrt{2}}\sin(-\theta)=8$ .....(1) $\frac{16}{\sqrt{2}}\cos(-\theta)-\frac{16}{\sqrt{2}}\sin(-\theta)=-8\sqrt{3}$ .....(2) (1) – (2): $-\frac{32}{\sqrt{2}}\cos(-\theta) = 8+8\sqrt{3}$ $\cos\theta=\frac{8+8\sqrt{3}}{-\frac{32}{\sqrt{2}}}=\frac{-\sqrt{6}-\sqrt{2}}{4}=-0{,}96592...$ $\theta=180°+15°$ *or* $\theta=180°-15°$ $=195°$	✓ substitution into *x* image of rotation ✓ substitution into *y* image of rotation ✓ subtraction of equations ✓ value of cos θ ✓ 180° + 15° (5)
	OR Clockwise /Kloksgewys: $-\frac{16}{\sqrt{2}}\cos(\theta)+\frac{16}{\sqrt{2}}\sin(\theta)=8$ .....(1) $\frac{16}{\sqrt{2}}\cos(\theta)+\frac{16}{\sqrt{2}}\sin(\theta)=-8\sqrt{3}$ .....(2) (1) +(2): $\frac{32}{\sqrt{2}}\sin(\theta) = 8-8\sqrt{3}$ $\sin(\theta)=\frac{8-8\sqrt{3}}{\frac{32}{\sqrt{2}}}$ $\sin\theta=\frac{-\sqrt{6}+\sqrt{2}}{4}=-0{,}258819...$ $\theta=180°+15°$ *or* $\theta=360°-15°$ $=195°$	✓ substitution into *x* image of rotation ✓ substitution into *y* image of rotation ✓ addition of equations ✓ value of sin θ ✓ 180° + 15° (5)

	OR $-\frac{16}{\sqrt{2}}\cos(\theta)+\frac{16}{\sqrt{2}}\sin(\theta)=8$ .....(1) $\frac{16}{\sqrt{2}}\cos(\theta)+\frac{16}{\sqrt{2}}\sin(\theta)=-8\sqrt{3}$ .....(2) (1) – (2): $-\frac{32}{\sqrt{2}}\cos(\theta) = 8+8\sqrt{3}$ $\cos\theta=\frac{8+8\sqrt{3}}{-\frac{32}{\sqrt{2}}}=\frac{-\sqrt{6}-\sqrt{2}}{4}=-0{,}96592...$ $\theta=180°+15° \quad or \quad \theta=180°-15°$ $=195°$	✓ substitution into x image of rotation ✓ substitution into y image of rotation ✓ subtraction of equations ✓ value of cos θ ✓ 180° + 15° (5)
	OR  $\tan\alpha=\frac{-\frac{16}{\sqrt{2}}}{\frac{16}{\sqrt{2}}}=-1$ α = 135° $\tan\beta=\frac{-8\sqrt{3}}{8}=-\sqrt{3}$ $\beta=-60°$ ∴θ = 135° + 60° = 195°	✓ tan α = – 1 ✓ 135° ✓ tan $\beta=-\sqrt{3}$ ✓ – 60° ✓ 195° (5)
	OR  tan α = 1 α = 45° tan β = $\sqrt{3}$ β = 60° ∴θ = (180° – 45°) + 60° = 195°	✓ tan α = 1 ✓ 45° ✓ tan $\beta=\sqrt{3}$ ✓ 60° ✓ 195° (5)

9.2	195° in 1,3 secs $\therefore$ 1 revolution/*omwenteling* in $\frac{360}{195}\times 1{,}3$ secs = 2,4 secs/*sek* 1 minute = 60 sec: $\therefore \frac{60}{2{,}4} = 25$ revolutions/*omwentelings* $\therefore$ 25 rev/min or 25 *omw/min* Answer only: 1 mark OR $\text{speed/sec} = \frac{195°}{1{,}3} = 150°/\text{sec}$ $\text{speed/minute} = 150° \times 60 = 9000°/\text{min}$ $\text{no of revolutions} = \frac{9000}{360} = 25\ \text{rev/min}$	✓ $\frac{360}{195}$ ✓ $\frac{360}{195}\times 1{,}3$ ✓ 2,4 secs ✓ $\frac{60}{2{,}4}$ ✓ 25 rev/min (5) ✓ $\frac{195}{1{,}3}$ or 150 ✓ 150×60 ✓ 9000 ✓ $\frac{9000}{360}$ ✓ 25 rev/min (5) **[10]**

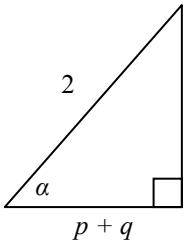
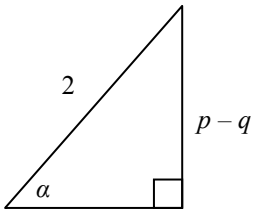
QUESTION/*VRAAG* 10

10.1	$OP^2 = 25 + 144 = 169$ $OP = 13$ $\cos\alpha = -\frac{5}{13}$ **OR** $r^2 = x^2 + y^2$ $25 + 144 = 169$ $r = 13$ $\cos\alpha = -\frac{5}{13}$ Answer only: full marks	✓ $OP^2 = 25 + 144$ ✓ $OP = 13$ ✓ answer (3) ✓ $r^2 = 25 + 144$ ✓ $r = 13$ ✓ answer (3)
10.2	$\tan(180° - \alpha)$ $= -\tan\alpha$ $= -\frac{12}{5}$	✓ $-\tan\alpha$ ✓ answer (2)
10.3	$\sin(30° - \alpha)$ $= \sin 30° \cos\alpha - \cos 30° \sin\alpha$ $= \left(\frac{1}{2}\right)\left(\frac{-5}{13}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{-12}{13}\right)$ $= \frac{-5 + 12\sqrt{3}}{26}$	✓ expansion ✓ $\left(\frac{1}{2}\right)\left(\frac{-5}{13}\right)$ ✓ $\left(\frac{\sqrt{3}}{2}\right)\left(\frac{-12}{13}\right)$ (3) **[8]**

QUESTION/*VRAAG* 11

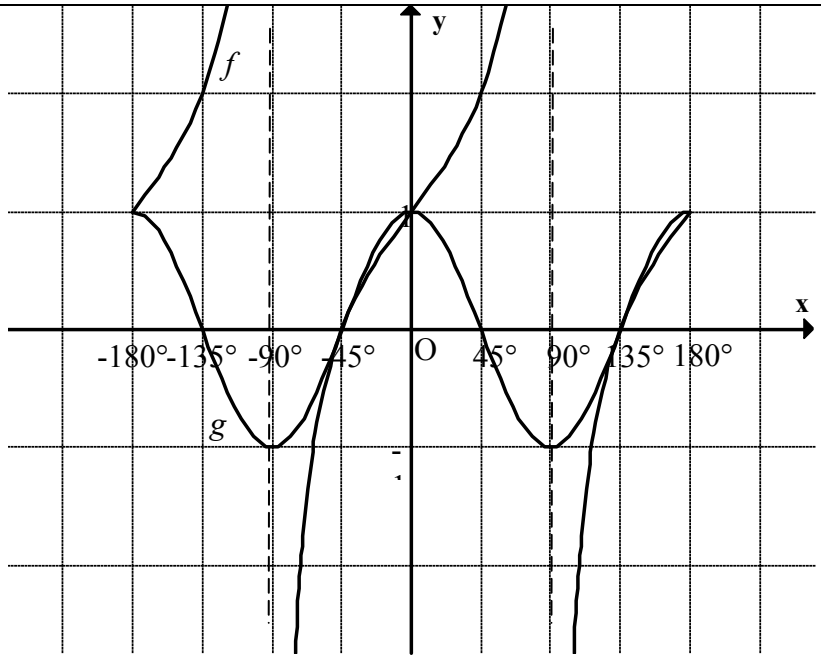
11.1	$LHS = \dfrac{\cos^2(90° + \theta)}{\cos(-\theta) + \sin(90° - \theta).\cos\theta}$ $= \dfrac{(-\sin\theta)^2}{\cos\theta + \cos\theta.\cos\theta}$ $= \dfrac{1 - \cos^2\theta}{\cos\theta + \cos^2\theta}$ $= \dfrac{(1-\cos\theta)(1+\cos\theta)}{\cos\theta(1+\cos\theta)}$ $= \dfrac{1-\cos\theta}{\cos\theta}$ $= \dfrac{1}{\cos\theta} - 1$ $= RHS$	✓ $\cos^2(90°+\theta) = \sin^2\theta$ ✓ $\sin(90° - \theta) = \cos\theta$ ✓ $\cos(-\theta) = \cos\theta$ ✓ $1 - \cos^2\theta$ ✓ factors ✓ $\dfrac{1-\cos\theta}{\cos\theta}$ (6)
11.2	$\tan x.\sin x + \cos x.\tan x = 0$ $\tan x(\sin x + \cos x) = 0$ $\tan x = 0$ *or* $\sin x + \cos x = 0$ $\sin x = -\cos x$ $\tan x = -1$ $x = 0° + k.180°;\ k \in Z$ *or* $x = 135° + k.180°;\ k \in Z$ **OR** $\dfrac{\sin x}{\cos x}.\sin x + \cos x.\dfrac{\sin x}{\cos x} = 0$ $\dfrac{\sin^2 x}{\cos x} + \dfrac{\cos x.\sin x}{\cos x} = 0$ $\sin^2 x + \cos x.\sin x = 0$ $\sin x(\sin x + \cos x) = 0$ $\sin x + \cos x = 0$ $\sin x = 0$ *or* $\tan x = -1$ $x = 0° + k.180°;\ k \in Z$ *or* $x = 135° + k.180°;\ k \in Z$ $(\sin x + \cos x)^2 = 0$ $1 + \sin 2x = 0$ $\sin 2x = -1$ ✓ $x = 135° + k.180°$ **OR** $\tan x.\sin x + \cos x.\tan x = 0$ $(\cos x \neq 0)$ $\tan^2 x + \tan x = 0$ $\tan x(\tan x + 1) = 0$ $\tan x = 0$ *or* $\tan x + 1 = 0$ $\tan x = -1$ $x = 0° + k.180°;\ k \in Z$ *or* $x = 135° + k.180°;\ k \in Z$	✓ factorising ✓ $\tan x = 0$ and $\sin x + \cos x = 0$ ✓ $\tan x = -1$ ✓ $x = 0°$ ✓ $x = 135°$ or $-45°$ ✓ $k.180°$ ✓ $k \in Z$ (7) ✓ factorising ✓ $\sin x = 0$ and ✓ $\tan x = -1$ ✓ $x = 0°$ ✓ $x = 135°$ or $-45°$ ✓ $k.180°$ ✓ $k \in Z$ (7) ✓ factorising ✓ $\tan x = 0$ and $\tan x + 1 = 0$ ✓ $\tan x = -1$ ✓ $x = 0°$ ✓ $x = 135°$ or $-45°$ ✓ $k.180°$ ✓ $k \in Z$ (7)

11.3.1	$2\sin^2 3x - \sin^2 x - \cos^2 x$ $= 2\sin^2 3x - (\sin^2 x + \cos^2 x)$ $= 2\sin^2 3x - 1$ $= -\cos 6x$ OR $2\sin^2(2x+x) - \sin^2 x - \cos^2 x$ $= 2(\sin 2x.\cos x + \cos 2x.\sin x)^2 - (\sin^2 x + \cos^2 x)$ $= 2((2\sin x.\cos x)\cos x + (1-2\sin^2 x)\sin x)^2 - 1$ $= 2(2\sin x.\cos^2 x + \sin x - 2\sin^3 x)^2 - 1$ $= 2(2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x)^2 - 1$ $= 2(2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x)^2 - 1$ $= 2(-4\sin^3 x + 3\sin x)^2 - 1$ $= 2(16\sin^6 x - 24\sin^4 x + 9\sin^2 x) - 1$ $= 32\sin^6 x - 48\sin^4 x + 18\sin^2 x - 1$	✓ $-(\sin^2 x + \cos^2 x)$ ✓ 1 ✓ $2\sin^2 3x - 1$ (3) ✓ $-(\sin^2 x + \cos^2 x)$ ✓1 ✓ answer (3)
11.3.2	Max value = 1	✓1 (1)
11.4.1 (a)	$p = \cos\alpha + \sin\alpha$ $q = \cos\alpha - \sin\alpha$ $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $= (\cos\alpha + \sin\alpha)(\cos\alpha - \sin\alpha)$ $= pq$ **OR** $\sin\alpha = \frac{p-q}{2}$ $\cos 2\alpha = 1 - 2\sin^2\alpha$ $= 1 - 2(\frac{p-q}{2})^2$ **OR** $\cos\alpha = \frac{p+q}{2}$ $\cos 2\alpha = 2\cos^2\alpha - 1$ $= 2(\frac{p+q}{2})^2 - 1$ **OR** $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $= (\frac{p+q}{2})^2 - (\frac{p-q}{2})^2$	✓ expansion ✓ factorise ✓ answer (3) ✓ $\frac{p-q}{2}$ ✓ expansion ✓ answer (3) ✓ $\frac{p+q}{2}$ ✓ expansion ✓ answer (3)

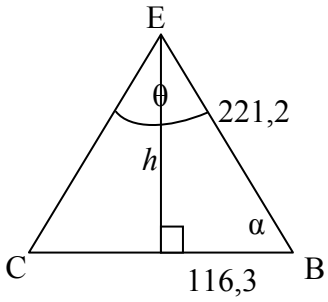
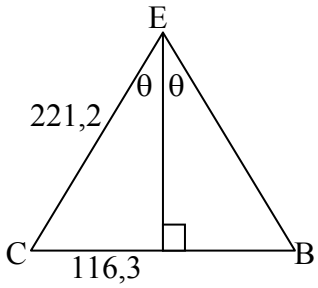
		✓ expansion ✓ $\frac{p-q}{2}$ ✓ $\frac{p+q}{2}$ (3)
11.4.1 (b)	$p+q=2\cos\alpha \quad \therefore \cos\alpha=\frac{p+q}{2}$ $p-q=2\sin\alpha \quad \therefore \sin\alpha=\frac{p-q}{2}$ $\tan\alpha=\frac{\sin\alpha}{\cos\alpha}$ $=\frac{2\sin\alpha}{2\cos\alpha}$ $=\frac{p-q}{p+q}$ **OR** $\cos\alpha+\sin\alpha=p$ $\cos\alpha-\sin\alpha=q$ $\Rightarrow 2\cos\alpha=p+q$ $\cos\alpha=\frac{p+q}{2}$ $y^2=2^2-(p+q)^2$ $y=\sqrt{4-(p+q)^2}$ $\therefore \tan\alpha=\frac{\sqrt{4-(p+q)^2}}{p+q}$  **OR** $\cos\alpha+\sin\alpha=p$ $\cos\alpha-\sin\alpha=q$ $\Rightarrow 2\sin\alpha=p-q$ $\sin\alpha=\frac{p-q}{2}$  $x^2=2^2-(p-q)^2$ $x=\sqrt{4-(p-q)^2}$ $\therefore \tan\alpha=\frac{p-q}{\sqrt{4-(p-q)^2}}$ OR	✓ $p+q$ ✓ $p-q$ ✓ identity ✓ answer (4) ✓ $2\cos\alpha=p+q$ ✓ sketch ✓ $y=\sqrt{4-(p+q)^2}$ ✓ answer (4) ✓ $2\sin\alpha=p-q$ ✓ sketch ✓ $x=\sqrt{4-(p-q)^2}$ ✓ answer (4)

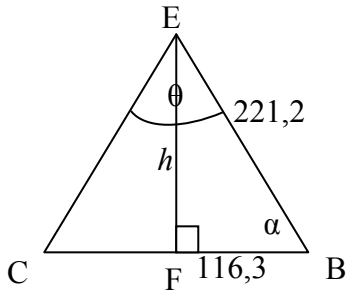
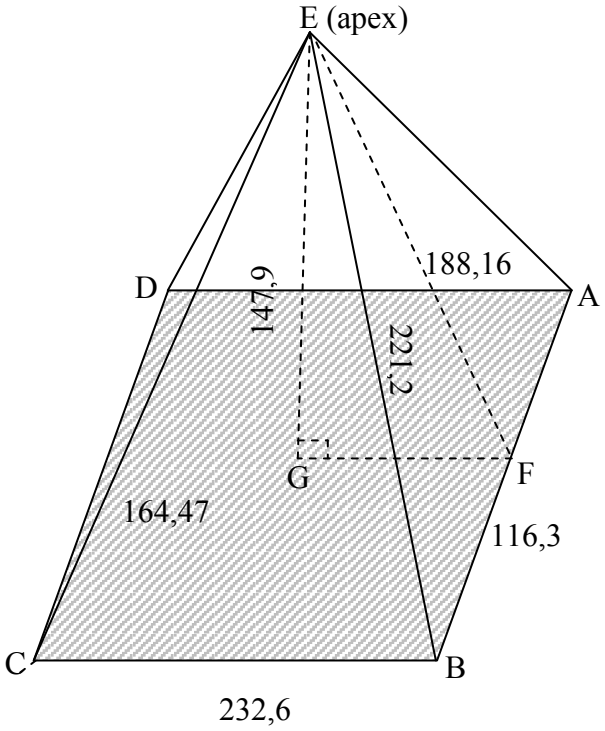
	$\cos 2\alpha = 1 - 2\sin^2\alpha$ $\sin^2\alpha = \frac{1-pq}{2}$ $\cos 2\alpha = 2\cos^2\alpha - 1$ $\cos^2\alpha = \frac{pq+1}{2}$ $(\tan\alpha)^2 = \left(\frac{\sin\alpha}{\cos\alpha}\right)^2$ $\tan^2\alpha = \frac{1-pq}{1+pq}$ $\therefore \tan\alpha = \sqrt{\frac{1-pq}{1+pq}}$	✓ $\sin^2\alpha = \frac{1-pq}{2}$ ✓ $\cos^2\alpha = \frac{pq+1}{2}$ ✓ $(\tan\alpha)^2 = \left(\frac{\sin\alpha}{\cos\alpha}\right)^2$ ✓ $\tan\alpha = \sqrt{\frac{1-pq}{1+pq}}$ (4)
11.4.2	$\frac{p}{2q} - \frac{q}{2p}$ $= \frac{p^2-q^2}{2pq}$ $= \frac{(p+q)(p-q)}{2pq}$ $= \frac{(2\cos\alpha)(2\sin\alpha)}{2\cos 2\alpha}$ $= \frac{4\sin\alpha\cos\alpha}{2\cos 2\alpha}$ $= \frac{2\sin 2\alpha}{2\cos 2\alpha}$ $= \tan 2\alpha$ **OR** $\frac{p}{2q} - \frac{q}{2p}$ $= \frac{\cos\alpha+\sin\alpha}{2(\cos\alpha-\sin\alpha)} - \frac{\cos\alpha-\sin\alpha}{2(\cos\alpha+\sin\alpha)}$ $= \frac{(\cos\alpha+\sin\alpha)^2-(\cos\alpha-\sin\alpha)^2}{2(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)}$ $= \frac{\cos^2\alpha+2\sin\alpha\cos\alpha+\sin^2\alpha-(\cos^2\alpha-2\sin\alpha\cos\alpha+\sin^2\alpha)}{2(\cos^2\alpha-\sin^2\alpha)}$ $= \frac{4\sin\alpha\cos\alpha}{2\cos 2\alpha}$ $= \frac{2\sin 2\alpha}{2\cos 2\alpha}$ $= \tan 2\alpha$	✓ $\frac{p^2-q^2}{2pq}$ ✓ factorising substituting ✓ from 11.4.1(a) ✓ and 11.4.1(b) ✓ 2 sin 2α ✓ tan 2α (6) ✓ substitution ✓ single fraction ✓ 4 sinα cosα ✓ 2 cos2α ✓ 2sin2α ✓ tan2α (6) **[30]**

QUESTION/*VRAAG* 12

12.1		$y = \tan x + 1$ ✓ asymptotes and shape for whole domain ✓ y intercept ✓ x intercepts $y = \cos 2x$ ✓ x intercepts ✓ y intercept/TP ✓ minimum values (6)
12.2	Period of g is 180°.	✓ 180° (1)
12.3	Reflected about the x-axis and then translated by 10° to the left/ *refleksie om die x-as en dan 'n translasie van 10° links*. **OR** Translated by 10° to the left and then reflected about the x-axis/ *Translasie van 10° links en dan 'n refleksie om die x-as.*	✓ reflected about x –axis/*refleksie om x-as* ✓ 10° to the left *10° na links* (2) ✓ 10° to the left *10° na links* ✓ reflected about x –axis/*refleksie om x-as* (2)
12.4	f is always increasing $\therefore f'(x) > 0$ always $\therefore g(x) > 0$ $\therefore 0° < x < 45°$ or $135° < x \leq 180°$ **OR** $\therefore x \in (0°; 45°)$ *or* $(135°; 180°]$	✓ critical values 0° and 45° ✓ inequality ✓ critical values 135° and 180° ✓ inequality (4) **[13]**

QUESTION/*VRAAG* 13

13.1	In ΔCEB: $BC^2 = EC^2 + EB^2 - 2(EC)(EB)\cos C\hat{E}B$ $(232,6)^2 = (221,2)^2 + (221,2)^2 - 2(221,2)(221,2)\cos C\hat{E}B$ $\cos C\hat{E}B = \dfrac{2(221,2)^2 - (232,6)^2}{2(221,2)^2}$ $= 0,447.....$ $C\hat{E}B = 63,44°$	✓ substitution into correct formula ✓ 0,447.... ✓ 63,44° (3)
	OR $\cos\alpha = \dfrac{116,3}{221,2}$ $\alpha = 58,28°$ $\theta = 180° - 2(58,28°) = 63,44°$  	✓ substitution into correct definition ✓ α = 58,28° ✓ 63,44° (3)
	OR $\sin\theta = \dfrac{116,3}{221,2}$ $\theta = 31,72°$ $A\hat{E}B = 2\theta = 2(31,72°) = 63,44°$  	✓ substitution into correct formula ✓ θ = 31,72° ✓ 63,44° (3)
13.2	$EF^2 = EB^2 - BF^2$ $= (221,2)^2 - (116,3)^2$ $= 35403,75$ $EF = 188,16\,m$ $\cos E\hat{F}G = \dfrac{GF}{EF}$ $= \dfrac{116,3}{188,16}$ $= 0,618....$ $E\hat{F}G = 51,82°$ **OR**	✓ using Pythagoras correctly ✓ BF = 116,3 ✓ 188,16 ✓ using $\cos E\hat{F}G$ ✓ $\dfrac{116,3}{188,16}$ ✓ 51,82° (6)

	$\sin\alpha = \dfrac{h}{221{,}2}$ $h = 221{,}2\sin 58{,}28°$ $= 188{,}158...$ In ΔEFG : $\cos E\hat{F}G = \dfrac{116{,}3}{188{,}158...}$ $= 0{,}61809...$ $E\hat{F}G = 51{,}82°$   	✓ $\sin\alpha = \dfrac{h}{221{,}2}$ ✓ h subject ✓ $h = 188{,}158...$ ✓ using $\cos E\hat{F}G$ ✓ $\dfrac{116{,}3}{188{,}16}$ ✓ $51{,}82°$ (6) **[9]**

TOTAL/*TOTAAL*: 150

PolyMathic

Vraestel 3

Mei/Junie

Eksamen

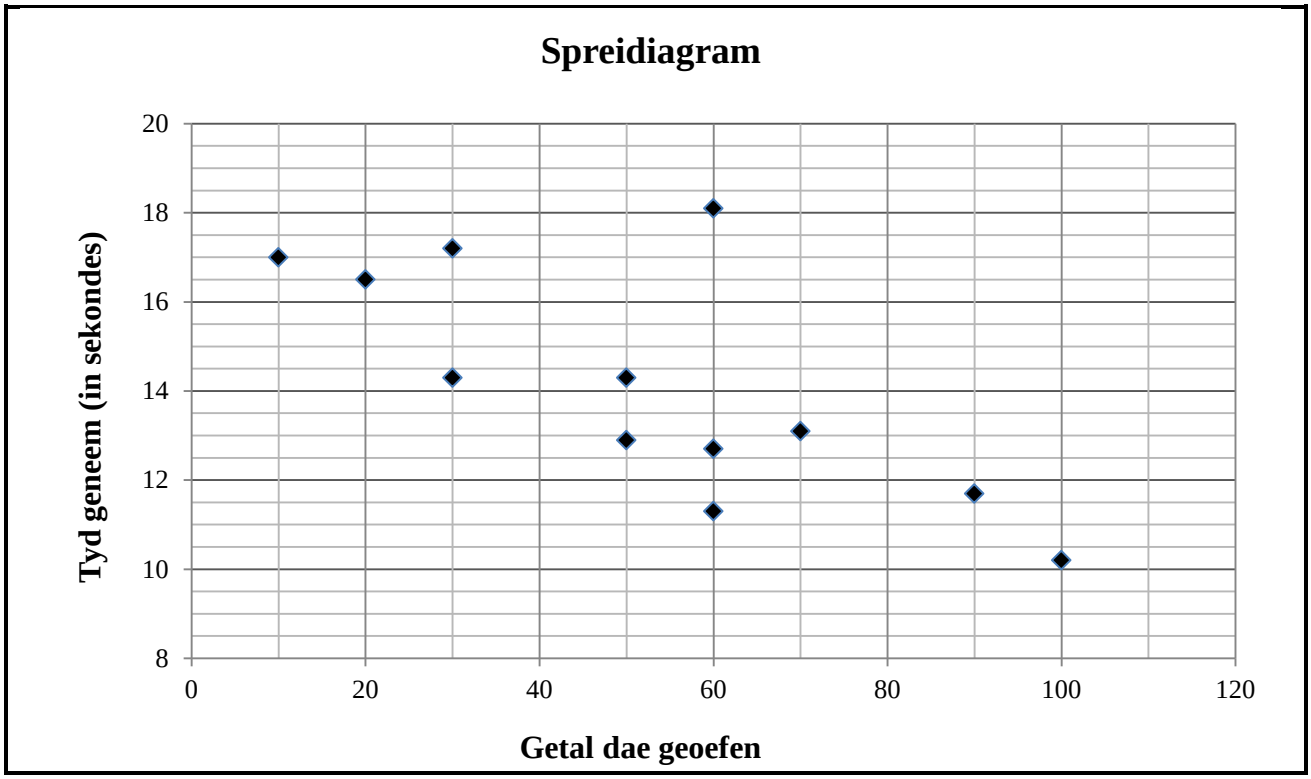
PolyMathic

Graad 12 Junie Eksamen
Totaal: 150
Tyd: 3ure

VRAAG 1

Twaalf atlete het geoefen om aan die proewe van die plaaslike atletiekklub se 100 m-naelloop-item deel te neem. Sommige van hulle het die oefeninge meer ernstig as die ander opgeneem. Die volgende tabel en spreidiagram toon die getal dae wat 'n atleet geoefen het en die tyd wat dit geneem het om die naelloop te voltooi. Die tye wat aangeteken is, in sekondes, is tot een desimale plek afgerond.

Getal dae geoefen	50	70	10	60	60	20	50	90	100	60	30	30
Tyd geneem (in sekondes)	12,9	13,1	17,0	11,3	18,1	16,5	14,3	11,7	10,2	12,7	17,2	14,3



1.1 Bespreek die neiging van die data wat versamel is. (1)

1.2 Identifiseer enige uitskieter(s) in die data. (1)

1.3 Bereken die vergelyking van die kleinstekwadrate-regressielyn. (4)

1.4 Voorspel in watter tyd 'n atleet wat 45 dae geoefen het, die 100 m-naelloop sal voltooi. (2)

1.5 Bereken die korrelasiekoëffisiënt. (2)

1.6 Lewer kommentaar op die sterkte van die verband tussen die veranderlikes. (1)

[11]

VRAAG 2

Die tabel hieronder toon die tyd (in uur) wat leerders tussen 14 en 18 jaar gedurende 3 weke van die vakansie voor die televisie deurgebring het.

Tyd (uur)	**Kumulatiewe frekwensie**
$0 \leq t < 20$	25
$20 \leq t < 40$	69
$40 \leq t < 60$	129
$60 \leq t < 80$	157
$80 \leq t < 100$	166
$100 \leq t < 120$	172

2.1 Skets 'n ogief (kumulatiewe frekwensiekurwe) op DIAGRAMVEL 1 om die data hierbo voor te stel. (3)

2.2 Skryf die modale klas van die data neer. (1)

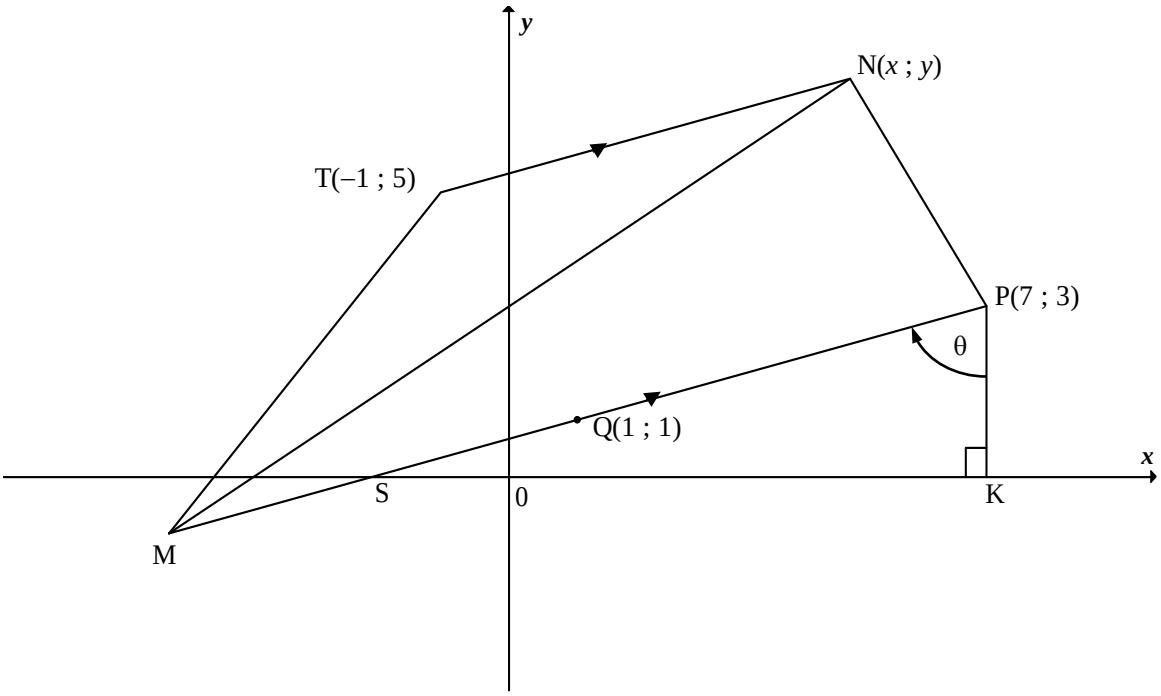
2.3 Gebruik die ogief (kumulatiewe frekwensiekurwe) om die getal leerders te bepaal wat meer as 80% van die tyd televisie gekyk het. (2)

2.4 Bepaal die gemiddelde tyd (in uur) wat die leerders tydens 3 weke van die vakansie voor die televisie deurgebring het. (4)

[10]

VRAAG 3

In die diagram hieronder is M, T(–1 ; 5), N(x ; y) en P(7 ; 3) hoekpunte van trapesium MTNP met TN | | MP. Q(1 ; 1) is die middelpunt van MP. PK is 'n vertikale lyn en $S\hat{P}K = \theta$. Die vergelyking van NP is $y = -2x + 17$.



3.1 Skryf die koördinate van K neer. (1)

3.2 Bepaal die koördinate van M. (2)

3.3 Bepaal die gradiënt van PM. (2)

3.4 Bereken die grootte van θ. (3)

3.5 Vervolgens, of andersins, bereken die lengte van PS. (3)

3.6 Bepaal die koördinate van N. (5)

3.7 As A(a ; 5) in die Cartesiese vlak lê:

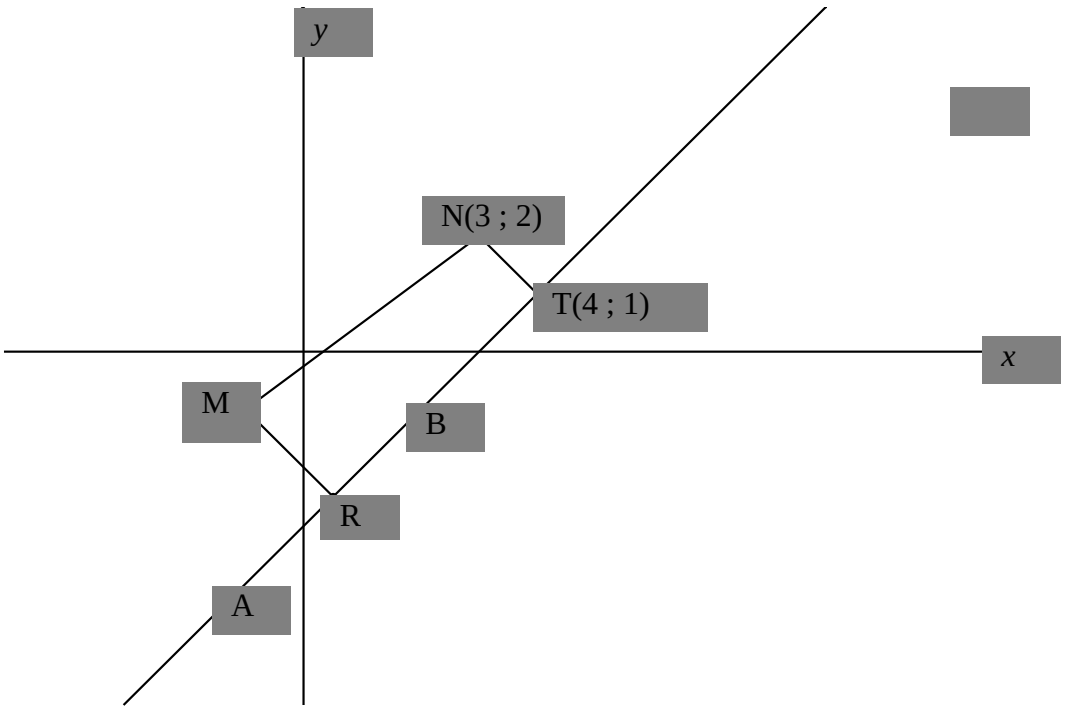
3.7.1 Skryf die vergelyking neer van die reguitlyn wat die moontlike posisies van A voorstel. (1)

3.7.2 Vervolgens, of andersins, bereken die waarde(s) van a waarvoor $T\hat{A}Q = 45°$. (5)

[22]

VRAAG 4

In die diagram hieronder word die sirkel, met middelpunt M, se vergelyking gegee deur $(x + 1)^2 + (y + 1)^2 = 9$. R is 'n punt op koord AB sodat MR koord AB halveer. ABT is 'n raaklyn aan die sirkel met middelpunt N(3 ; 2) by punt T(4 ; 1).



4.1 Skryf die koördinate van M neer. (1)

4.2 Bepaal die vergelyking van AT in die vorm $y = mx + c$. (5)

4.3 Indien verder gegee word dat MR = $\frac{\sqrt{10}}{2}$ eenhede, bereken die lengte van AB.
Laat jou antwoord in vereenvoudigde wortelvorm. (4)

4.4 Bereken die lengte van MN. (2)

4.5 'n Ander sirkel met middelpunt N raak die sirkel met middelpunt M by punt K. Bepaal die vergelyking van die nuwe sirkel. Skryf jou antwoord in die vorm $x^2 + y^2 + Cx + Dy + E = 0$. (3)

[15]

VRAAG 5

5.1 Gegee dat $\sin\alpha = -\frac{4}{5}$ en $90° < \alpha < 270°$.

SONDER om 'n sakrekenaar te gebruik, bepaal die waarde van elk van die volgende, in die eenvoudigste vorm:

5.1.1 $\sin(-\alpha)$ (2)

5.1.2 $\cos\alpha$ (2)

5.1.3 $\sin(\alpha - 45°)$ (3)

5.2 Beskou die identiteit: $\frac{8\sin(180° - x)\cos(x - 360°)}{\sin^2 x - \sin^2(90° + x)} = -4\tan 2x$

5.2.1 Bewys die identiteit. (6)

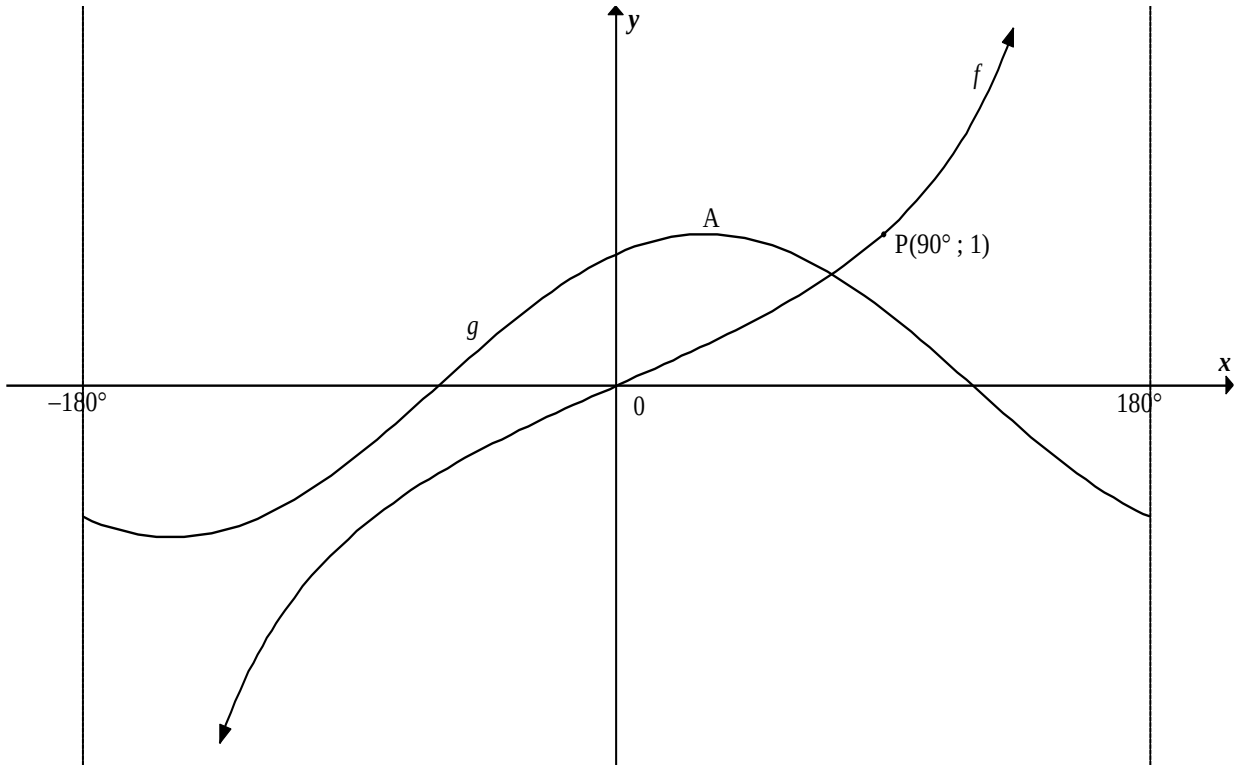
5.2.2 Vir watter waarde(s) van x in die interval $0° < x < 180°$ is die identiteit nie gedefinieerd nie? (2)

5.3 Bepaal die algemene oplossing van $\cos 2\theta + 4\sin^2\theta - 5\sin\theta - 4 = 0$. (7)

[22]

VRAAG 6

In die diagram hieronder is die grafieke van $f(x) = \tan bx$ en $g(x) = \cos(x - 30°)$ op dieselfde assestelsel geskets vir $-180° \leq x \leq 180°$. Die punt P(90° ; 1) lê op f. Gebruik die diagram om die volgende vrae te beantwoord.



6.1 Bepaal die waarde van b. (1)

6.2 Skryf die koördinate van A, 'n draaipunt van g, neer. (2)

6.3 Skryf die vergelyking van die asimptoot/asimptote van $y = \tan b(x + 20°)$, vir $x \in [-180°;180°]$, neer. (1)

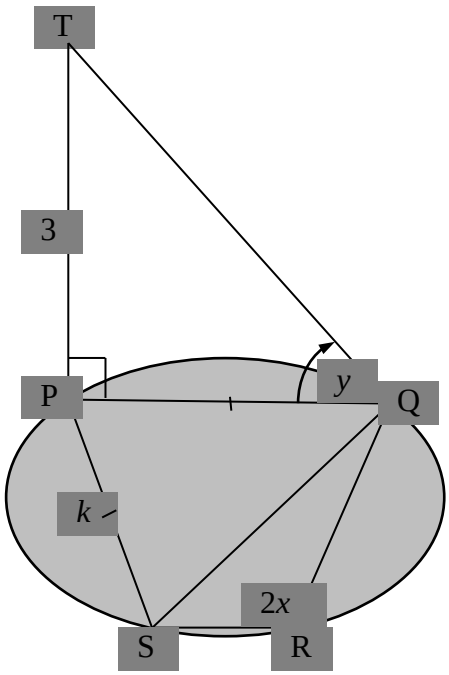
6.4 Bepaal die waardeversameling van h as $h(x) = 2g(x) + 1$. (2)

[6]

VRAAG 7

7.1 Bewys dat in enige skerphoekige ΔABC is $\frac{\sin A}{a} = \frac{\sin B}{b}$. (5)

7.2 Die raamwerk van 'n konstruksie bestaan uit 'n koordevierhoek PQRS in die horisontale vlak en 'n vertikale paal TP soos in die figuur aangetoon. Die hoogtehoek van T, soos gemeet vanaf Q, is $y°$. PQ = PS = k eenhede, TP = 3 eenhede en $S\hat{R}Q = 2x°$.



7.2.1 Toon aan, met redes, dat $P\hat{S}Q = x$. (2)

7.2.2 Bewys dat $SQ = 2k \cos x$. (4)

7.2.3 Bewys vervolgens dat $SQ = \frac{6\cos x}{\tan y}$. (2)

[13]

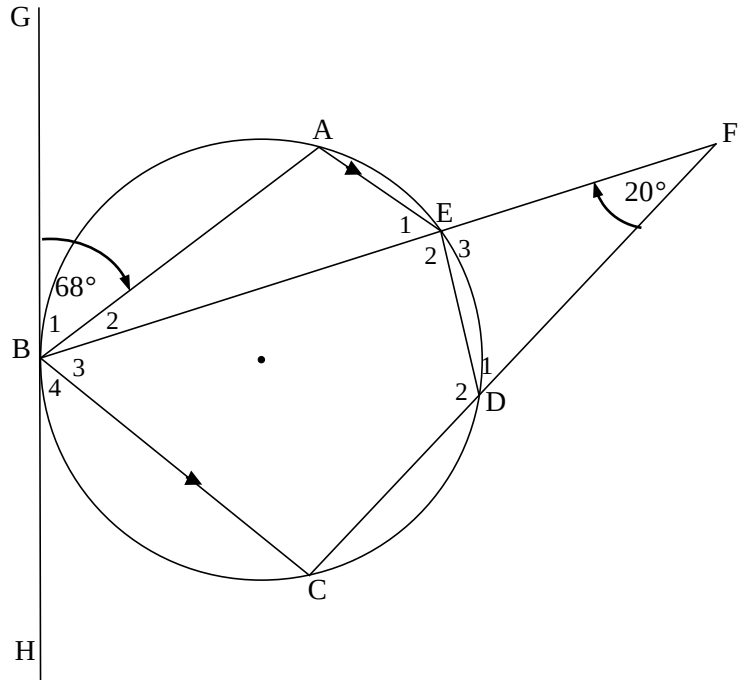
Gee redes vir jou bewerings in VRAAG 8, 9 en 10.

VRAAG 8

8.1 Voltooi die volgende bewering:

Die hoek tussen 'n raaklyn en 'n koord by die raakpunt is gelyk aan ... (1)

8.2 In die diagram is A, B, C, D en E punte op die omtrek van die sirkel sodat AE | | BC. BE en CD verleng ontmoet in F. GBH is 'n raaklyn aan die sirkel by B. $\hat{B}_1 = 68°$ en $\hat{F} = 20°$.



Bepaal die grootte van elk van die volgende:

8.2.1 $\hat{E}_1$ (2)

8.2.2 $\hat{B}_3$ (1)

8.2.3 $\hat{D}_1$ (2)

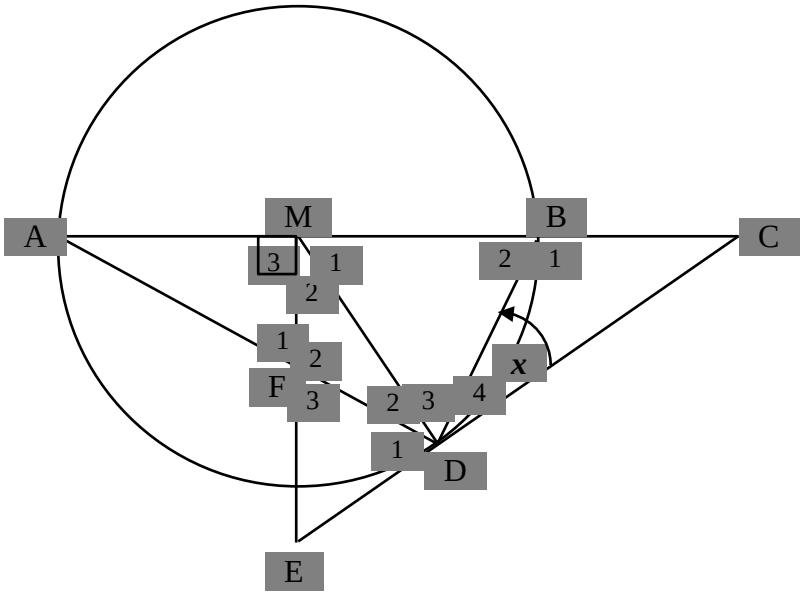
8.2.4 $\hat{E}_2$ (1)

8.2.5 $\hat{C}$ (2)

[9]

VRAAG 9

In die diagram is M die middelpunt van die sirkel en middellyn AB is verleng na C. ME is loodreg op AC getrek sodat CDE 'n raaklyn aan die sirkel by D is. ME en koord AD sny in F. MB = 2BC.



9.1 As $\hat{D}_4 = x$, skryf, met redes, TWEE ander hoeke neer wat gelyk is aan x. (3)

9.2 Bewys dat CM 'n raaklyn by M is aan die sirkel wat deur M, E en D gaan. (4)

9.3 Bewys dat FMBD 'n koordevierhoek is. (3)

9.4 Bewys dat $DC^2 = 5BC^2$. (3)

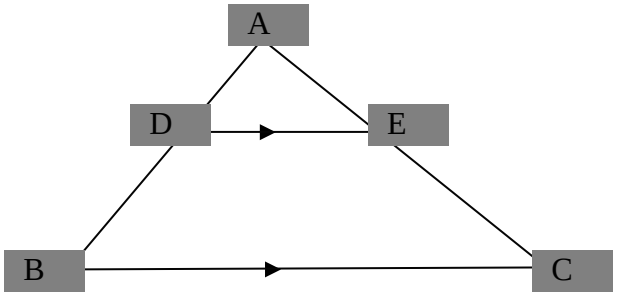
9.5 Bewys dat $\Delta DBC \,|||\, \Delta DFM$. (4)

9.6 Bepaal vervolgens die waarde van $\frac{DM}{FM}$. (2)

[19]

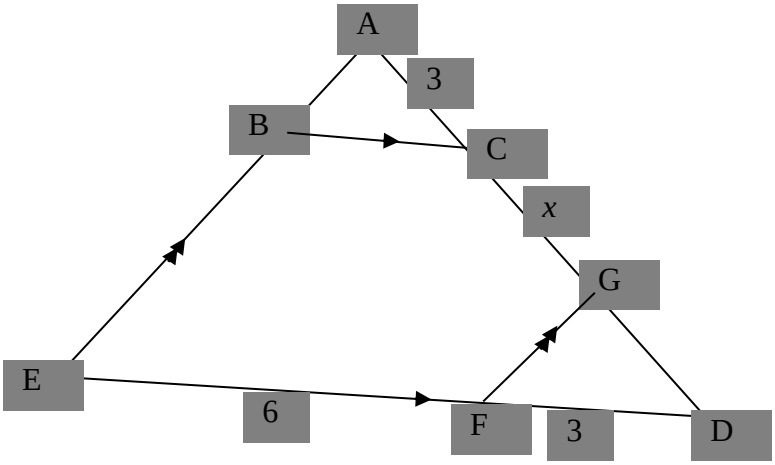
VRAAG 10

10.1 In die diagram lê punt D en E op onderskeidelik sy AB en AC van ΔABC sodat DE | | BC. Gebruik Euklidiesemeetkunde-metodes om die stelling te bewys wat beweer dat $\frac{AD}{DB} = \frac{AE}{EC}$.



(6)

10.2 In die diagram is ADE 'n driehoek met BC | | ED en AE | | GF. Verder word ook gegee dat AB : BE = 1 : 3, AC = 3 eenhede, EF = 6 eenhede, FD = 3 eenhede en CG = x eenhede.



Bereken, met redes:

10.2.1 Die lengte van CD (3)

10.2.2 Die waarde van x (4)

10.2.3 Die lengte van BC (5)

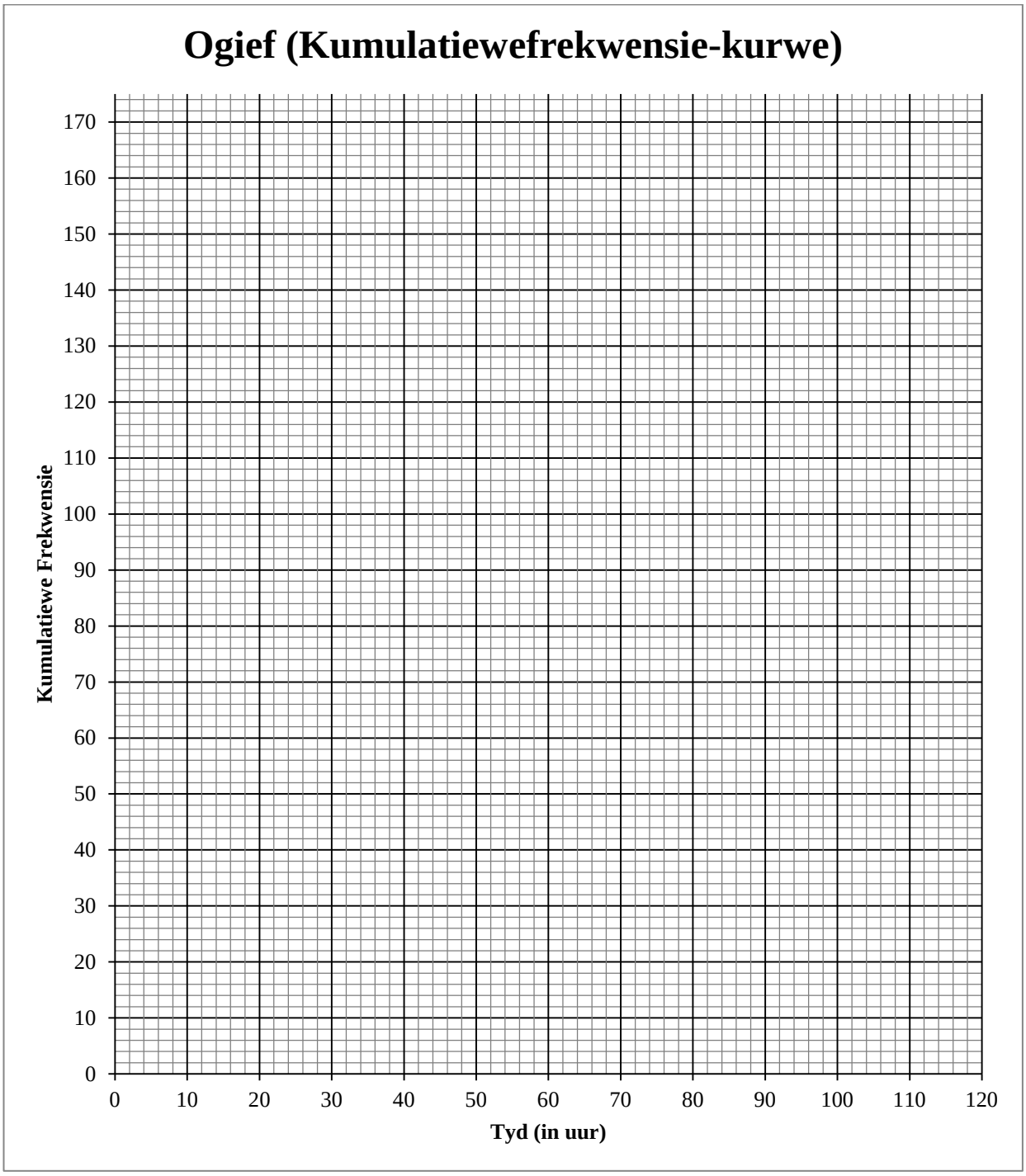
10.2.4 Die waarde van $\frac{\text{area } \Delta ABC}{\text{area } \Delta GFD}$ (5)

[23]

TOTAAL: 150

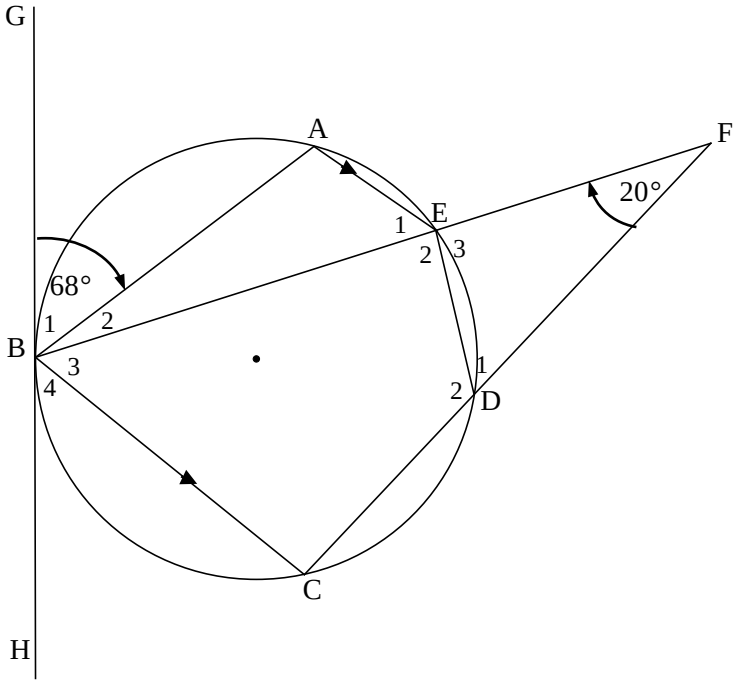
DIAGRAMVEL 1

VRAAG 2.1

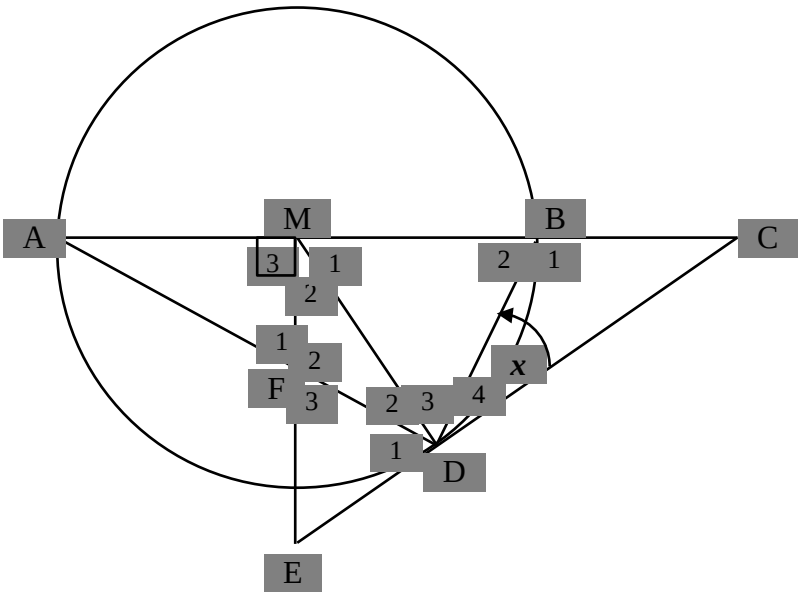


DIAGRAMVEL 2

VRAAG 8.2

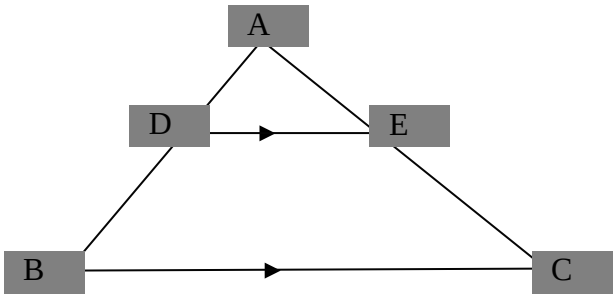


VRAAG 9

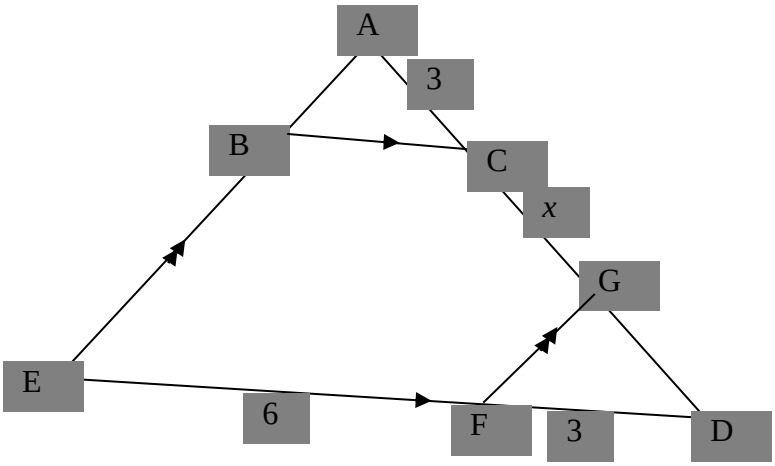


DIAGRAMVEL 3

VRAAG 10.1



VRAAG 10.2



Memo

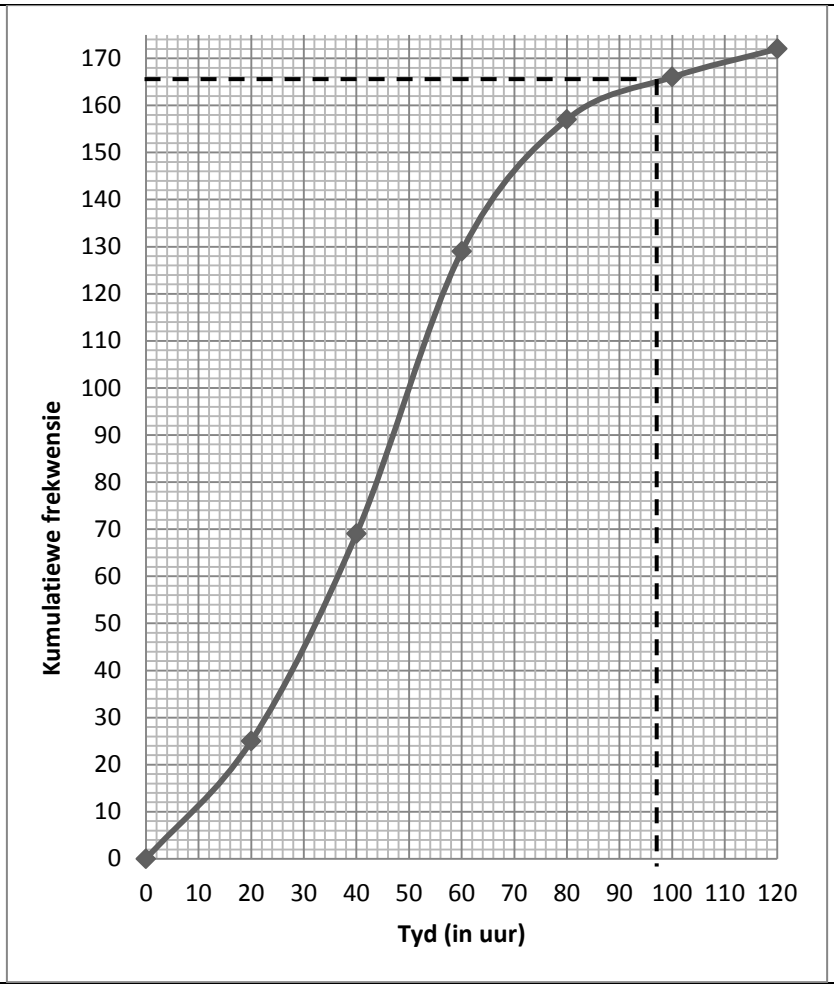
LET WEL:

- *Indien ‘n kandidaat ‘n vraag TWEE keer beantwoord het, merk slegs die EERSTE poging.*
- *Indien ‘n kandidaat ‘n poging om ‘n vraag te beantwoord gekanselleer het en die vraag nie weer gedoen het nie, merk die gekanselleerde poging.*
- *Volgehoue akkuraatheid is van toepassing in **ALLE** aspekte van die nasien-memorandum.*
- *Aanvaar van antwoorde/waardes om ‘n probleem op te los, is ONAANVAARBAAR.*

VRAAG 1

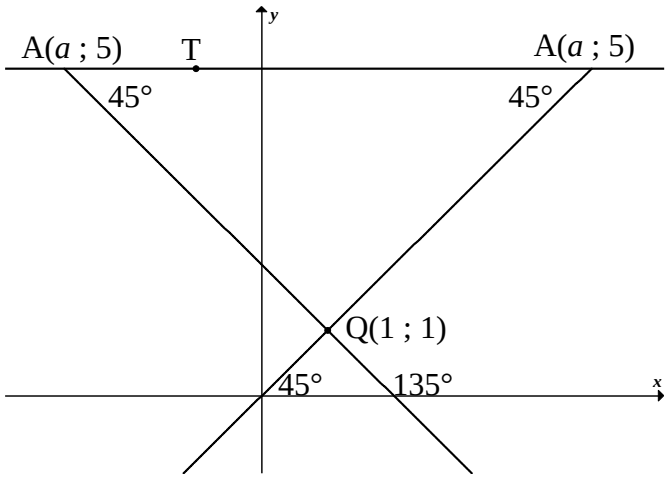
1.1	Indien die aantal dae wat ‘n atleet oefen toeneem, neem die tyd waarin die 100m naelloop afgelê word, af. **OF** Indien die aantal dae wat ‘n atleet oefen afneem, neem die tyd waarin die 100m naelloop afgelê word, toe. **OF** Hoe meer dae ‘n atleet oefen, hoe korter is die tyd wat hy die 100m naelloop aflê.	✓ verduideliking (1)
1.2	(60 ; 18,1)	✓ (1)
1.3	$a = 17{,}81931464\ldots$ $b = -0{,}070685358\ldots$ $\therefore\ \hat{y} = -0{,}07x + 17{,}82$	✓✓ a ✓ b ✓ vergelyking (4)
1.4	$\therefore\ \hat{y} \approx -0{,}07(45) + 17{,}82$ $\approx 14{,}67$ sekondes	✓ substitusie ✓ antwoord (2)
1.5	$r = -0{,}74$ (–0,740772594...)	✓✓ r (2)
1.6	Daar is ‘n redelike sterk verwantskap tussen die veranderlikes.	✓ redelik sterk (1) **[11]**

VRAAG 2

2.1		✓anker by 0 ✓plot by boonste limiete ✓ gladde kurwe (3)
2.2	$40 \leq t < 60$	✓ klas (1)
2.3	(96 ; 164) $\therefore$ 172 – 164 = 8 leerders	✓164 ✓ 8 (2)
2.4	Frekwensie: 25; 44; 60; 28; 9; 6 gemiddelde = $\frac{25\times10+44\times30+60\times50+28\times70+9\times90+6\times110}{172}$ $= \frac{8000}{172}$ = 46,51 uur	✓frekwensie ✓ middelpunte ✓ $\frac{8000}{172}$ ✓ antwoord (4) **[10]**

VRAAG 3

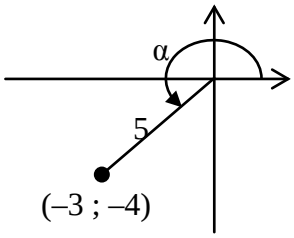
3.1	K(7 ; 0)	✓ antwoord (1)
3.2	$1 = \frac{x_M + 7}{2}$ *en* $1 = \frac{y_M + 3}{2}$ $\therefore$ M(– 5 ; – 1)	✓ x ✓ y (2)
3.3	$m_{PM} = \frac{3-1}{7-1}$ $= \frac{1}{3}$	✓ substitusie ✓antwoord (2)
3.4	$\tan \text{P}\hat{\text{S}}\text{K} = m_{PM} = \frac{1}{3}$ $\text{P}\hat{\text{S}}\text{K} = \tan^{-1}\left(\frac{1}{3}\right) = 18{,}43°$ $\therefore\ \theta = 180° - 90° - 18{,}43° = 71{,}57°$	✓ $\tan \text{P}\hat{\text{S}}\text{K} = m_{PM}$ ✓ $\text{P}\hat{\text{S}}\text{K}$ ✓ θ (3)
3.5	$\cos 71{,}57° = \frac{\text{PK}}{\text{PS}} = \frac{3}{\text{PS}}$ $\text{PS} = \frac{3}{\cos 71{,}57°}$ = 9,49 eenhede **OF** $\sin 18{,}43° = \frac{\text{PK}}{\text{PS}} = \frac{3}{\text{PS}}$ $\text{PS} = \frac{3}{\sin 18{,}43°}$ = 9,49 eenhede	✓ korrekte verhouding ✓ PS onderwerp ✓ antwoord (3) ✓ korrekte verhouding ✓ PS onderwerp ✓ antwoord (3)
3.6	N(x ; –2x + 17) $m_{TN} = m_{PM}$ (TN | | PM) $\frac{-2x + 17 - 5}{x - (-1)} = \frac{1}{3}$ $-6x + 36 = x + 1$ $-7x = -35$ $x = 5$ $\therefore\ y = -2(5) + 17 = 7$ $\therefore$ N(5 ; 7) **OF**	✓ N in terme van x ✓ gelyke gradiënte ✓ substitusie ✓ x -waarde ✓ y -waarde (5)

	$m_{TM} = \frac{1}{3}$ (TN | | PM) vergelyking van TM: $y - y_1 = \frac{1}{3}(x - x_1)$ $y - 5 = \frac{1}{3}(x - (-1))$ $y - 5 = \frac{1}{3}x + \frac{1}{3}$ $y = \frac{1}{3}x + 5\frac{1}{3}$ **OF** $y = \frac{1}{3}x + c$ $5 = \frac{1}{3}(-1) + c$ $5\frac{1}{3} = c$ $y = \frac{1}{3}x + 5\frac{1}{3}$ $-2x + 17 = \frac{1}{3}x + 5\frac{1}{3}$ $-2\frac{1}{3}x = -11\frac{2}{3}$ $x = 5$ $\therefore y = -2(5) + 17 = 7$ $\therefore$ N(5 ; 7)	✓ m_{TM} ✓ vergelyking van TM ✓ stel gelyk aan mekaar ✓ x -waarde ✓ y -waarde (5)
3.7.1	$y = 5$	✓ vergelyking (1)
3.7.2	 gradiënt van AQ = tan 45° of tan 135° = 1 of –1 $m_{AQ} = \frac{5-1}{a-1} = \pm 1$ $\therefore a - 1 = 4$ of -4 $\therefore a = 5$ of -3	✓ $m_{AQ} = 1$ of ✓ $m_{AQ} = -1$ ✓ substitusie in gradiëntformule ✓ x-waarde ✓ y-waarde (5) **[22]**

VRAAG 4

4.1	M(–1 ; –1)	✓ antwoord (1)
4.2	$m_{NT} = \frac{2-1}{3-4} = -1$ $\therefore\ m_{AT} = 1$ (radius ⊥ raaklyn) $y - 1 = 1(x - 4)$ $y = x - 3$	✓ m_{NT} ✓ m_{AT} ✓ rede ✓ substitusie van m en (4 ; 1) ✓ vergelyking (5)
4.3	MR ⊥ AB (lyn vanaf midpt na midpt van koord) $MB^2 = MR^2 + RB^2$ (Stelling van Pythagoras) $9 = (\frac{\sqrt{10}}{2})^2 + RB^2$ $RB^2 = \frac{13}{2}$ $RB = \sqrt{\frac{13}{2}}$ $AB = 2\left(\sqrt{\frac{13}{2}}\right) = \sqrt{26}$ eenhede	✓ MR ⊥ AB ✓MB = 3 ✓ substitusie in stelling van Pythagoras ✓ AB in wortelvorm (4)
4.4	$MN^2 = (-1 - 3)^2 + (-1 - 2)^2$ $= 16 + 9$ $= 25$ MN = 5 eenhede	✓ substitusie in afstandformule ✓ antwoord (2)
4.5	$r = 5 - 3 = 2$ eenhede $\therefore\ (x - 3)^2 + (y - 2)^2 = 4$ $\therefore\ x^2 + y^2 - 6x - 4y + 9 = 0$	✓r ✓substitusie in sirkelvergelyking ✓vergelyking (3) **[15]**

VRAAG 5

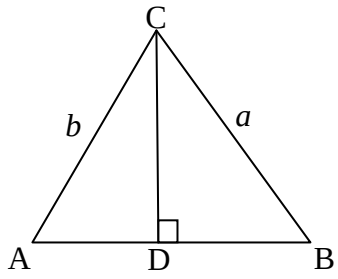
5.1.1	$-\sin\alpha$ $= -(-\frac{4}{5}) = \frac{4}{5}$	✓ reduksie ✓ antwoord (2)
5.1.2	$(-4)^2 + b^2 = 5^2$ $b^2 = 25 - 16 = 9$ $b = -3$ $\cos\alpha = \frac{-3}{5}$  	✓ $b = -3$ ✓antwoord (2)
5.1.3	$\sin(\alpha - 45°)$ $= \sin\alpha\cos 45° - \cos\alpha\sin 45°$ $= -\frac{4}{5}\cdot\frac{1}{\sqrt{2}} - (-\frac{3}{5})\cdot\frac{1}{\sqrt{2}}$ $= -\frac{1}{5\sqrt{2}}$ **OF** $\sin(\alpha - 45°)$ $= \sin\alpha\cos 45° - \cos\alpha\sin 45°$ $= -\frac{4}{5}\cdot\frac{\sqrt{2}}{2} - (-\frac{3}{5})\cdot\frac{\sqrt{2}}{2}$ $= -\frac{\sqrt{2}}{10}$	✓ uitbreiding ✓ $\frac{1}{\sqrt{2}}$ ✓ antwoord in eenvoudigste vorm (3) ✓ uitbreiding ✓ $\frac{\sqrt{2}}{2}$ ✓ antwoord in eenvoudigste vorm (3)
5.2.1	$LHS = \frac{8\sin x.\cos x}{\sin^2 x - \cos^2 x}$ $= \frac{4(2\sin x.\cos x)}{\sin^2 x - \cos^2 x}$ $= \frac{4\sin 2x}{-(\cos^2 x - \sin^2 x)}$ $= \frac{4\sin 2x}{-\cos 2x}$ $= -4\tan 2x$	✓ $\sin x$ ✓ $\cos x$ ✓ $\cos^2 x$ ✓ $4\sin 2x$ ✓ faktoriseer ✓ $-\cos 2x$ (6)
5.2.2	Ongedefinieer as $\cos 2x = 0$ of $\tan 2x = \infty$: $x = 45°$ en $x = 135°$	✓ 45° ✓ 135° (2)

5.3	$1-2\sin^2\theta+4\sin^2\theta-5\sin\theta-4=0$ $2\sin^2\theta-5\sin\theta-3=0$ $(2\sin\theta+1)(\sin\theta-3)=0$ $\therefore\ \sin\theta=-\frac{1}{2}$ of $\sin\theta=3$ (geen oplossing) $\therefore\theta=210°+360°k$ of $\theta=330°+360°k$; $k\in Z$ **OF** $\therefore\theta=210°+360°k$ of $\theta=30°+360°k$; $k\in Z$	✓ $1-2\sin^2\theta$ ✓ standaardvorm ✓ faktore ✓ geen oplossing ✓ 210° ✓ 330° ✓ $+360°k$; $k\in Z$ (7) **[22]**

VRAAG 6

6.1	$b=\frac{1}{2}$	✓ waarde van b (1)
6.2	A(30° ; 1)	✓ 30° ✓ 1 (2)
6.3	$x=160°$	✓ $x=160°$ (1)
6.4	$h(x)=2\cos(x-30°)+1$ $y\in[-1\ ;\ 3]$ **OF** $-1\le y\le 3$	✓ kritiese waardes ✓ notasie (2) **[6]**

VRAAG 7

7.1	Trek CD ⊥ AB In ΔACD: $\sin A = \frac{CD}{b}$ ∴ CD = b. sin A  In ΔCBD: $\sin B = \frac{CD}{a}$ ∴ CD = a. sin B ∴ b . sin A = a. sin B $\therefore \frac{\sin A}{a} = \frac{\sin B}{b}$	✓ konstruksie ✓ sin A ✓ maak CD die onderwerp ✓ sin B ✓ b. sin A = a. sin B (5)
7.2.1	$S\hat{P}Q = 180° - 2x$ (teenoorst ∠e van koordevierh) $P\hat{S}Q + P\hat{Q}S = 2x$ (som van ∠e in Δ) $P\hat{S}Q = P\hat{Q}S = x$ (∠e teenoor gelyke sye)	✓ $S\hat{P}Q = 180° - 2x$ (S/R) ✓ rede (2)
7.2.2	$\frac{\sin S\hat{P}Q}{SQ} = \frac{\sin P\hat{S}Q}{PQ}$ $\frac{\sin(180° - 2x)}{SQ} = \frac{\sin x}{k}$ $SQ = \frac{k \sin 2x}{\sin x}$ $SQ = \frac{k(2\sin x.\cos x)}{\sin x} = 2k\cos x$ **OF** $SQ^2 = PQ^2 + PS^2 - 2PQ.PS.\cos S\hat{P}Q$ $= k^2 + k^2 - 2.k.k.\cos(180° - 2x)$ $= 2k^2 + 2k^2 \cos 2x$ $= 2k^2 + 2k^2(2\cos^2 x - 1)$ $= 4k^2\cos^2 x$ $SQ = 2k\cos x$	✓ substitusie in korrekte formule ✓ sin $2x$ ✓ SQ onderwerp ✓ $2\sin x.\cos x$ (4) ✓ substitusie in korrekte formule ✓ – cos $2x$ ✓ $2\cos^2 x - 1$ ✓ vereenvoudig (4)
7.2.3	$\tan y = \frac{3}{k}$ $k = \frac{3}{\tan y}$ ∴ $SQ = 2\cos x\left(\frac{3}{\tan y}\right)$ $= \frac{6\cos x}{\tan y}$	✓ tan-verhouding ✓ k onderwerp en substitusie (2) **[13]**

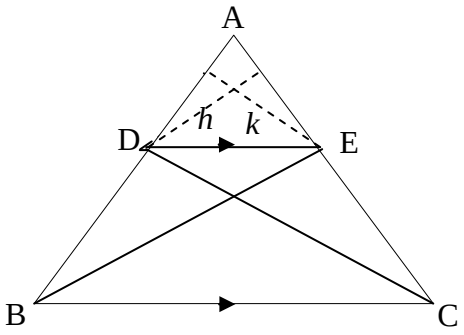
VRAAG 8

8.1	die hoek onderspan in die teenoorstaande sirkelsegment	✓korrekte stelling (1)
8.2.1	$\hat{B}_1 = \hat{E}_1 = 68°$ (rkl-koordst)	✓ $\hat{E}_1 = 68°$ ✓ rede (2)
8.2.2	$\hat{E}_1 = \hat{B}_3 = 68°$ (verwiss∠e; AE | | BC)	✓ $\hat{B}_3 = 68°$ (S/R) (1)
8.2.3	$\hat{D}_1 = \hat{B}_3 = 68°$ (buite∠ v koordevh)	✓ $\hat{D}_1 = 68°$ ✓ rede (2)
8.2.4	$\hat{E}_2 = 20° + 68°$ $= 88°$ (buite∠ v Δ)	✓ $\hat{E}_2 = 88°$ (S/R) (1)
8.2.5	$\hat{C} = 180° - 88°$ $= 92°$ (tos ∠e v koordevh)	✓ $\hat{C} = 92°$ ✓ rede (2) **[9]**

VRAAG 9

9.1	$\hat{D}_4 = \hat{A} = x$ (rkl-koordstelling) $\hat{A} = \hat{D}_2 = x$ (∠e tos gelyke sye)	✓ $\hat{A} = x$ ✓ rede ✓ $\hat{A} = \hat{D}_2 = x$ (S/R) (3)
9.2	$\hat{M}_1 = 2x$ (buite ∠vΔ) OF (∠ by midpt = 2∠ by omtr) $M\hat{D}E = 90°$ (radius ⊥ rkl) $\hat{M}_2 = 90° - 2x$ $\therefore \hat{E} = 180° - (90° + 90° - 2x)$ (som v ∠e in ΔMDE) $= 2x$ ∴ CM is 'n rkl (omgek rkl-koordst)	✓ $\hat{M}_1 = 2x$ (S/R) ✓ $M\hat{D}E = 90°$ (S/R) ✓ $\hat{E} = 2x$ ✓ rede (4)
9.3	$\hat{M}_3 = 90°$ (EM ⊥ AC) $A\hat{D}B = 90°$ (∠ in halfsirkel) ∴ FMBD is koordevh (buite∠ v vh = tos binne ∠) **OF** $E\hat{M}C = 90°$ (EM ⊥ AC) $A\hat{D}B = 90°$ (∠ in halfsirkel) ∴ FMBD is koordevh (tos ∠e v vh suppl)	✓ $\hat{M}_3 = 90°$ ✓ $A\hat{D}B = 90°$ (S/R) ✓ rede (3) ✓ $E\hat{M}C = 90°$ ✓ $A\hat{D}B = 90°$ (S/R) ✓ rede (3)
9.4	$DC^2 = MC^2 - MD^2$ (Pythagoras) $= (3BC)^2 - (2BC)^2$ (MB = MD = radii) $= 9BC^2 - 4BC^2$ $= 5BC^2$	✓ Pythagoras ✓ substitusie ✓ $9BC^2 - 4BC^2$ (3)
9.5	In ΔDBC en ΔDFM: $\hat{D}_4 = \hat{D}_2 = x$ (bewys in 9.1) $\hat{B}_1 = \hat{F}_2$ (buite ∠ v koordevh) $\hat{C} = \hat{M}_2$ ∴ΔDBC ||| ΔDFM (∠; ∠; ∠)	✓ $\hat{D}_4 = \hat{D}_2$ ✓ $\hat{B}_1 = \hat{F}_2$ ✓ rede ✓ $\hat{C} = \hat{M}_2$ of (∠; ∠; ∠) (4)
9.6	$\frac{DM}{FM} = \frac{DC}{BC}$ (ΔDBC ||| ΔDFM) $= \frac{\sqrt{5}BC}{BC}$ $= \sqrt{5}$	✓ S ✓ antwoord (2) **[19]**

VRAAG 10

10.1	 Konstruksie: Verbind DC en BE en trek hoogtes *k* en *h* $\frac{\text{opp } \Delta\text{ADE}}{\text{opp } \Delta\text{DEB}} = \frac{\frac{1}{2}.\text{AD}.k}{\frac{1}{2}.\text{DB}.k} = \frac{\text{AD}}{\text{DB}}$ (gelyke hoogtes) $\frac{\text{opp } \Delta\text{ADE}}{\text{opp } \Delta\text{DEC}} = \frac{\frac{1}{2}.\text{AE}.h}{\frac{1}{2}.\text{E}C.h} = \frac{\text{AE}}{\text{EC}}$ (gelyke hoogtes) Maar Opp ΔDEB = Opp ΔDEC (dies basis, dies hoogte) $\therefore \frac{\text{opp } \Delta\text{ADE}}{\text{opp } \Delta\text{DEB}} = \frac{\text{opp } \Delta\text{ADE}}{\text{opp } \Delta\text{DEC}}$ $\therefore \frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$	✓ konstruksie ✓ $\frac{\text{opp } \Delta\text{ADE}}{\text{opp } \Delta\text{DEB}} = \frac{\text{AD}}{\text{DB}}$ ✓ rede ✓ $\frac{\text{opp } \Delta\text{ADE}}{\text{opp } \Delta\text{DEC}} = \frac{\text{AE}}{\text{EC}}$ ✓ Area ΔDEB = Area ΔDEC (S/R) ✓ $\frac{\text{opp } \Delta\text{ADE}}{\text{opp } \Delta\text{DEB}} = \frac{\text{opp } \Delta\text{ADE}}{\text{opp } \Delta\text{DEC}}$ (6)

10.2.1	$\frac{AB}{BE} = \frac{AC}{CD}$ (Ewered st; BC | | ED) $\frac{1}{3} = \frac{3}{CD}$ $\therefore$ CD = 9 eenhede	✓ $\frac{AB}{BE} = \frac{AC}{CD}$ (S/R) ✓ substitusie ✓ antwoord (3)
10.2.2	$\frac{DG}{GA} = \frac{FD}{FE}$ (Ewered st; FG | | EA) $\frac{9-x}{3+x} = \frac{3}{6}$ $54 - 6x = 9 + 3x$ $-9x = -45$ $x = 5$	✓ $\frac{DG}{GA} = \frac{FD}{FE}$ (S/R) ✓ substitusie ✓ vereenvoudig ✓ antwoord (4)
10.2.3	In ΔABC en ΔAED: $\hat{A}$ is gemeen $A\hat{B}C = \hat{E}$ (ooreenk $\angle$s; BC | | ED) $A\hat{C}B = \hat{D}$ (ooreenk $\angle$s; BC | | ED) ΔABC | | | ΔAED ($\angle, \angle, \angle$) $\therefore \frac{BC}{ED} = \frac{AC}{AD}$ $\frac{BC}{9} = \frac{3}{12}$ BC = $2\frac{1}{4}$ eenhede	✓ $\hat{A}$ is gemeen ✓ $A\hat{B}C = \hat{E}$ (S/R) ✓ $A\hat{C}B = \hat{D}$ (S/R) of ($\angle$; $\angle$; $\angle$) ✓ $\frac{BC}{ED} = \frac{AC}{AD}$ ✓ antwoord (5)
10.2.4	$\frac{\text{opp } \Delta ABC}{\text{opp } \Delta GFD} = \frac{\frac{1}{2}AC.BC.\sin A\hat{C}B}{\frac{1}{2}GD.FD.\sin \hat{D}}$ $= \frac{\frac{1}{2}(3)(2\frac{1}{4})\sin \hat{D}}{\frac{1}{2}(4)(3)\sin \hat{D}}$ (ooreenk $\angle$s; BC | | ED) $= \frac{9}{16}$	✓ gebruik v opp reël ✓ korrekte sye en $\angle$e ✓ substitusie v waardes ✓ $\sin A\hat{C}B = \sin \hat{D}$ (S/R) ✓ antwoord (5) **[23]**

TOTAAL: 150

PolyMathic

Vraestel 4

Okt/Nov

Eksamen

PolyMathic

Graad 12 Eindeksamen
Totaal: 150
Tyd: 3ure

VRAAG 1

Die snoepwinkel by Great Future Hoërskool verkoop blikkies gaskoeldrank. Die Omgewingsklub by die skool het besluit om vir drie weke 'n blikkie-insamelingsprojek te hou om leerders van die effek van rommel op die omgewing bewus te maak.

Die data hieronder toon die getal blikkies wat op elke skooldag van die drie-week-projek ingesamel is.

58	83	85	89	94
97	98	100	105	109
112	113	114	120	145

1.1 Bereken die gemiddelde getal blikkies wat oor die tydperk van drie weke ingesamel is. (2)

1.2 Bereken die standaardafwyking. (2)

1.3 Bepaal die onderste en boonste kwartiele van die data. (2)

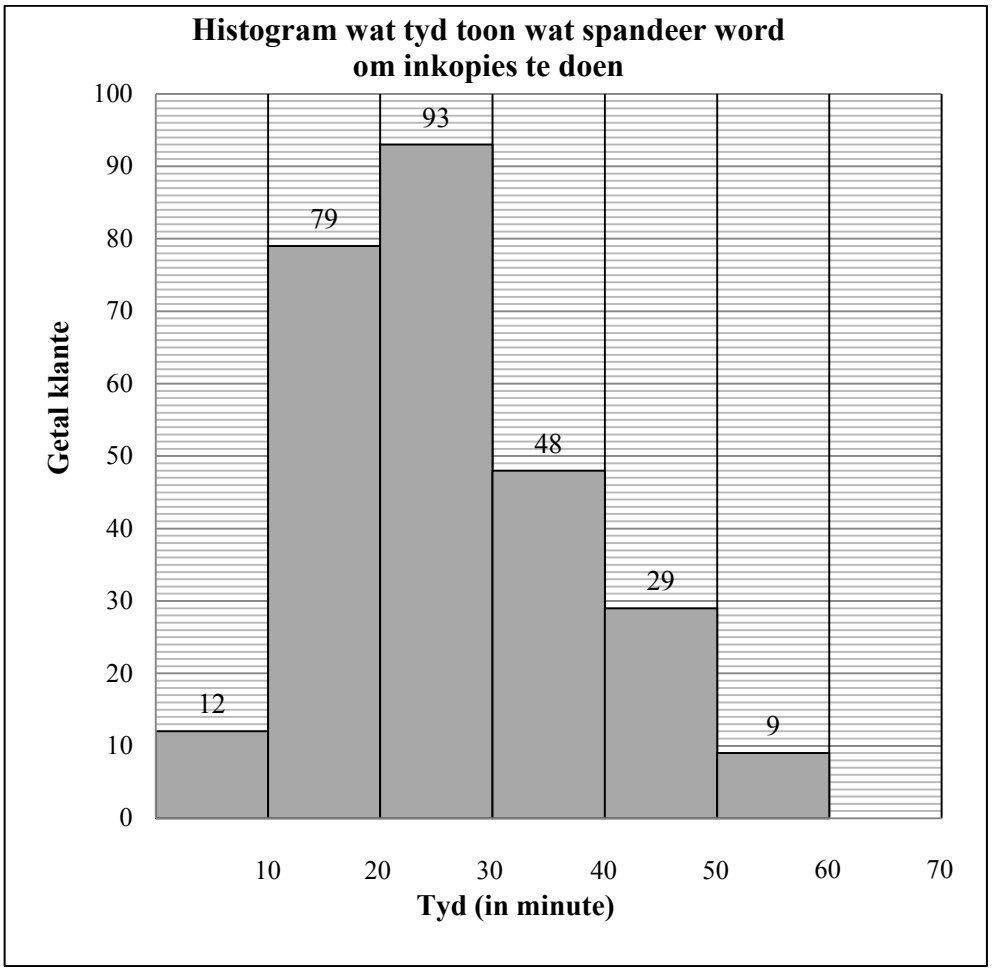
1.4 Gebruik die getallelyn op DIAGRAMVEL 1 om 'n mond-en-snordiagram, wat die data voorstel, te teken. (3)

1.5 Op hoeveel dae het die getal blikkies wat ingesamel is, buite EEN standaardafwyking vanaf die gemiddeld gelê? (3)

[12]

VRAAG 2

Die histogram hieronder toon die tyd, in minute, wat klante spandeer om hulle inkopies by Excellent Supermark te doen.



2.1 Voltooi die frekwensiekolom en kumulatiewefrekwensie-kolom in die tabel op DIAGRAMVEL 1. (3)

2.2 Gebruik die rooster op DIAGRAMVEL 2 om 'n ogief van die data hierbo te skets. (4)

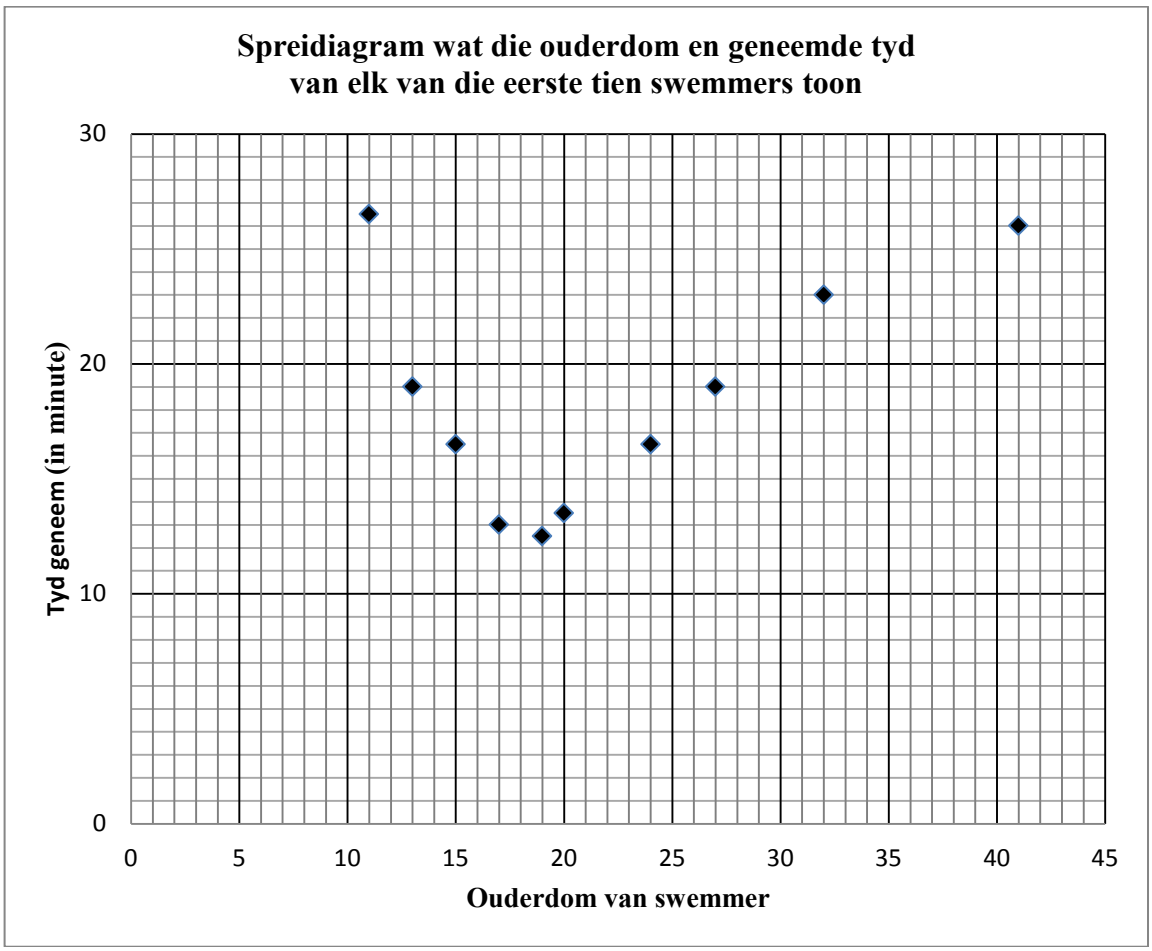
2.3 Gebruik die ogief om die mediaantyd wat klante by hierdie supermark spandeer het te skat. (2)

2.4 Lewer kommentaar op die skeefheid van die data. (1)

[10]

VRAAG 3

Die spreidiagram hieronder toon die ouderdom en die geneemde tyd van elk van die eerste tien swemmers van 'n swemklub om 'n oopwater-swemitem te voltooi. Die geneemde tyd is tot die naaste halfminuut afgerond.



3.1 Skryf die koördinate van 'n uitskieter in die spreidiagram neer. (1)

3.2 Watter van die volgende funksies sal die data die beste pas: lineêr, kwadraties of eksponensieel? (1)

3.3 Gee 'n verduideliking vir die tendens wat in hierdie dataversameling waargeneem word. (2)

3.4 Indien die twee swakste (langste) tye uit die dataversameling weggelaat word, hoe sal dit die volgende beïnvloed:

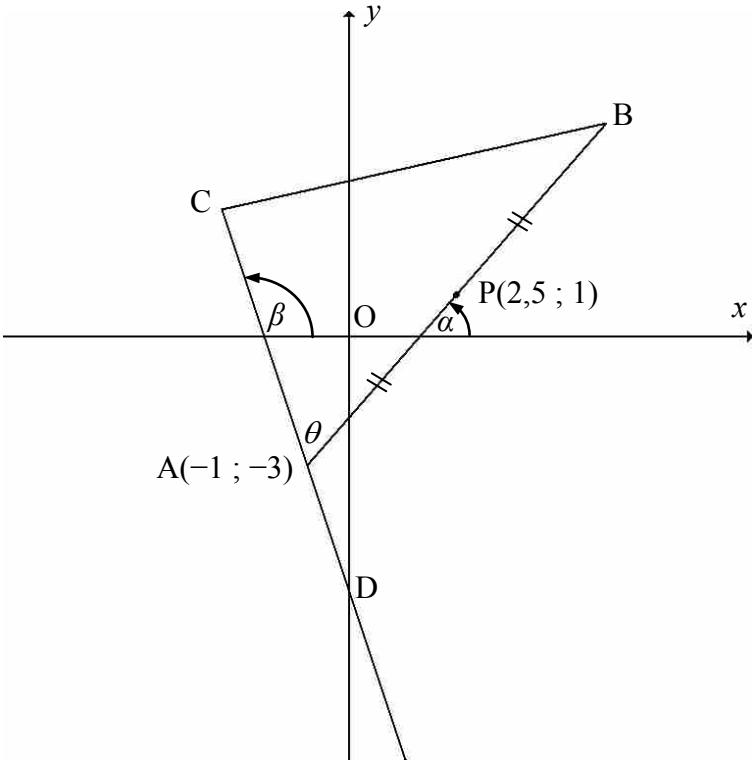
3.4.1 Die standaardafwyking van die oorspronklike dataversameling (1)

3.4.2 Die gemiddeld van die oorspronklike dataversameling (1)

[6]

VRAAG 4

In die diagram hieronder is A(−1 ; −3), B en C die hoekpunte van 'n driehoek.
P(2,5 ; 1) is die middelpunt van AB. CA verleng sny die y-as by D.
Die vergelyking van CD is $y = -3x + k$. $C\hat{A}B = \theta$.
α en β is die hoeke wat AB en AC onderskeidelik met die x-as maak.



4.1 Bepaal die waarde van k. (2)

4.2 Bepaal die koördinate van B. (2)

4.3 Bepaal die gradiënt van AB. (2)

4.4 Bereken die grootte van θ. (5)

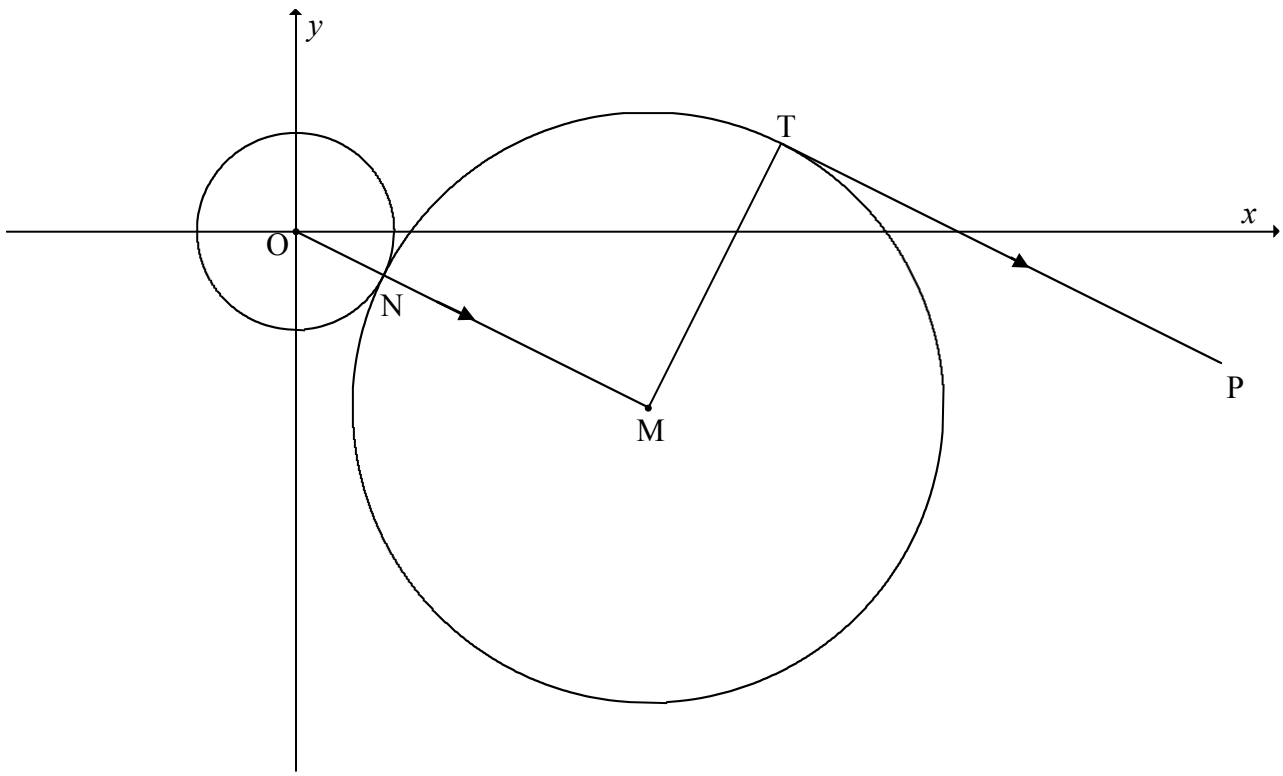
4.5 Bereken die lengte van AD. Laat jou antwoord in wortelvorm. (2)

4.6 Indien AC = 2AD en $AB = \sqrt{113}$, bereken die lengte van CB. (5)

[18]

VRAAG 5

In die diagram hieronder is $(x - 8)^2 + (y + 4)^2 = 45$ die vergelyking van die sirkel met middelpunt M.
PT is 'n raaklyn aan hierdie sirkel by T en PT is ewewydig aan OM. 'n Ander sirkel, met middelpunt O, raak die sirkel met middelpunt M by N.



5.1 Skryf die koördinate van M neer. (1)

5.2 Bereken die lengte van OM. Laat jou antwoord in die eenvoudigste wortelvorm. (2)

5.3 Bereken die lengte van ON. Laat jou antwoord in die eenvoudigste wortelvorm. (3)

5.4 Bereken die grootte van $O\hat{M}T$. (2)

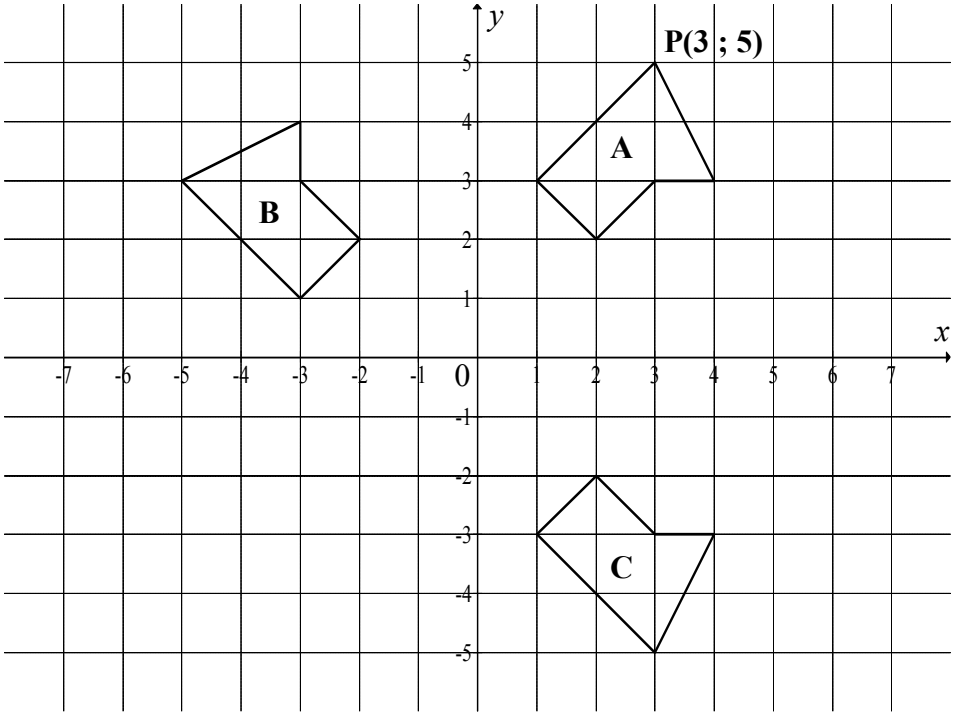
5.5 Bepaal die vergelyking van MT in die vorm $y = mx + c$. (5)

5.6 Bereken die koördinate van T. (6)

[19]

VRAAG 6

6.1 Die diagram hieronder toon veelhoeke A, B en C.
P(3 ; 5) is 'n hoekpunt van veelhoek A.



6.1.1 Beskryf volledig die transformasie van veelhoek A na veelhoek B. (2)

6.1.2 Skryf die reël neer wat veelhoek B na veelhoek C transformeer. (2)

6.1.3 $P^{/}$ is die beeld van P wanneer veelhoek A om die oorsprong deur 180° geroteer word. Skryf die koördinate van $P^{/}$ neer. (2)

6.2 P(3 ; 2), Q(−1 ; 2), R(−2 ; 1) en S(−1 ; 0) is die hoekpunte van vierhoek PQRS.

6.2.1 PQRS word deur die oorsprong met 'n skaalfaktor van 2 vergroot om $P^{/}Q^{/}R^{/}S^{/}$ te verkry. Skryf die koördinate van $Q^{/}$ neer. (1)

6.2.2 $P^{//}Q^{//}R^{//}S^{//}$ is die beeld wanneer $P^{/}Q^{/}R^{/}S^{/}$ om die y-as gereflekteer en dan 3 eenhede na regs en 1 eenheid opwaarts getransleer word.

(a) Skryf die enkelreël neer wat PQRS na $P^{//}Q^{//}R^{//}S^{//}$ transformeer. (3)

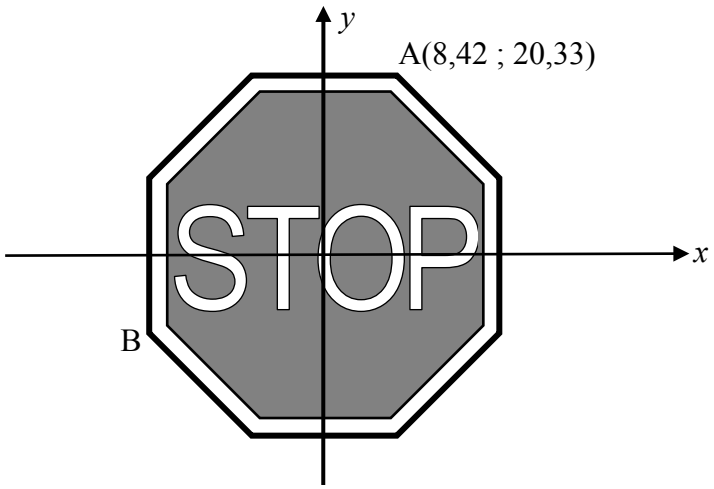
(b) Gebruik die rooster op DIAGRAMVEL 3 om $P^{//}Q^{//}R^{//}S^{//}$ te skets. Benoem die hoekpunte. (5)

6.2.3 Los vir t op in die vergelyking:

Omtrek van PQRS = t × omtrek van $P^{//}Q^{//}R^{//}S^{//}$ (2)

[17]

VRAAG 7



Die STOP-teken is 'n reëlmatige oktagoon (agthoek). Beskou A(8,42 ; 20,33) as 'n hoekpunt op 'n STOP-teken met sy middelpunt by die oorsprong.

Indien die STOP-teken om die oorsprong in 'n antikloksgewyse rigting geroteer word sodat A met punt B saamval, bepaal die koördinate van B. **[6]**

VRAAG 8

8.1 Indien $\sin A = \frac{3}{5}$ en $\cos A < 0$, bepaal, SONDER om 'n sakrekenaar te gebruik, die waarde van:

8.1.1 $\sin(-A)$ (2)

8.1.2 $\tan A$ (3)

8.2 Indien $\cos 34° = p$, skryf die volgende, SONDER om 'n sakrekenaar te gebruik, in terme van p neer:

8.2.1 $\cos 214°$ (2)

8.2.2 $\cos 68°$ (2)

8.2.3 $\tan 56°$ (4)

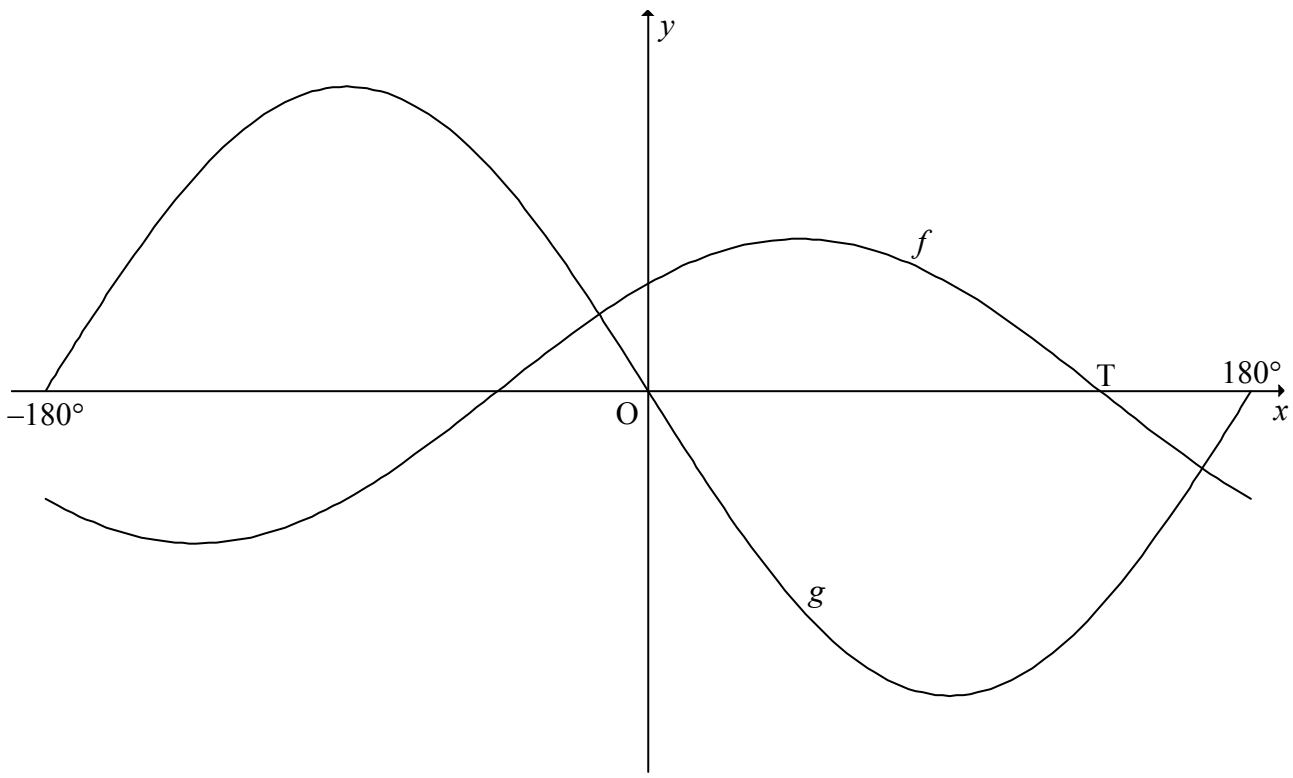
8.3 Bepaal die waarde van die volgende uitdrukking, SONDER om 'n sakrekenaar te gebruik:

$\cos 350° \sin 40° - \cos 440° \cos 40°$ (5)

[18]

VRAAG 9

Die grafieke van $f(x) = \cos(x - 45°)$ en $g(x) = -2\sin x$ is hieronder geskets vir $x \in [-180° ; 180°]$. Die punt T is 'n x-afsnit van f, soos in die diagram aangetoon.



9.1 Toon aan dat $\cos(x - 45°) = -2\sin x$ geskryf kan word as $\tan x = -0{,}2612$. (4)

9.2 Los die vergelyking op: $\cos(x - 45°) = -2\sin x$ vir $x \in [-180° ; 180°]$. (3)

9.3 Skryf die koördinate van punt T neer. (2)

9.4 Skryf die interval neer waarvoor $f(x) \geq g(x)$. (2)

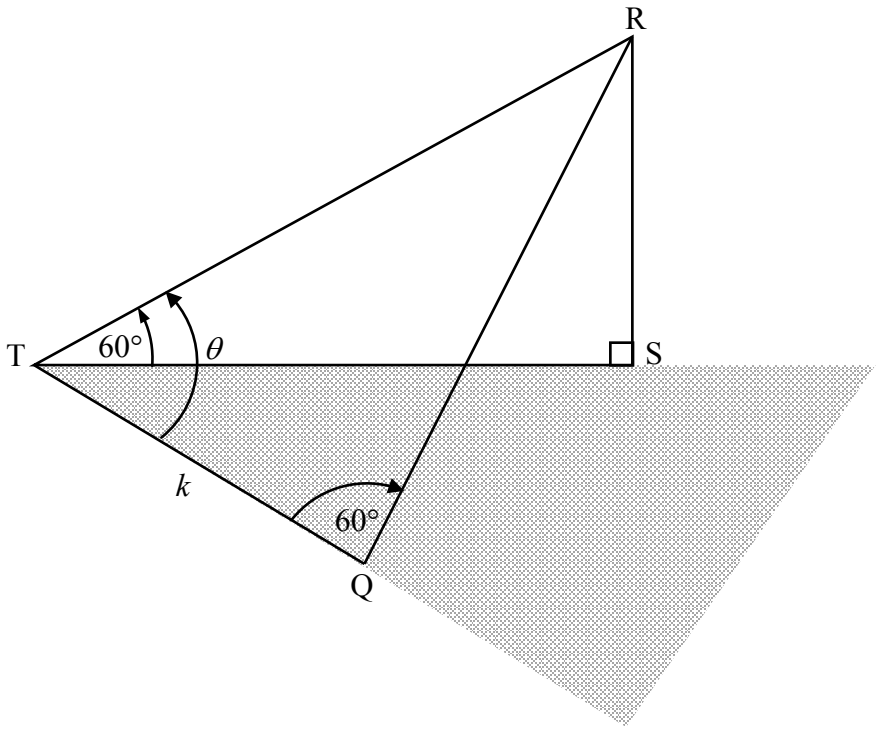
9.5 Skryf die interval neer waarvoor beide f en g streng stygend is. (3)

9.6 Die grafiek h word verkry as die grafiek f met 45° na regs geskuif word. Skryf die vergelyking van h in sy eenvoudigste vorm neer. (2)

[16]

VRAAG 10

In die diagram hieronder is RS die hoogte van 'n vertikale toring. T en Q is twee punte in dieselfde horisontale vlak as die voet S van die toring. Vanaf punt T is die hoogtehoek na die toppunt van die toring 60°. $R\hat{T}Q = \theta$, $R\hat{Q}T = 60°$ en $TQ = k$ meter.



10.1 Druk TR in terme van θ en k uit. (3)

10.2 Toon dat $RS = \dfrac{3k}{2(\sqrt{3}\cos\theta + \sin\theta)}$. (7)

[10]

VRAAG 11

11.1 Beskou die funksie: $f(x) = 3 - 2\sin^2 x$

11.1.1 Bepaal die waardeversameling van f. (4)

11.1.2 Vir watter waarde(s) van x, $x \in [-180° ; 180°]$, sal f 'n minimum waarde hê? (3)

11.2 11.2.1 Toon aan dat $1 - \cos 2Q = 2\sin^2 Q$. (1)

11.2.2 Gegee: $\hat{P} + \hat{Q} + \hat{R} = 180°$

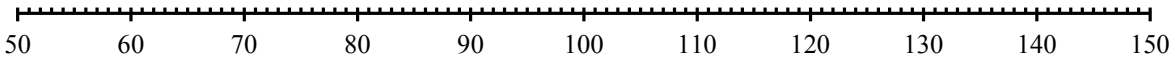
(a) Toon aan dat $\sin 2R = -\sin(2P + 2Q)$. (3)

(b) Toon vervolgens aan dat
$\sin 2P + \sin 2Q + \sin 2R = 4 \sin P \sin Q \sin R$. (7)
[18]

TOTAAL: 150

DIAGRAMVEL 1

VRAAG 1.4

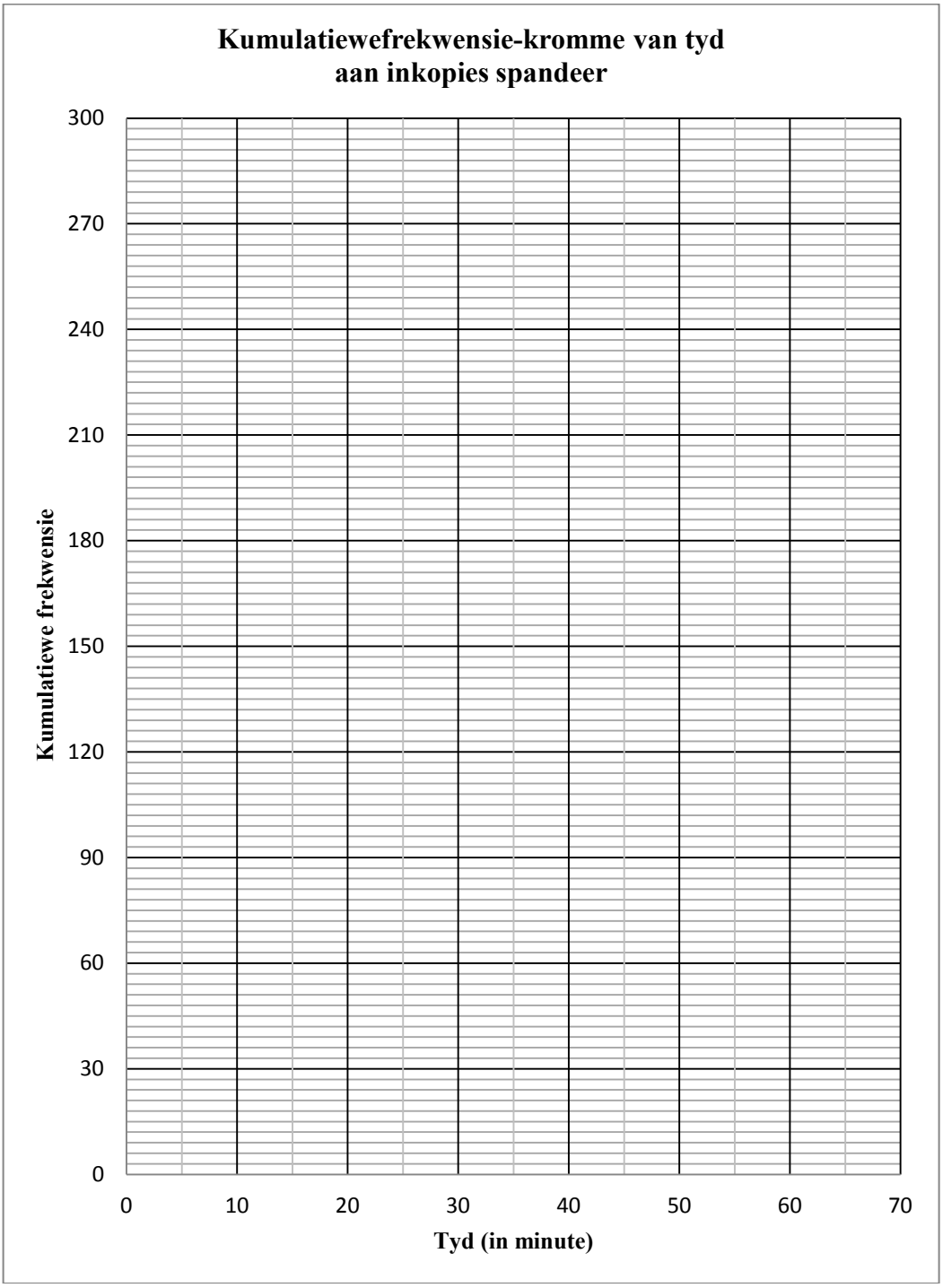


VRAAG 2.1

Tyd (in minute)	Frekwensie	Kumulatiewe frekwensie
$0 < x \leq 10$		
$10 < x \leq 20$		
$20 < x \leq 30$		
$30 < x \leq 40$		
$40 < x \leq 50$		
$50 < x \leq 60$		

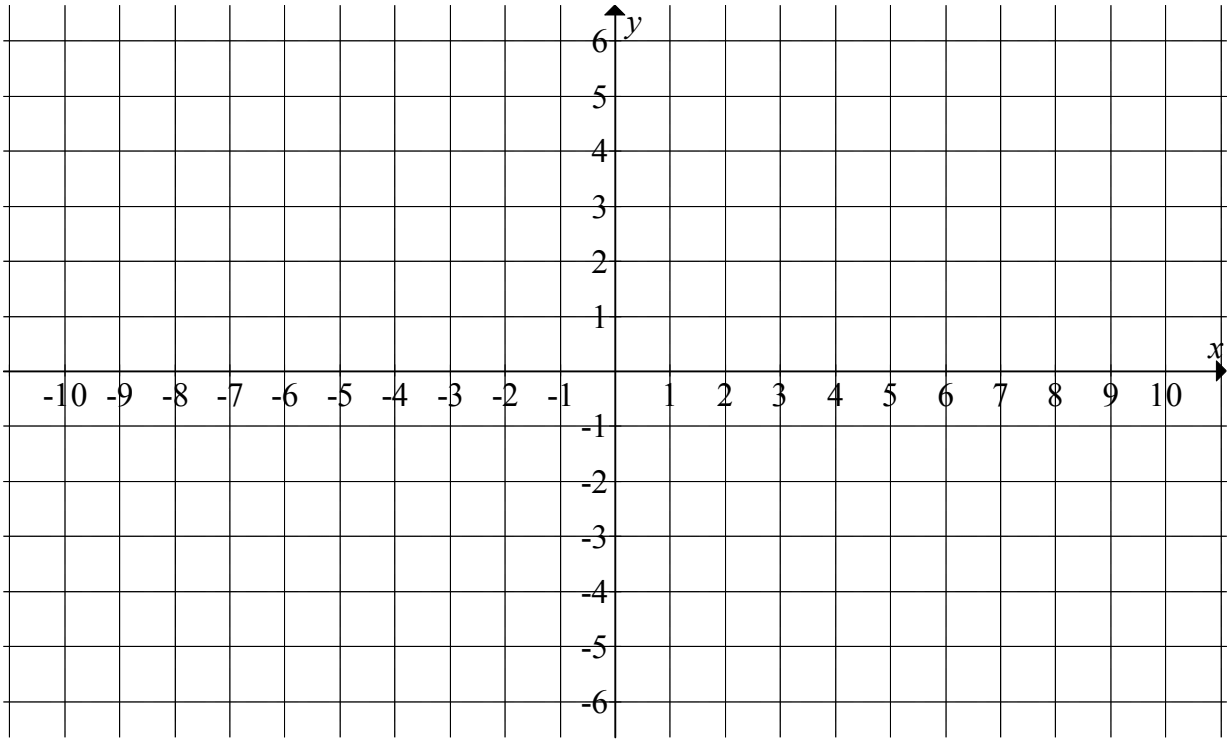
DIAGRAMVEL 2

VRAAG 2.2



DIAGRAMVEL 3

VRAAG 6.2.2(b)

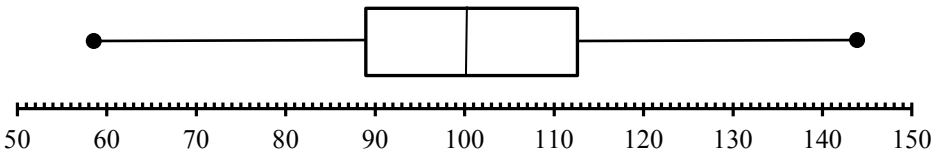


Memo

LET WEL:

- *As 'n kandidaat 'n vraag TWEE keer beantwoord het, merk slegs die eerste poging.*
- *As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, merk die deurgehaalde antwoord.*
- *Volgehoue akkuraatheid is DEURGAANS in ALLE aspekte van die memorandum van toepassing.*
- *Aanvaarding van waardes/antwoorde om 'n problem op te los, is onaanvaarbaar.*

QUESTION/*VRAAG* 1

1.1	Mean/*Gemiddelde* = $\frac{\sum x}{n} = \frac{1522}{15} = 101{,}47$	✓ 1522 ✓ 101,47 (2)
1.2	Standard deviation/*standaardafwyking* = 19,07	✓19,07 ✓2 decimal places (2)
1.3	lower quartile/*onderste (eerste) kwartiel* = 89 upper quartile/*boonste (derde) kwartiel* = 113	✓ 89 ✓ 113 (2)
1.4		✓ M at 100 ✓ min = 58 and max = 145 ✓ Q_1 = 89 and Q_2 = 113 (3)
1.5	$(\bar{x} - 1\sigma\ ;\ \bar{x} + 1\sigma) = (82{,}4\ ;\ 120{,}54)$ $\therefore$ 2 days/*dae* Answer only: full marks	✓✓ interval ✓ answer (3) **[12]**

QUESTION/*VRAAG* 2

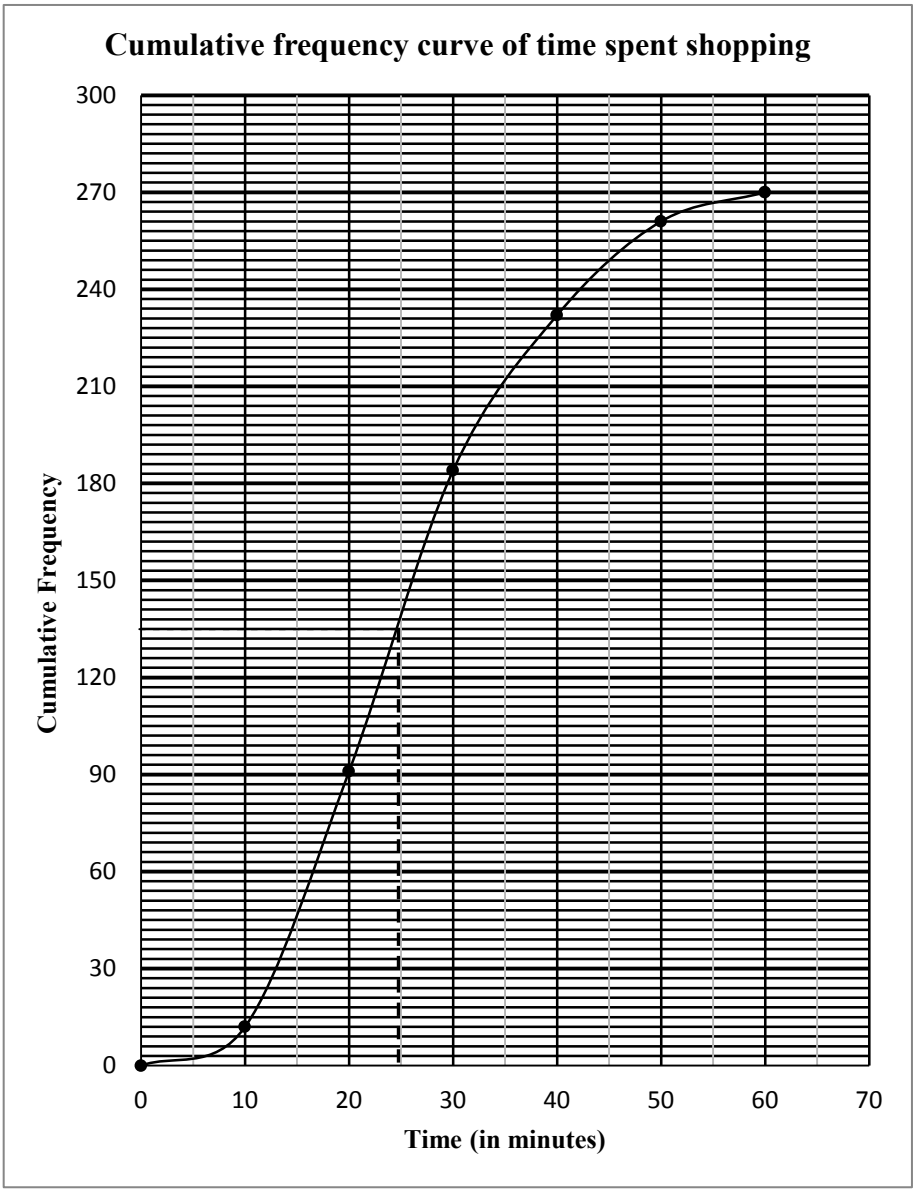
2.1

TIME IN MINUTES	NUMBER OF CUSTOMERS (frequency)	CUMULATIVE FREQUENCY
$0 < x \leq 10$	12	12
$10 < x \leq 20$	79	91
$20 < x \leq 30$	93	184
$30 < x \leq 40$	48	232
$40 < x \leq 50$	29	261
$50 < x \leq 60$	9	270

✓ completing frequency column
✓ ✓ cumulative frequency

(3)

2.2



✓ plot against upper limit

✓ cumulative frequency

✓ anchored
✓ smooth curve

(4)

2.3

Median time spent shopping is approximately 25 minutes.
(Allow 24–25 minutes)

✓ ✓ answer

(2)

2.4

The data is skewed to the right or positively skewed/
Die data is skeef na regs of positief skeef.

✓ correct skewness

(1)
[10]

QUESTION/*VRAAG* 3

3.1	(41 ; 26)	✓ correct outlier (1)
3.2	quadratic/*kwadraties*	✓ correct answer (1)
3.3	The younger or older the participants are, the longer they will take to complete the item. They do not have the required strength, fitness and stamina. *Hoe jonger of ouer die deelnemers is, hoe langer sal hulle neem om die item te voltooi. Hulle het nie die vereiste krag, fiksheid en stamina (energie) nie.* **OR** It would appear that swimmers close to 19 years completed the item in the shortest time. Swimmers of that age are normally in good physical condition and have lots of stamina. *Dit wil voorkom of swemmers rondom 19 jaar die item in die kortste tyd voltooi het. Swemmers van daardie ouderdom is normaalweg in goeie fisiese kondisie en het baie energie en stamina.*	✓ younger/older *jonger/ouer* ✓ lack of strength/ *tekort aan krag* (2) ✓ 19 years/*jaar* ✓ peak fitness/ *top fiks* (2)
3.4.1	The standard deviation will become smaller/decrease./ *Die standaardafwyking sal kleiner word/verminder.*	✓ decrease/ *verminder* (1)
3.4.2	The mean will become smaller/decrease./ *Die gemiddelde sal kleiner word/verminder.*	✓ decrease/ *verminder* (1) **[6]**

QUESTION/*VRAAG* 4

4.1	$y = -3x + k$ $-3 = (-3)(-1) + k$ $k = -6$ **OR** By inspection, using the gradient: $k = -6$	✓ substitution of (–1 ; –3) ✓ $k = -6$ (2)
4.2	$\frac{x_A + x_B}{2} = x_P$ and $\frac{y_A + y_B}{2} = y_P$ $\frac{-1 + x_B}{2} = \frac{5}{2}$ and $\frac{-3 + y_B}{2} = 1$ $x_B = 6$ $y_B = 5$ $\therefore$B (6 ; 5) **OR** By using translation: B(6 ; 5)	✓ 6 ✓ 5 (2)
4.3	$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5-(-3)}{6-(-1)}$ $= \frac{8}{7}$ **OR** $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{1-(-3)}{2{,}5-(-1)}$ $= \frac{8}{7}$	✓ substitution ✓ gradient (2)

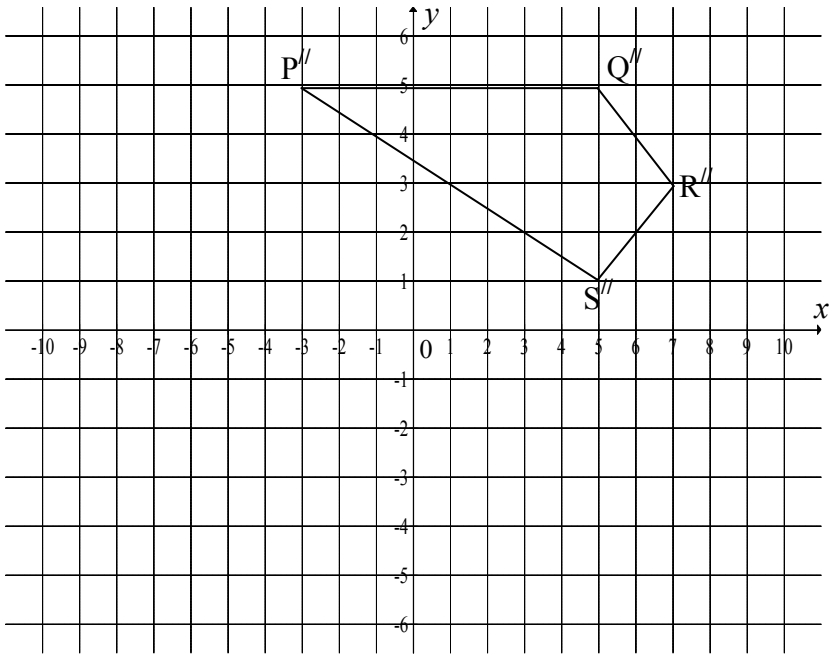
4.4	$\tan\beta = m_{AD} = -3$ $\beta = 108{,}43°$ $\tan\alpha = m_{AB} = \frac{8}{7}$ $\alpha = 48{,}81°$ $\theta = 108{,}43° - 48{,}81°$ $\theta = 59{,}62°$ **OR** $\tan\beta = m_{AD} = -3$ $\beta = 108{,}43°$ $C\hat{D}O = 18{,}43°$ $\tan\alpha = m_{AB} = \frac{8}{7}$ $\alpha = 48{,}81°$ $\theta = 18{,}43° + (90° - 48{,}81°)$ $\theta = 59{,}62°$	✓ $\tan\beta = -3$ ✓ $\beta = 108{,}43°$ ✓ $\tan\alpha = \frac{8}{7}$ ✓ $\alpha = 48{,}81°$ ✓ $\theta = 59{,}62°$ (5) ✓ $\tan\beta = -3$ ✓ $\beta = 108{,}43°$ ✓ $\tan\alpha = \frac{8}{7}$ ✓ $\alpha = 48{,}81°$ ✓ $\theta = 59{,}62°$ (5)
4.5	$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(0+1)^2 + (-6+3)^2}$ $= \sqrt{10}$	✓ substitution into distance formula ✓ $\sqrt{10}$ (2)
4.6	AC = 2 AD $= 2\sqrt{10}$ $CB^2 = AC^2 + AB^2 - 2AC.AB.\cos\theta$ $= (2\sqrt{10})^2 + (\sqrt{113})^2 - 2(2\sqrt{10})(\sqrt{113})\cos 59{,}62°$ $= 84{,}998...$ CB = 9,22 units. **OR** D(0 ; –6), A(–1 ; –3), AC = 2AD So $x_c - x_A = 2(x_A - x_D)$ $x_C + 1 = 2(-1 - 0)$, $x_C = -3$ $y_c - y_A = 2(y_A - y_D)$ $y_C + 3 = 2(-3 + 6)$, $y_C = 3$ The coordinates of C are (–3 ; 3). $CB = \sqrt{(6-(-3))^2 + (5-3)^2}$ $= 9{,}22$ units	✓ $AC = 2\sqrt{10}$ ✓ using cosine rule ✓ substitution ✓ 84,998... ✓ 9,22 (5) ✓✓✓ C(–3 ; 3) ✓ substitution into distance formula ✓ 9,22 (5) **[18]**

QUESTION/*VRAAG* 5

5.1	M(8 ; – 4)	✓ coordinates (1)
5.2	$OM = \sqrt{(8-0)^2 + (-4-0)^2}$ $= \sqrt{80}$ or $4\sqrt{5}$ units	✓ substitution into distance formula ✓ $\sqrt{80}$ *or* $4\sqrt{5}$ (2)
5.3	$ON = OM - NM$ $= \sqrt{80} - \sqrt{45}$ $= 4\sqrt{5} - 3\sqrt{5}$ $= \sqrt{5}$ *units*	✓ $ON = OM - NM$ ✓ length of NM ✓ answer (3)
5.4	$M\hat{T}P = 90°$ (tangent/*raaklyn* ⊥ radius) $\therefore\ O\hat{M}T = 90°$ (alternate ∠'s /*verwissellende* ∠'*e* ; TP | | OM)	✓ Statement + reason ✓ answer (2)
5.5	$m_{MT} \cdot m_{OM} = -1$ $m_{OM} = \frac{-4-0}{8-0} = -\frac{1}{2}$ $m_{MT} = 2$ $y + 4 = 2(x-8)$ $y = 2x - 20$ **OR** $y = 2x + c$ $-4 = 2(8) + c$ $c = -20$ $y = 2x - 20$	✓ ✓ m_{OM} ✓ m_{MT} ✓ substitution of m and (8 ; –4) ✓ equation MT (5)
5.6	$(x-8)^2 + (y+4)^2 = 45$ $(x-8)^2 + (2x-20+4)^2 = 45$ $(x-8)^2 + (2x-16)^2 = 45$ $x^2 - 16x + 64 + 4x^2 - 64x + 256 - 45 = 0$ $5x^2 - 80x + 275 = 0$ $x^2 - 16x + 55 = 0$ $(x-11)(x-5) = 0$ $x = 11$ $y = 2(11) - 20$ $y = 2$ $\therefore$ T(11 ; 2)	✓ substitution ✓ expansion ✓ standard form ✓ factors ✓ $x = 11$ ✓ substitution (6) **[19]**

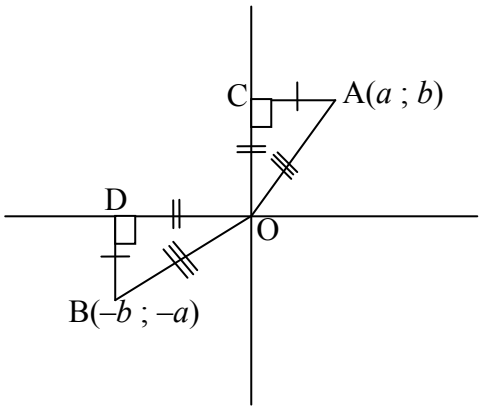
QUESTION/*VRAAG* 6

6.1.1	Rotation by 90° anti-clockwise about the origin/ *Rotasie van 90° antikloksgewys om die oorsprong.*	✓ rotation/*rotasie* ✓ 90° anti-clockwise/ *antikloksgewys* (2)
6.1.2	$(x;y) \rightarrow (y;x)$	✓ y ✓ x (2)
6.1.3	$P^{/}(-3;-5)$	✓ x value ✓ y value (2)
6.2.1	$Q^{/}(-2;4)$	✓ x value and y value (1)
6.2.2(a)	$(x;y) \rightarrow (2x;2y) \rightarrow (-2x;2y) \rightarrow (-2x+3\ ;\ 2y+1)$	✓ $(2x;2y)$ ✓ $(-2x;2y)$ ✓ $(-2x+3\ ;\ 2y+1)$ (3)

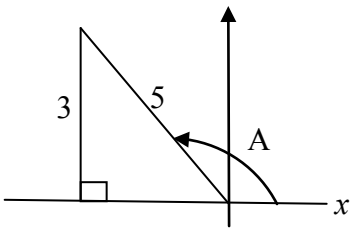
6.2.2(b)	
	
	$P^{/}(6;4) \rightarrow P^{//}(-3;5)$
	$Q^{/}(-2;4) \rightarrow Q^{//}(5;5)$
	$R^{/}(-4;2) \rightarrow R^{//}(7;3)$
	$S^{/}(-2;0) \rightarrow S^{//}(5;1)$
		✓ $P^{//}(-3;5)$
		✓ $Q^{//}(5;5)$
		✓ $R^{//}(7;3)$
		✓ $S^{//}(5;1)$
		✓ joining the points
		(5)

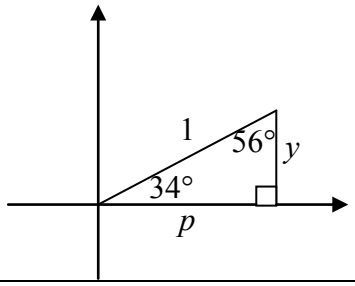
6.2.3	Perimeter/*Omtrek* PQRS = $t \times$ Perimeter/*Omtrek* $P^{//}Q^{//}R^{//}S^{//}$ Perimeter/*Omtrek* PQRS = $t \times 2$ Perimeter/*Omtrek* PQRS $\therefore \quad t = \frac{1}{2}$	✓ 2 Perimeter PQRS/ 2 *omtrek PQRS* ✓ $\frac{1}{2}$ (2) **[17]**

QUESTION/*VRAAG* 7

	A regular octagon has 8 equal sides. Therefore each side subtends an angle of 45º at the centre of the octagon./*'n Reëlmatige oktagoon het 8 gelyke sye. Dus elke sy onderspan 'n hoek van 45° by die middelpunt van die oktagoon.* Angle of rotation is/*Hoek van rotasie is*: $A\hat{O}B = 135°$ $x' = x\cos\theta - y\sin\theta$ $= 8{,}42\cos 135° - 20{,}33\sin 135°$ $= -20.33$ $y' = y\cos\theta + x\sin\theta$ $= 20{,}33\cos 135° + 8{,}42\sin 135°$ $= -8{,}42$ B (– 20,33 ; – 8,42) **OR** Draw a sketch:  ΔAOC ≡ ΔBOD (S ; ∠ ; S *or* S ; S ; S) ∴ B(– *b* ; – *a*) ∴ B(–20,33 ; –8,42)	✓ $\frac{360°}{8} = 45°$ ✓ rotation of 135°/ *rotasie van 135°* ✓ substitution into correct formula ✓ – 20,33 ✓ substitution into correct formula ✓ – 8,42 **[6]** ✓ ✓✓ Sketch drawn showing the equal parts **OR** proving the Δ's ≡ ✓ B(– *b* ; – *a*) ✓ – 20,33 ✓ –8,42 **[6]**

QUESTION/*VRAAG* 8

8.1.1	$\sin A = \frac{3}{5}$ (given) $\sin(-A)$ $= -\sin A$ $= -\frac{3}{5}$	✓ sin A ✓ value (2)
8.1.2	$\sin^2 A + \cos^2 A = 1$ $\cos^2 A = 1 - \frac{9}{25} = \frac{16}{25}$ $\cos A = -\frac{4}{5}$ $\tan A = \frac{\sin A}{\cos A}$ $= \frac{3}{5} \times -\frac{5}{4}$ $= -\frac{3}{4}$ **OR** $x = -4$ $\tan A = -\frac{3}{4}$  	✓ $\cos^2 A = \frac{16}{25}$ ✓ $\cos A = -\frac{4}{5}$ ✓ ratio (3) ✓ sketch in correct quadrant ✓ $x = -4$ ✓ ratio (3)
8.2.1	cos 214° = cos (180° + 34°) = – cos 34° = – p	✓ – cos 34° ✓ – p (2)
8.2.2	cos 68° = cos[2(34°)] = $2\cos^2 34° - 1$ = $2p^2 - 1$	✓ cos [2(34°)] ✓ $2p^2 - 1$ (2)

8.2.3	$\tan 56° = \dfrac{\sin 56°}{\cos 56°}$ $= \dfrac{\cos 34°}{\sin 34°}$ $= \dfrac{\cos 34°}{\sqrt{1-\cos^2 34°}}$ $= \dfrac{p}{\sqrt{1-p^2}}$ **OR** $y^2 = 1 - p^2$ $y = \sqrt{1-p^2}$ $\therefore \tan 56° = \dfrac{p}{\sqrt{1-p^2}}$  	✓ identity ✓ co-functions ✓ $\sqrt{1-\cos^2 34°}$ ✓ answer (4) ✓ sketch with 34° ✓ $y = \sqrt{1-p^2}$ ✓ 90° – 34° = 56° ✓ answer (4)
8.3	$\cos 350° \sin 40° - \cos 440° \cos 40°$ $= \cos 10° \sin 40° - \cos 80° \cos 40°$ $= \cos 10° \sin 40° - \sin 10° \cos 40°$ $= \sin 40° \cos 10° - \cos 40° \sin 10°$ $= \sin(40° - 10°)$ $= \sin 30°$ $= \dfrac{1}{2}$ **OR** $\cos 350° \sin 40° - \cos 440° \cos 40°$ $= \cos 10° \sin 40° - \cos 80° \cos 40°$ $= \cos 10° \cos 50° - \sin 10° \sin 50°$ $= \cos(10° + 50°)$ $= \cos 60°$ $= \dfrac{1}{2}$ **NOTE:** There are many solutions.	✓ cos 10° ✓ cos 80° ✓ sin10° ✓ sin 30° ✓ answer (5) ✓ cos 10° ✓ cos 80° ✓ cos 50° and sin 50° ✓ cos 60° ✓ answer (5) **[18]**

QUESTION/*VRAAG* 9

9.1	$\cos(x-45°) = -2\sin x$ $\cos x\cos 45° + \sin x \sin 45° = -2\sin x$ $\frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x = -2\sin x$ $\sqrt{2}\cos x = (-4-\sqrt{2})\sin x$ $\sqrt{2} = \frac{(-4-\sqrt{2})\sin x}{\cos x}$ $\tan x = \frac{\sqrt{2}}{-4-\sqrt{2}} = -0{,}2612$	✓ expansion ✓ $\frac{\sqrt{2}}{2}$ ✓ simplification ✓ dividing by $\cos x$ (4)
9.2	$\tan x = -0{,}2612...$ $x = 165{,}36° + 180°\,k;\ k \in Z$ $x = -14{,}64°$ *or* $165{,}36°$	✓ general solution ✓ ✓ values of x (3)
9.3	T (135° ; 0)	✓ x value ✓ y value (2)
9.4	$f(x) \geq g(x)$ $-14{,}64° \leq x \leq 165{,}36°$ **OR** $x \in [-14{,}64°\ ;\ 165{,}36°]$	✓ extreme values ✓ notation (2)
9.5	$-135° < x < -90°$ **OR** $x \in (-135°\ ;\ -90°)$	✓✓ extreme values ✓ notation (3)
9.6	$h(x) = \cos(x-45°-45°)$ $= \cos(x-90°)$ $= \sin x$ Answer only: full marks	✓ $\cos(x-90°)$ ✓ $\sin x$ (2) **[16]**

QUESTION/*VRAAG* 10

10.1	In ΔTRQ: $\frac{TR}{\sin R\hat{Q}T} = \frac{TQ}{\sin Q\hat{R}T}$ $\frac{TR}{\sin 60°} = \frac{k}{\sin[180° - (\theta + 60°)]}$ $TR = \frac{k \sin 60°}{\sin[180° - (\theta + 60°)]}$ *or* $TR = \frac{k \sin 60°}{\sin(120° - \theta)}$	✓ using sine rule ✓ correct substitution ✓ rewrite TR as subject (3)
10.2	In ΔTRS: $\frac{RS}{TR} = \sin R\hat{T}S$ $RS = TR.\sin 60°$ $= \frac{k \sin 60°}{\sin[180° - (\theta + 60°)]}.\sin 60°$ $= \frac{k \sin 60°}{\sin(120° - \theta)}.\sin 60°$ $= \frac{k(\frac{\sqrt{3}}{2})^2}{\sin 120° \cos\theta - \cos 120° \sin\theta}$ $= \frac{3k}{4\left(\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta\right)}$ $= \frac{3k}{2(\sqrt{3}\cos\theta + \sin\theta)}$	✓ using sine ratio ✓ substitution of TR ✓ simplification ✓ $k(\frac{\sqrt{3}}{2})^2$ or $\frac{3k}{4}$ ✓ expansion of denominator ✓ value of sin 120° ✓ value of cos 120° (7) **[10]**

QUESTION/*VRAAG* 11

11.1.1	$f(x) = y = 3 - 2\sin^2 x$ $0 \le \sin^2 x \le 1$ $-2 \le -2\sin^2 x \le 0$ $1 \le 3 - 2\sin^2 x \le 3$ $1 \le y \le 3$ **OR** $y \in [1\ ;\ 3]$ **OR** $f(x) = y = 3 - 2\sin^2 x$ $= 2 + (1 - 2\sin^2 x)$ $= \cos 2x + 2$ $-1 \le \cos 2x \le 1$ $1 \le \cos 2x + 2 \le 3$ $1 \le y \le 3$ **OR** $y \in [1\ ;\ 3]$ **OR** $f(x) = y = 3 - 2\sin^2 x$ $= 3 - 2(1 - \cos^2 x)$ $= 1 + 2\cos^2 x$ $0 \le 2\cos^2 x \le 2$ $1 \le 1 + 2\cos^2 x \le 3$ $1 \le y \le 3$ **OR** $y \in [1\ ;\ 3]$	✓ ✓ $-2 \le -2\sin^2 x \le 0$ ✓ ✓ $1 \le 3 - 2\sin^2 x \le 3$ (4) ✓ rewriting ✓ $\cos 2x + 2$ ✓ $-1 \le \cos 2x \le 1$ ✓ $1 \le \cos 2x + 2 \le 3$ (4) ✓ rewriting ✓ $1 + 2\cos^2 x$ ✓ $0 \le 2\cos^2 x \le 2$ ✓ $1 \le 1 + 2\cos^2 x \le 3$ (4)
11.1.2	f has a minimum when $\sin^2 x = 1$ $\therefore\ \sin x = \pm 1$ $\therefore\ x = 90°$ *or* $-90°$ **OR** f has a minimum when $\cos 2x = -1$ $\therefore\ 2x = 180°$ *or* $-180°$ $\therefore\ x = 90°$ *or* $-90°$	✓ $\sin^2 x = 1$ ✓ $90°$ ✓ $-90°$ (3) ✓ $\cos 2x = -1$ ✓ $90°$ ✓ $-90°$ (3)
11.2.1	$LHS = 1 - \cos 2Q$ $= 1 - (1 - 2\sin^2 Q)$ $= 2\sin^2 Q$ $= RHS$	✓ identity (1)
11.2.2(a)	$LHS = \sin 2R$ $= \sin 2[180° - (P + Q)]$ $= \sin[360° - 2(P + Q)]$ $= -\sin 2(P + Q)$ $= -\sin(2P + 2Q)$ $= RHS$	✓ $R = 180° - (P + Q)$ ✓ $360° - 2(P + Q)$ ✓ $-2(P + Q)$ (3)

11.2.2(b)	$LHS = \sin 2P + \sin 2Q + \sin 2R$ $= \sin 2P + \sin 2Q - \sin(2P + 2Q)$ $= \sin 2P + \sin 2Q - [\sin 2P \cos 2Q + \cos 2P \sin 2Q]$ $= \sin 2P + \sin 2Q - \sin 2P \cos 2Q - \cos 2P \sin 2Q$ $= \sin 2P(1 - \cos 2Q) + \sin 2Q(1 - \cos 2P)$ $= \sin 2P(2\sin^2 Q) + \sin 2Q(2\sin^2 P)$ $= 2\sin P \cos P.2\sin^2 Q + 2\sin Q \cos Q.2\sin^2 P$ $= 4\sin P \sin Q(\sin Q \cos P + \cos Q \sin P)$ $= 4\sin P \sin Q(\sin(Q + P))$ $= 4\sin P \sin Q(\sin[180° - (Q + P)])$ $= 4\sin P \sin Q \sin R$ $= RHS$	✓ substitution ✓ expansion ✓ factorising ✓ substitution ✓ identities ✓ factorising ✓ $[180° - (Q + P)]$ (7) **[18]**

TOTAL/*TOTAAL*: 150

PolyMathic

Vraestel 5
Okt/Nov
Eksamen

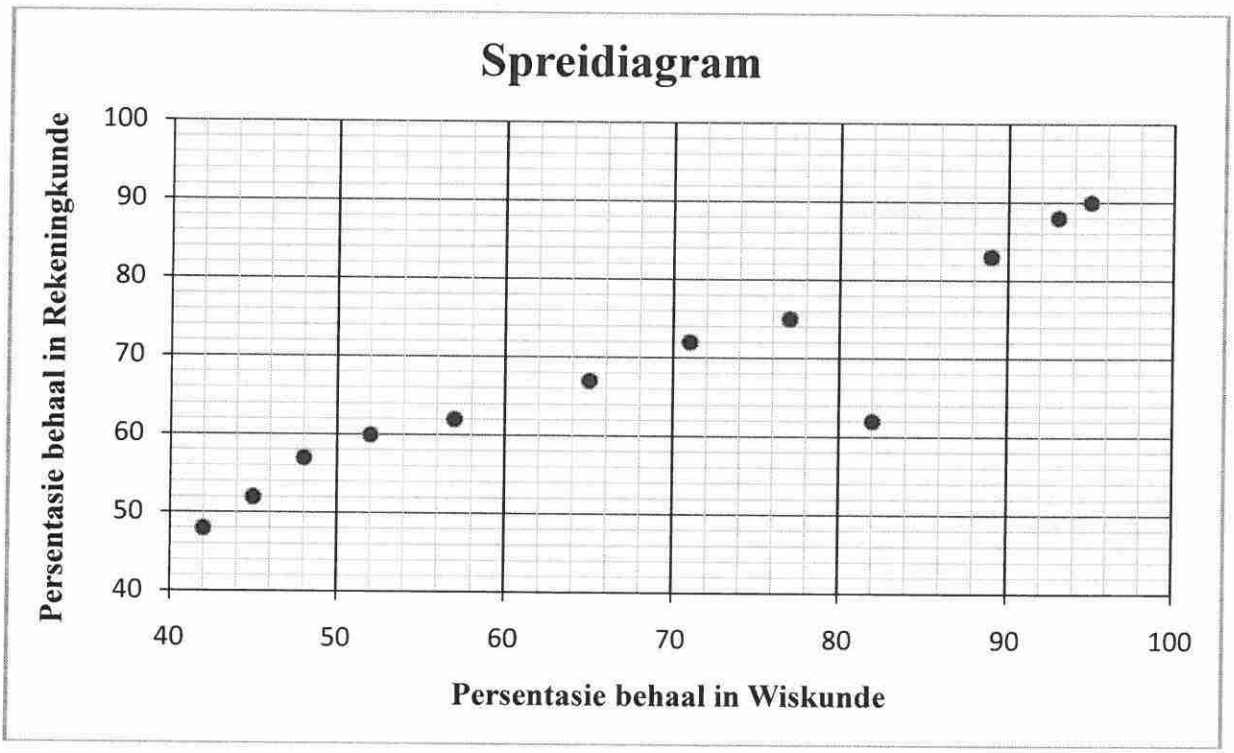
PolyMathic

Graad 12 EindEksamen
Totaal: 150
Tyd: 3ure

VRAAG 1

By 'n sekere skool neem slegs 12 kandidate Wiskunde en Rekeningkunde. Die punte, as 'n persentasie, wat deur hierdie kandidate in die voorbereidende eksamen in Wiskunde en Rekeningkunde behaal is, word in die tabel en spreidiagram hieronder getoon.

Wiskunde	52	82	93	95	71	65	77	42	89	48	45	57
Rekeningkunde	60	62	88	90	72	67	75	48	83	57	52	62



1.1 Bereken die gemiddelde persentasie van die Wiskunde-data. (2)

1.2 Bereken die standaardafwyking van die Wiskunde-data. (1)

1.3 Bepaal die getal kandidate wie se persentasie in Wiskunde binne EEN standaardafwyking vanaf die gemiddelde is. (3)

1.4 Bereken 'n vergelyking vir die kleinstekwadrate-regressielyn (lyn van beste passing) vir die data. (3)

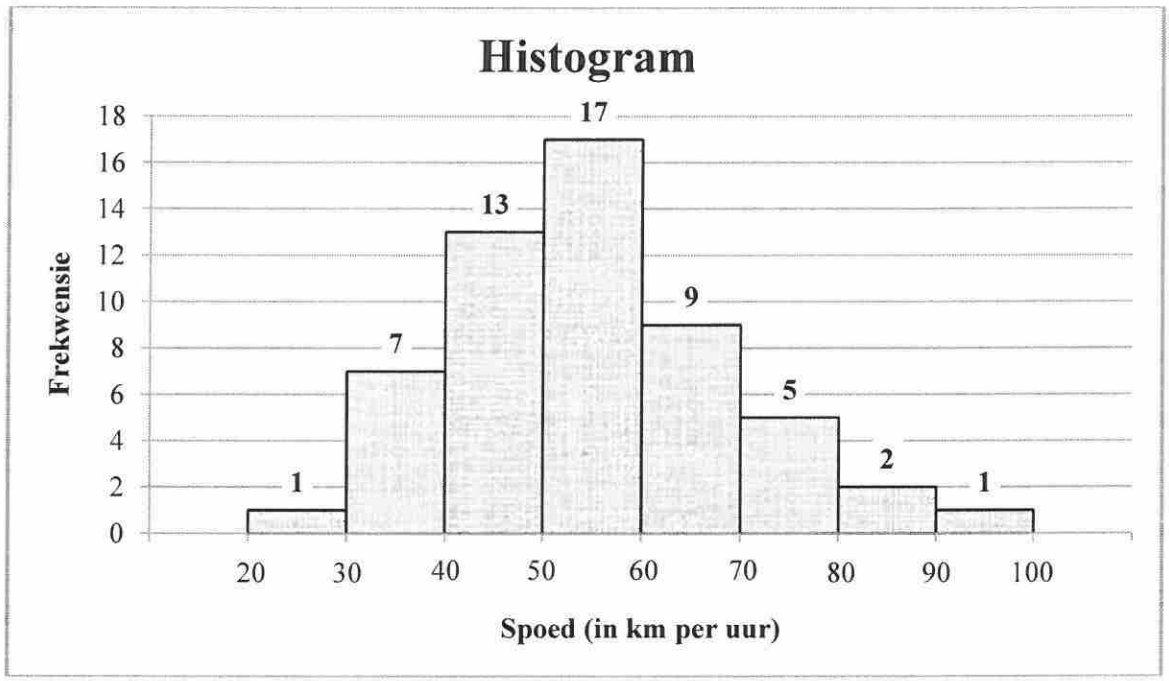
1.5 Indien 'n kandidaat uit hierdie groep 60% in die Wiskunde-eksamen behaal, maar vir die Rekeningkunde-eksamen afwesig was, voorspel, deur jou vergelyking in VRAAG 1.4 te gebruik, die persentasie wat hierdie kandidaat in die Rekeningkunde-eksamen sou behaal het. (Rond jou antwoord tot die NAASTE HEELGETAL af.) (2)

1.6 Gebruik die spreidiagram en identifiseer enige uitskieter(s) in die data. (1)

[12]

VRAAG 2

Die spoed van 55 motors wat deur 'n sekere gedeelte van 'n pad beweeg, word vir een uur gemonitor. Die spoedgrens op hierdie gedeelte van die pad is 60 km per uur. 'n Histogram is geskets om hierdie data voor te stel.



2.1 Identifiseer die modale klas van die data. (1)

2.2 Gebruik die histogram om:

2.2.1 Die kumulatiewefrekwensie-kolom in die tabel op DIAGRAMVEL 1 te voltooi (2)

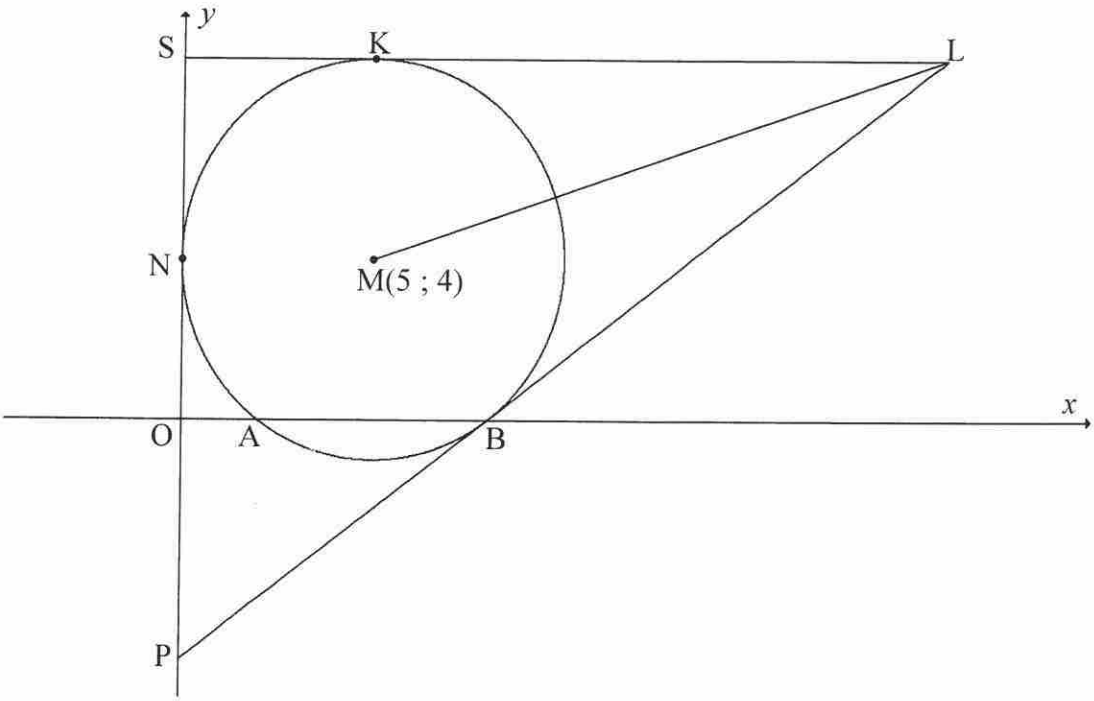
2.2.2 'n Ogief (kumulatiewefrekwensie-grafiek) van die data hierbo op die rooster op DIAGRAMVEL 1 te teken (3)

2.3 Die verkeersdepartement stuur spoedboetes aan alle motoriste wat 'n spoed van 66 km per uur oorskry. Skat die getal motoriste wat 'n spoedboete sal ontvang. (2)

[8]

VRAAG 3

In die diagram hieronder raak 'n sirkel met middelpunt M(5 ; 4) die y-as by N en sny die x-as by A en B. PBL en SKL is raaklyne aan die sirkel waar SKL ewewydig aan die x-as en P en S punte op die y-as is. LM is getrek.



3.1 Skryf die lengte van die radius van die sirkel met middelpunt M neer. (1)

3.2 Skryf die vergelyking van die sirkel met middelpunt M in die vorm $(x-a)^2+(y-b)^2=r^2$ neer. (1)

3.3 Bereken die koördinate van A. (3)

3.4 Indien (8 ; 0) die koördinate van B is, bereken:

3.4.1 Die gradiënt van MB (2)

3.4.2 Die vergelyking van die raaklyn PB in die vorm $y=mx+c$ (3)

3.5 Skryf die vergelyking van raaklyn SKL neer. (2)

3.6 Toon aan dat L die punt (20 ; 9) is. (2)

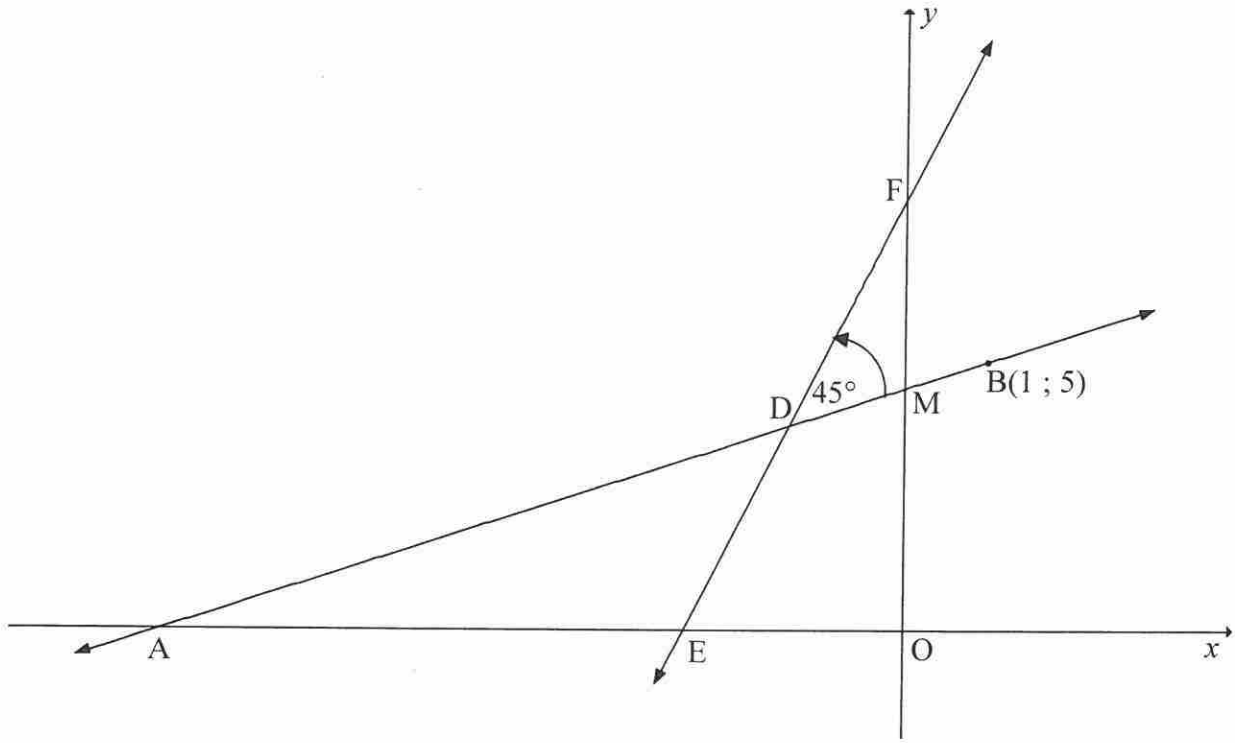
3.7 Bereken die lengte van ML in wortelvorm. (2)

3.8 Bepaal die vergelyking van die sirkel wat deur punt K, L en M gaan in die vorm $(x-p)^2+(y-q)^2=c^2$ (5)

[21]

VRAAG 4

In die diagram hieronder is E en F onderskeidelik die x- en y-afsnit van die lyn met vergelyking $y = 3x + 8$. Die lyn deur B(1 ; 5) wat 'n hoek van 45° met EF vorm, soos hieronder aangetoon, het x- en y-afsnitte by A en M onderskeidelik .



4.1 Bepaal die koördinate van E. (2)

4.2 Bereken die grootte van $D\hat{A}E$. (3)

4.3 Bepaal die vergelyking van AB in die vorm $y = mx + c$. (4)

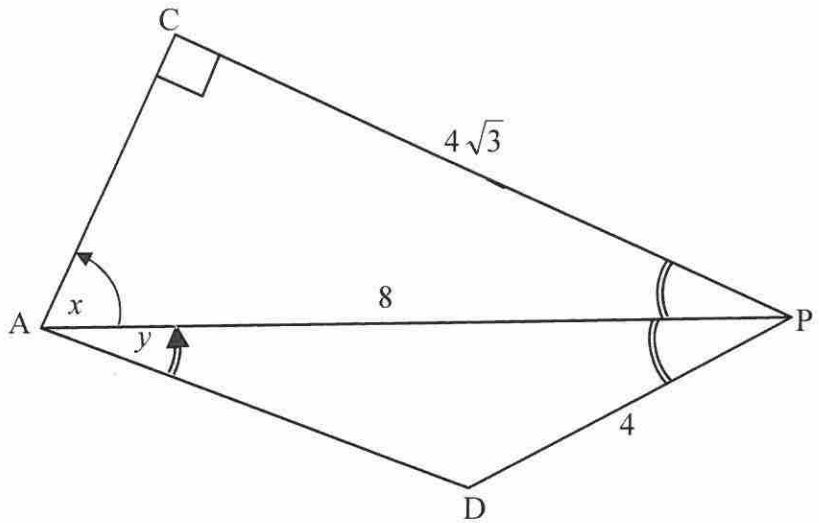
4.4 Indien $x - 2y + 9 = 0$ die vergelyking van AB is, bepaal die koördinate van D. (4)

4.5 Bereken die oppervlakte van vierhoek DMOE. (6)

[19]

VRAAG 5

In die figuur hieronder is ACP en ADP driehoeke met $\hat{C} = 90°$, $CP = 4\sqrt{3}$, $AP = 8$ en $DP = 4$. PA halveer $D\hat{P}C$. Gestel $C\hat{A}P = x$ en $D\hat{A}P = y$.



5.1 Toon aan, deur berekeninge, dat $x = 60°$. (2)

5.2 Bereken die lengte van AD. (4)

5.3 Bepaal y. (3)

[9]

VRAAG 6

6.1 Bewys die identiteit: $\cos^2(180° + x) + \tan(x - 180°)\sin(720° - x)\cos x = \cos 2x$ (5)

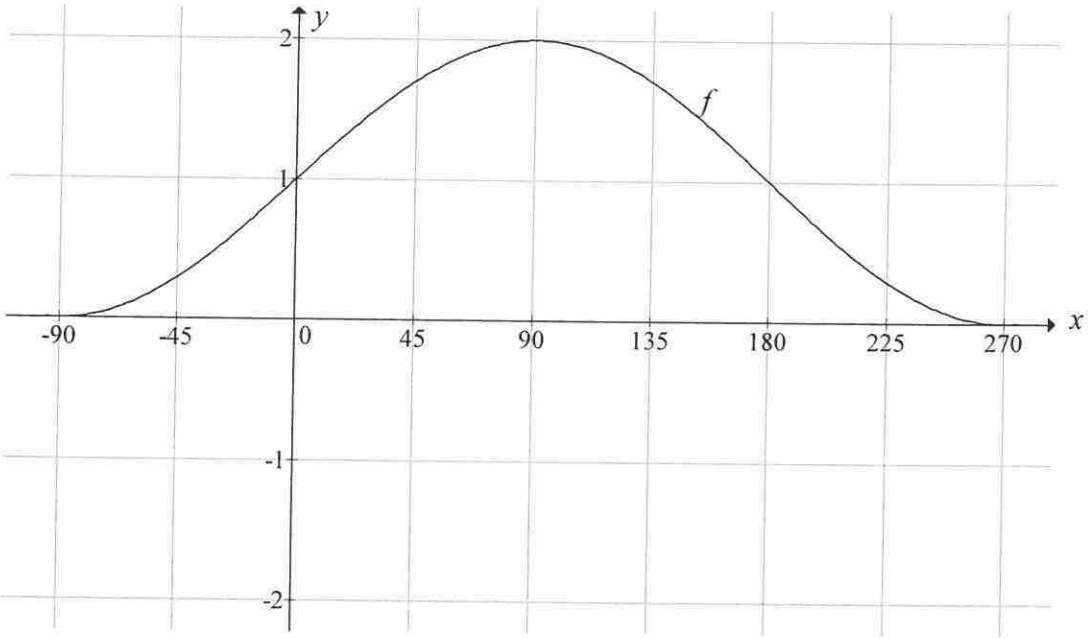
6.2 Gebruik $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ om die formule vir $\sin(\alpha - \beta)$ af te lei. (3)

6.3 Indien $\sin 76° = x$ en $\cos 76° = y$, toon aan dat $x^2 - y^2 = \sin 62°$. (4)

[12]

VRAAG 7

In die diagram hieronder is die grafiek van $f(x) = \sin x + 1$ geskets vir $-90° \leq x \leq 270°$.



7.1 Skryf die waardeversameling van f neer. (2)

7.2 Toon aan dat $\sin x + 1 = \cos 2x$ as $(2\sin x + 1)\sin x = 0$ herskryf kan word. (2)

7.3 Bepaal vervolgens die algemene oplossing van $\sin x + 1 = \cos 2x$. (4)

7.4 Gebruik die rooster op DIAGRAMVEL 2 om die grafiek van $g(x) = \cos 2x$ vir $-90° \leq x \leq 270°$ te teken. (3)

7.5 Bepaal die waarde(s) van x waarvoor $f(x + 30°) = g(x + 30°)$ in die interval $-90° \leq x \leq 270°$. (3)

7.6 Beskou die volgende meetkundige reeks:

$1 + 2\cos 2x + 4\cos^2 2x + \ldots$

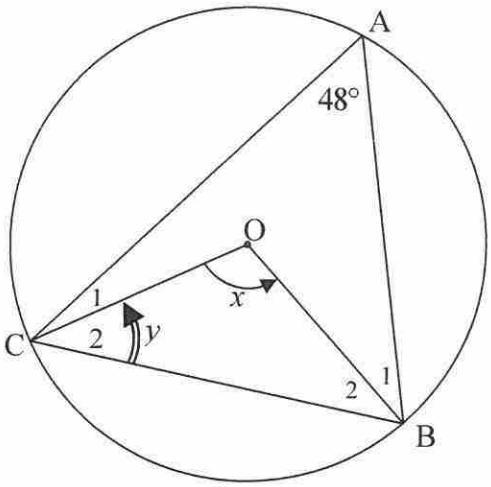
Gebruik die grafiek van g om te bepaal vir watter waarde(s) van x in die interval $0° \leq x \leq 90°$ hierdie reeks sal konvergeer. (5)

[19]

GEE REDES VIR JOU BEWERINGS IN VRAAG 8, 9 EN 10.

VRAAG 8

8.1 In die diagram is O die middelpunt van die sirkel wat deur A, B en C gaan. $C\hat{A}B = 48°$, $C\hat{O}B = x$ en $\hat{C}_2 = y$.

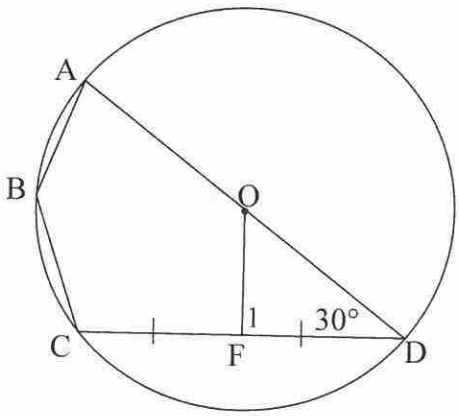


Bepaal, met redes, die grootte van:

8.1.1 x (2)

8.1.2 y (2)

8.2 In die diagram is O die middelpunt van die sirkel wat deur A, B, C en D gaan. AOD is 'n reguitlyn en F is die middelpunt van koord CD. $O\hat{D}F = 30°$ en OF is verbind.

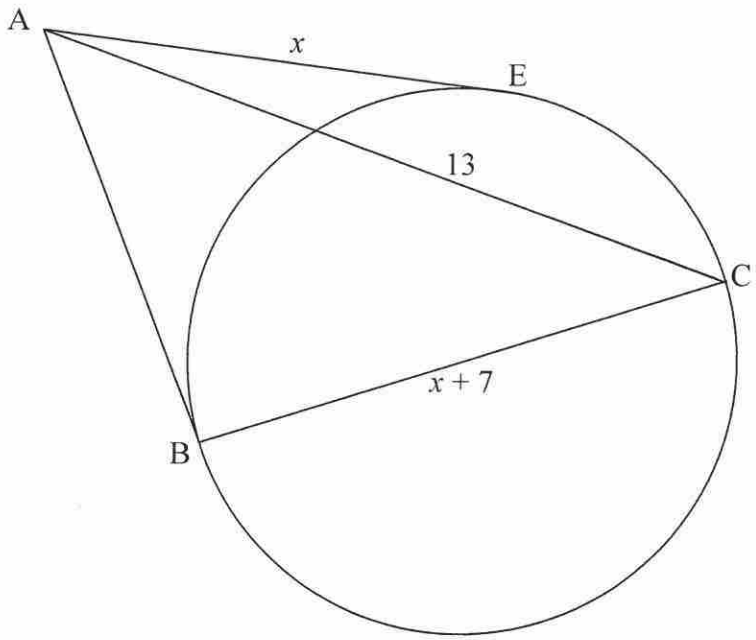


Bepaal, met redes, die grootte van:

8.2.1 $\hat{F}_1$ (2)

8.2.2 $A\hat{B}C$ (2)

8.3 In die diagram is AB en AE raaklyne aan die sirkel by B en E onderskeidelik. BC is 'n middellyn van die sirkel. AC = 13, AE = x en BC = $x + 7$.



8.3.1 Gee redes vir die bewerings hieronder.
Voltooi die tabel op DIAGRAMVEL 3.

	Bewering	**Rede**
(a)	$A\hat{B}C = 90°$	
(b)	AB = x	

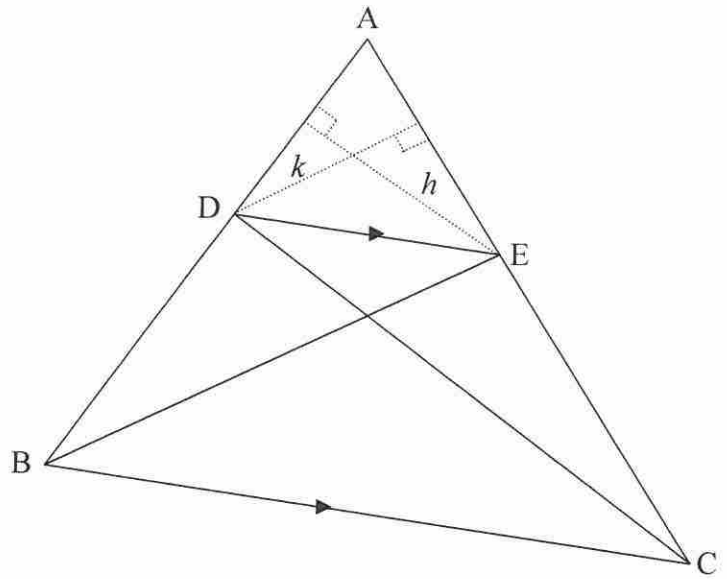
(2)

8.3.2 Bereken die lengte van AB. (4)

[14]

VRAAG 9

9.1 In die diagram lê punte D en E op sye AB en AC van ΔABC onderskeidelik sodat $DE \,||\, BC$. DC en BE is verbind.



9.1.1 Verduidelik waarom die oppervlaktes van ΔDEB en ΔDEC gelyk is. (1)

9.1.2 Hieronder verskyn die gedeeltelik voltooide bewys van die stelling wat beweer dat indien in enige ΔABC die lyn $DE \,||\, BC$ dan is $\frac{AD}{DB} = \frac{AE}{EC}$.

Gebruik die diagram hierbo en voltooi die bewys van die stelling op DIAGRAMVEL 4.

Konstruksie: Konstrueer die hoogtelyne (hoogtes) h en k in ΔADE.

$$\frac{\text{oppervlakte } \Delta ADE}{\text{oppervlakte } \Delta DEB} = \frac{\frac{1}{2}(AD)(h)}{\frac{1}{2}(BD)(h)} = \ldots\ldots$$

$$\frac{\text{oppervlakte } \Delta ADE}{\text{oppervlakte } \Delta DEC} = \ldots\ldots\ldots\ldots\ldots\ldots = \frac{AE}{EC}$$

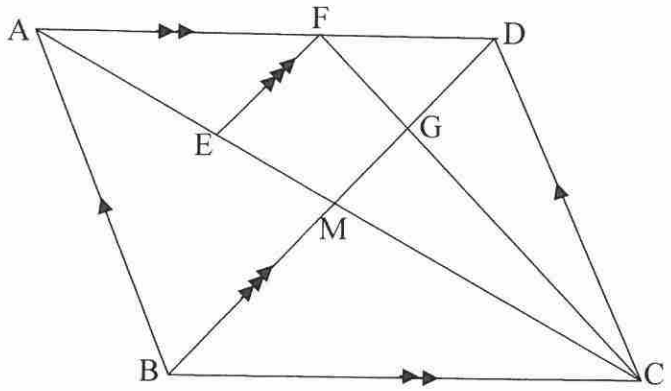
Maar oppervlakte ΔDEB = ... (rede: .....)

$$\therefore \frac{\text{oppervlakte } \Delta ADE}{\text{oppervlakte } \Delta DEB} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

(5)

9.2 In die diagram is ABCD 'n parallelogram. Die hoeklyne van ABCD sny by M. F is 'n punt op AD sodat AF : FD = 4 : 3. E is 'n punt op AM sodat EF | | BD. FC en MD sny by G.



Bereken, met redes, die verhouding van:

9.2.1 $\dfrac{EM}{AM}$ (3)

9.2.2 $\dfrac{CM}{ME}$ (3)

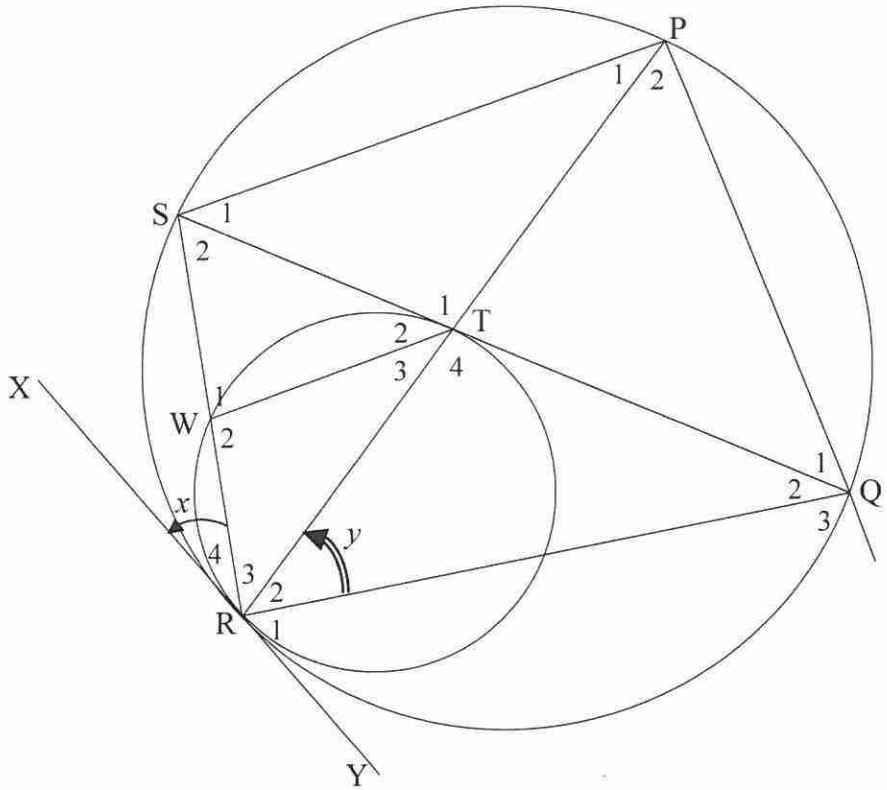
9.2.3 $\dfrac{\text{oppervlakte } \Delta FDC}{\text{oppervlakte } \Delta BDC}$ (4)

[16]

VRAAG 10

Die twee sirkels in die diagram het 'n gemeenskaplike raaklyn XRY by R. W is enige punt op die klein sirkel. Die reguitlyn RWS ontmoet die groter sirkel by S. Die koord STQ is 'n raaklyn aan die klein sirkel, met T as die raakpunt. Koord RTP is getrek.

Gestel $\hat{R}_4 = x$ en $\hat{R}_2 = y$



10.1 Gee redes vir die bewerings hieronder.
Voltooi die tabel op DIAGRAMVEL 6.

Gestel $\hat{R}_4 = x$ en $\hat{R}_2 = y$		
	Bewering	**Rede**
10.1.1	$\hat{T}_3 = x$	
10.1.2	$\hat{P}_1 = x$	
10.1.3	WT || SP	
10.1.4	$\hat{S}_1 = y$	
10.1.5	$\hat{T}_2 = y$	

(5)

10.2 Bewys dat $RT = \frac{WR.RP}{RS}$ (2)

10.3 Identifiseer, met redes, nog TWEE ander hoeke gelyk aan y. (4)

10.4 Bewys dat $\hat{Q}_3 = \hat{W}_2$. (3)

10.5 Bewys dat $\Delta RTS \,|||\, \Delta RQP$. (3)

10.6 Bewys vervolgens dat $\frac{WR}{RQ} = \frac{RS^2}{RP^2}$. (3)

[20]

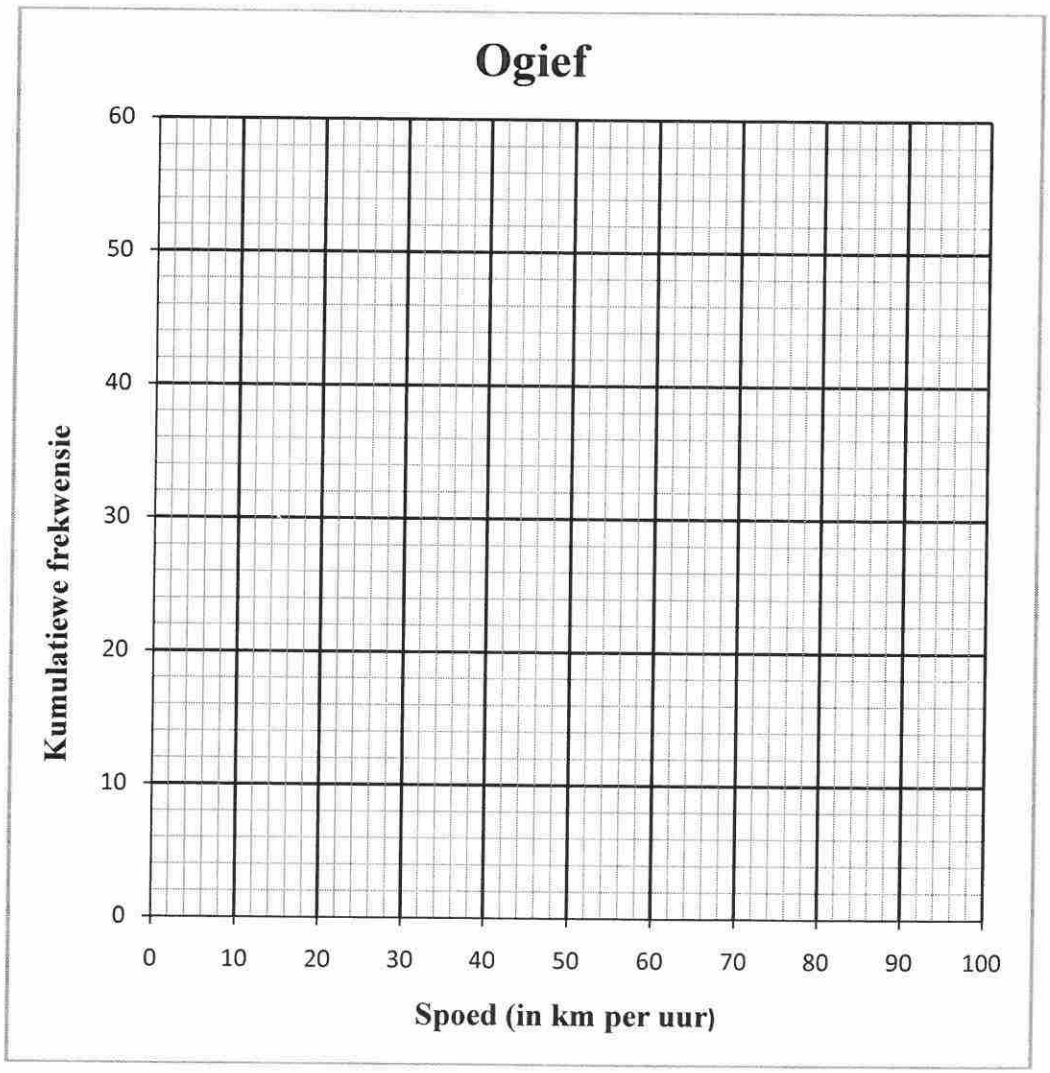
TOTAAL: 150

DIAGRAMVEL 1

VRAAG 2.2.1

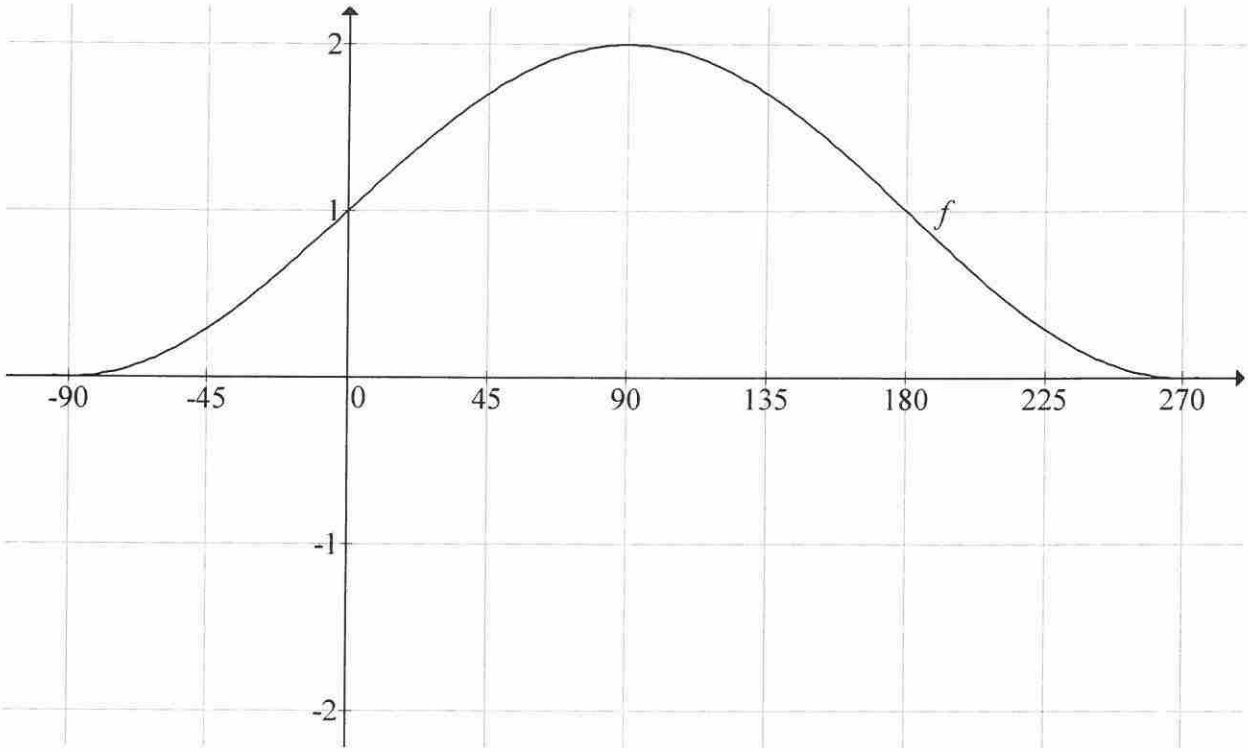
Klas	Frekwensie	Kumulatiewe frekwensie
$20 < x \leq 30$	1	
$30 < x \leq 40$	7	
$40 < x \leq 50$	13	
$50 < x \leq 60$	17	
$60 < x \leq 70$	9	
$70 < x \leq 80$	5	
$80 < x \leq 90$	2	
$90 < x \leq 100$	1	

VRAAG 2.2.2

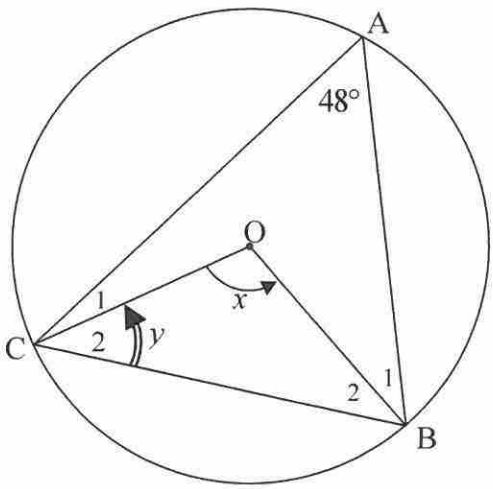


DIAGRAMVEL 2

VRAAG 7.4

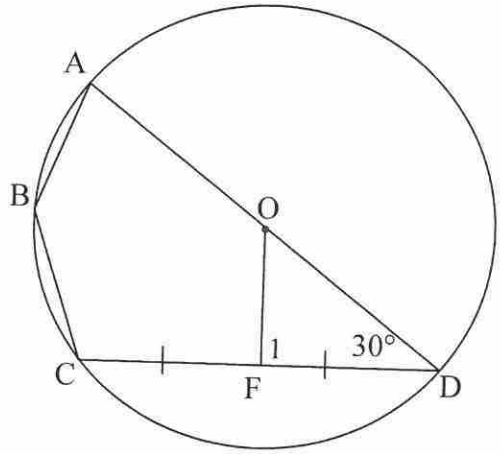


VRAAG 8.1

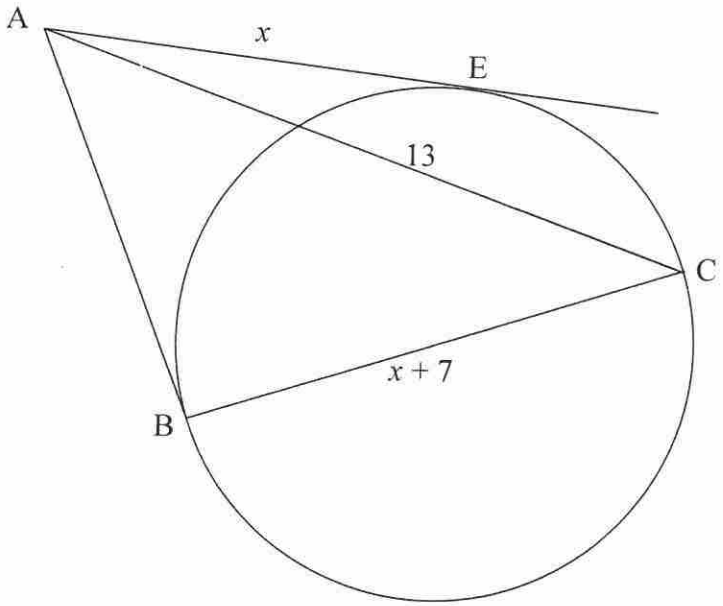


DIAGRAMVEL 3

VRAAG 8.2



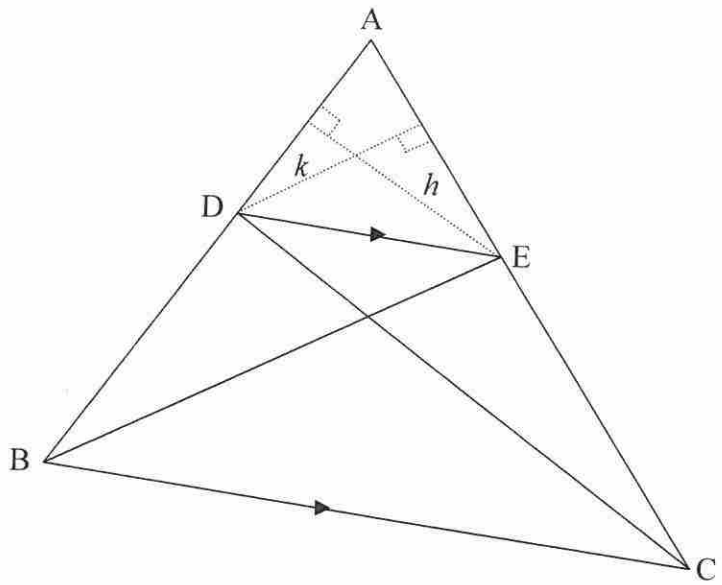
VRAAG 8.3



8.3.1	Bewering	Rede
(a)	$A\hat{B}C = 90°$	
(b)	$AB = x$	

DIAGRAMVEL 4

VRAAG 9.1



9.1.2 Konstruksie: Konstrueer hoogtelyne (hoogtes) h en k in ΔADE.

$$\frac{\text{oppervlakte } \Delta ADE}{\text{oppervlakte } \Delta DEB} = \frac{\frac{1}{2}(AD)(h)}{\frac{1}{2}(BD)(h)} = \ldots\ldots$$

$$\frac{\text{oppervlakte } \Delta ADE}{\text{oppervlakte } \Delta DEC} = \ldots\ldots\ldots\ldots\ldots\ldots = \frac{AE}{EC}$$

Maar oppervlakte ΔDEB = ...

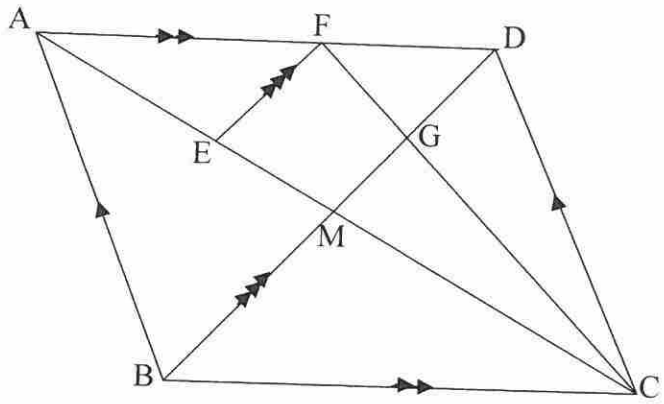
(rede: .....)

$$\therefore \frac{\text{oppervlakte } \Delta ADE}{\text{oppervlakte } \Delta DEB} = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

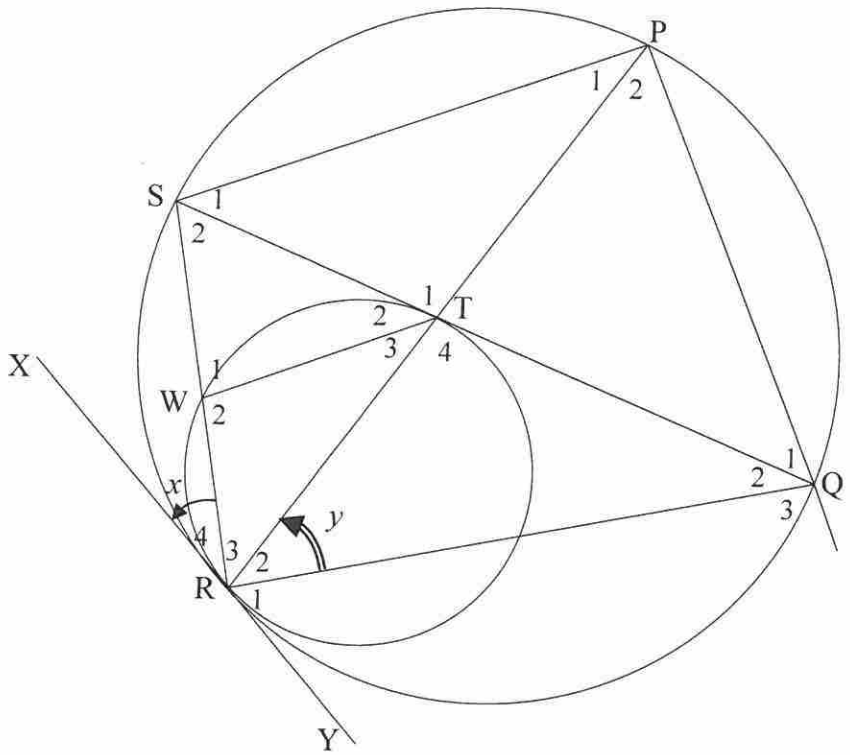
DIAGRAMVEL 5

VRAAG 9.2



DIAGRAMVEL 6

VRAAG 10



Gestel $\hat{R}_4 = x$ en $\hat{R}_2 = y$		
	Bewering	**Rede**
10.1.1	$\hat{T}_3 = x$	
10.1.2	$\hat{P}_1 = x$	
10.1.3	WT || SP	
10.1.4	$\hat{S}_1 = y$	
10.1.5	$\hat{T}_2 = y$	

Memo

NOTA:

- *As 'n kandidaat 'n vraag TWEEKEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n poging om die vraag te beantwoord, doodgetrek het en nie dit oorgedoen het nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienmemorandum toegepas.*
- *Aanvaarding van antwoorde/waardes om 'n probleem op te los, is ONaanvaarbaar.*

QUESTION/*VRAAG* 1

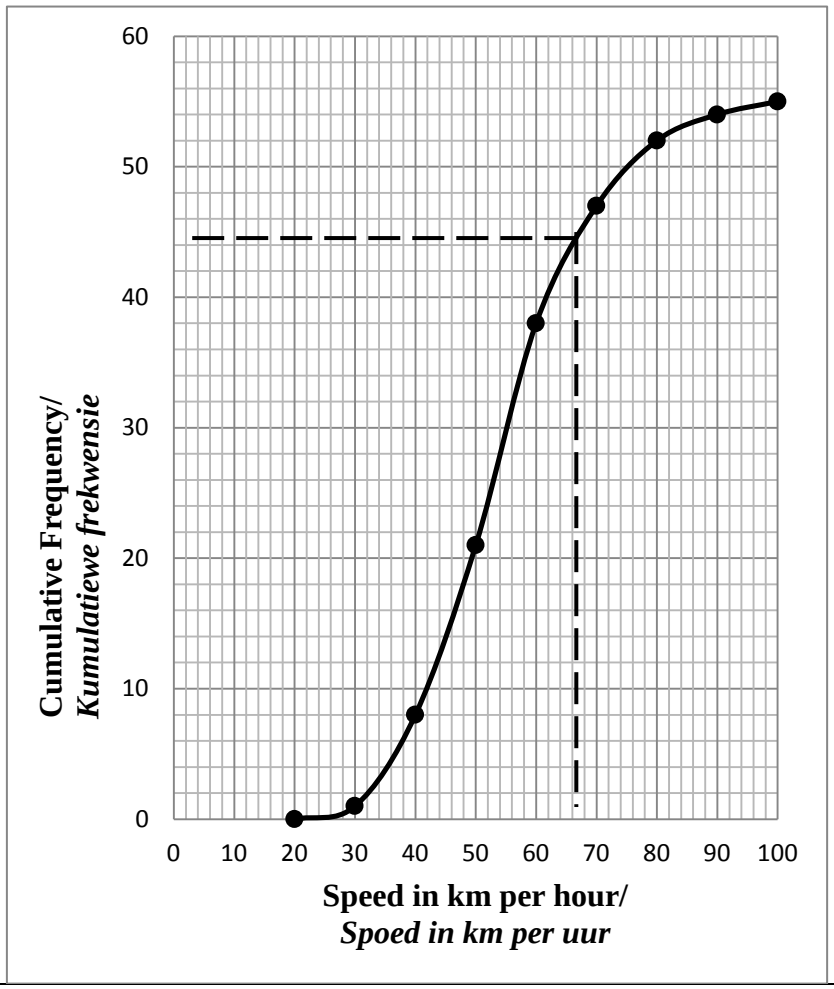
1.1	$\bar{x} = \frac{816}{12} = 68$	✓ $\frac{816}{12}$ ✓ 68 (2)
1.2	$\sigma = 18{,}42$	✓ answer/*antw* (1)
1.3	(68 – 18,42 ; 68 + 18,42) = (49,58 ; 86,42) $\therefore$ 6 candidates had a mark within one standard deviation of the mean/*6 kandidate het 'n punt binne een standaardafwyking vanaf die gemiddelde*.	✓✓ interval ✓ answer/*antw* (3)
1.4	$a = 22{,}828... = 22{,}83$ $b = 0{,}66429... = 0{,}66$ $\therefore \hat{y} = 0{,}66x + 22{,}83$ **OR/*OF*** $\hat{y} = 22{,}83 + 0{,}66x$	✓ value of *a/ waarde van a* ✓ value of *b/ waarde van b* ✓ equation/*vgl* (3)
1.5	$\hat{y} = 0{,}66x + 22{,}83$ $y = 0{,}66(60) + 22{,}83$ $62{,}43...\% \approx 62\%$ **OR/*OF*** $62{,}69\% \approx 63\%$	✓ subs of 60 into equation ✓ answer/*antw* (2) ✓✓ answer/*antw* (2)
1.6	(82 ; 62)	✓ answer/*antw* (1) **[12]**

QUESTION/*VRAAG* 2

2.1	$50 < x \leq 60$ **OR/*OF*** $50 \leq x < 60$ **OR/*OF*** between 50 and 60/*tussen 50 en 60*	✓ answer/*antw* (1)
2.2.1		✓ 8 ✓ 55 (2)

Class ***Klas***	**Frequency** ***Frekwensie***	**Cumulative frequency** ***Kumulatiewe frekwensie***
$20 < x \leq 30$	1	1
$30 < x \leq 40$	7	8
$40 < x \leq 50$	13	21
$50 < x \leq 60$	17	38
$60 < x \leq 70$	9	47
$70 < x \leq 80$	5	52
$80 < x \leq 90$	2	54
$90 < x \leq 100$	1	55

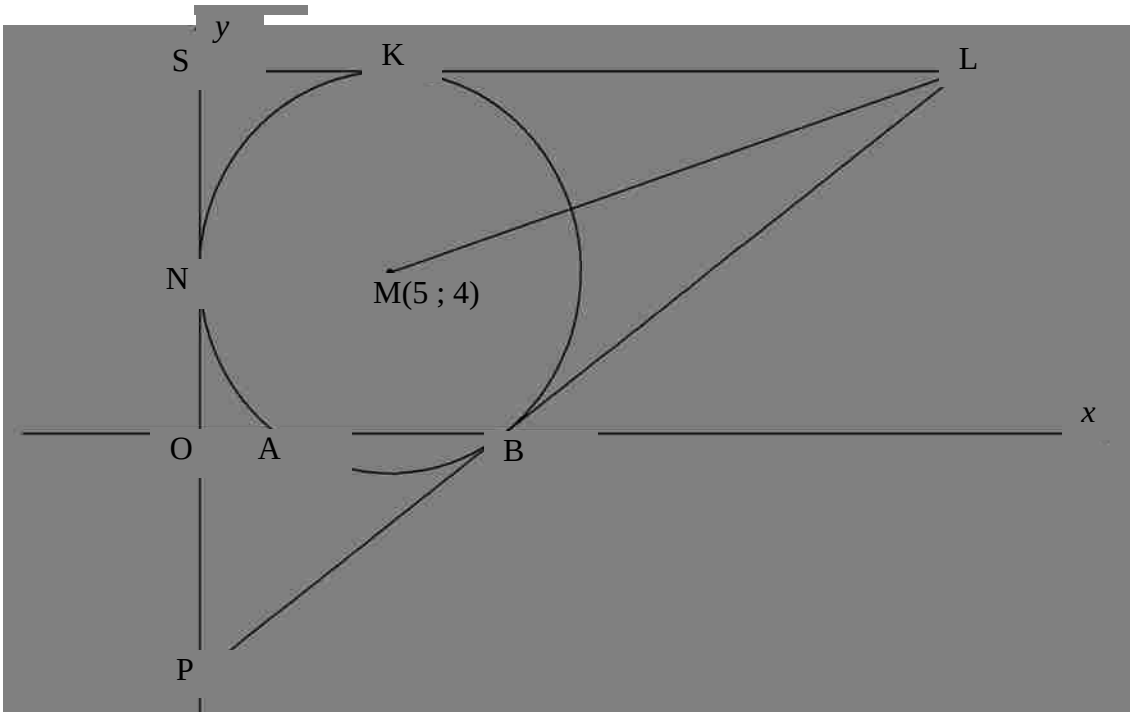
2.2.2



✓ grounding at (20 ; 0)/ *anker by* (20 ; 0)
✓ plotting at upper limits/ *plot by boonste limiete*
✓ smooth shape of curve/*gladde kurwe*
(3)

2.3	55 – 44 (accept/*aanvaar* 43 – 45) ≈ 11 motorists/*motoriste* (accept/*aanvaar* 10 – 12 motorists/*motoriste*)	✓ 44 ✓ 11 (2) **[8]**

QUESTION/*VRAAG* 3

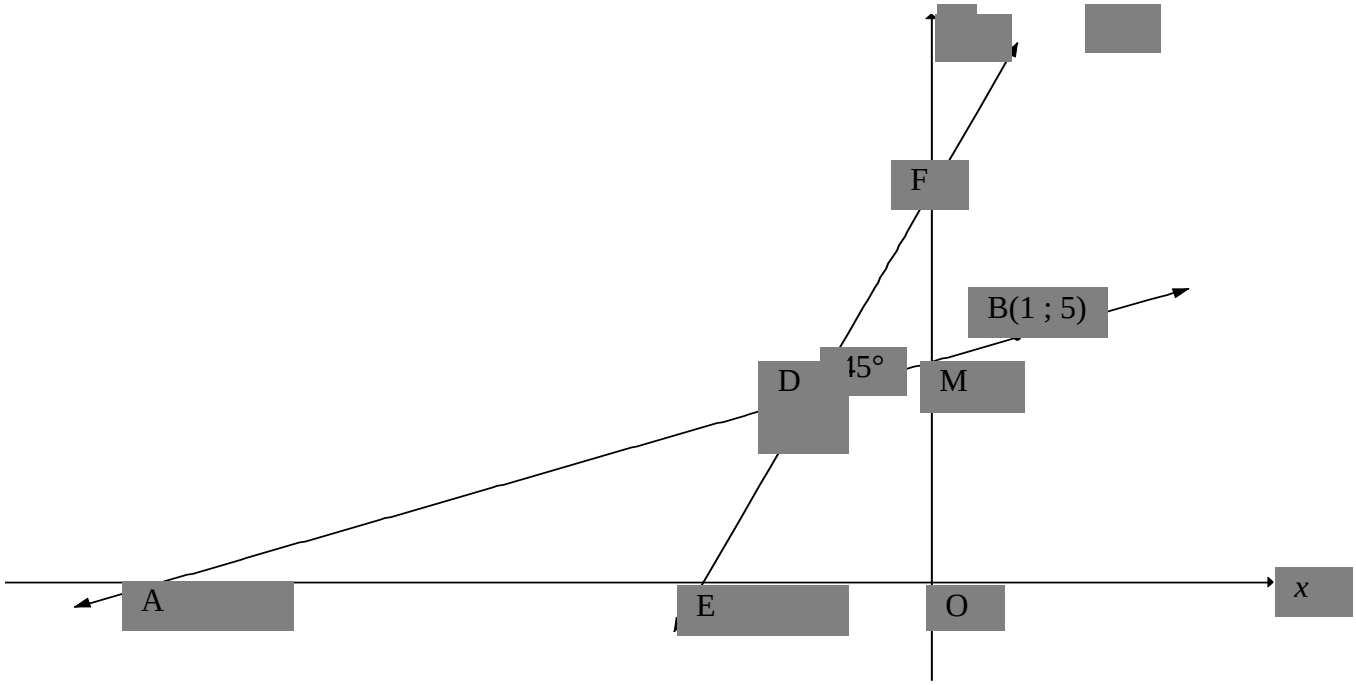


3.1	r = MN = 5	✓ answer/*antw* (1)
3.2	$(x-5)^2+(y-4)^2=25$	✓equation/*vgl* (1)
3.3	A(x ; 0) $(x-5)^2+(0-4)^2=25$ $x^2-10x+25+16=25$ $x^2-10x+16=0$ $(x-8)(x-2)=0$ $\therefore x=8$ or/*of* $x=2$ $\therefore$ A(2 ; 0) **OR/*OF*** $(x-5)^2+(0-4)^2=25$ $(x-5)^2+16=25$ $(x-5)^2=9$ $(x-5)=\pm 3$ $\therefore x=8$ or/*of* $x=2$ $\therefore$ A(2 ; 0)	✓ substitute into eq/ *vervang in vgl* $y=0$ ✓ standard form/ *standaardvorm* or perfect square form/*kwadr vorm* ✓ answer/*antw* (3)
3.4.1	$m_{MB}=\frac{4-0}{5-8}$ $=-\frac{4}{3}$	✓ subst M and B into form/*vervang M and B in form* ✓ $m_{MB}=-\frac{4}{3}$ (2)

3.4.2	$m_{MB} \times m_{PB} = -1$ (tangent ⊥ radius/ *rkl ⊥ radius*) $m_{PB} = \frac{3}{4}$ $y = \frac{3}{4}x + c$ **OR/*OF*** $y - y_1 = \frac{3}{4}(x - x_1)$ $0 = \frac{3}{4}(8) + c$ $y - 0 = \frac{3}{4}(x - 8)$ $y = \frac{3}{4}x - 6$ $y = \frac{3}{4}x - 6$	✓ $m_{MB} \times m_{PB} = -1$ ✓ $m_{PB} = \frac{3}{4}$ ✓ equation/*vgl* (3)
3.5	$y_K = y_M + r = 4 + 5$ $y = 9$	✓ 9 ✓ equation/*vgl* (2)
3.6	At/*By* L: $\frac{3}{4}x - 6 = 9$ $3x - 24 = 36$ $3x = 60$ $x = 20$ $\therefore$ L(20 ; 9)	✓ equating simultaneously ✓ simplification (2)
3.7	L(20 ; 9) $ML = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **OR/OF** $ML = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(20-5)^2 + (9-4)^2}$ $= \sqrt{(15)^2 + (5)^2}$ $= \sqrt{225 + 25}$ $= \sqrt{(5)^2(9+1)}$ $= \sqrt{250}$ *or / of* $5\sqrt{10}$ $= \sqrt{250}$ *or / of* $5\sqrt{10}$	✓ correct subst into distance formula/ *korrekte subst in afstand-formule* ✓ answer in surd form/*antw in wortelvorm* (2)
3.8	**MK ⊥ KL OR/OF $M\hat{K}L = 90°$** (radius ⊥ tangent/*radius ⊥ rkl*) $\therefore$ ML is a diameter as it subtends a right angle/*ML is middellyn* $r = \frac{ML}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}}$ or 7,91 Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML* $x = \frac{5+20}{2} = \frac{25}{2} = 12{,}5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6{,}5$ Centre/*midpt*: (12,5 ; 6,5) Equation of the circle KLM /*Vgl van sirkel KLM*: $\therefore (x - 12{,}5)^2 + (y - 6{,}5)^2 = \frac{250}{4} = \frac{125}{2} = 62{,}5$ **OR/*OF***	✓ S ✓ value of/*waarde van r* ✓ $x = 12{,}5$ ✓ $y = 6{,}5$ ✓ answer in correct form/ *antw in korrekte vorm* (5)

	MK ⊥ KL OR/OF $\mathbf{M\hat{K}L} = 90°$ (radius ⊥ tangent/*radius ⊥ rkl*) ∴ML is a diameter as it subtends a right angle/*ML is middellyn* Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML* $x = \frac{5+20}{2} = \frac{25}{2} = 12{,}5$ $\quad y = \frac{4+9}{2} = \frac{13}{2} = 6{,}5$ Centre/*midpt*: (12,5 ; 6,5) Equation of the circle KLM /*Vgl van sirkel KLM*: $(x-12{,}5)^2 + (y-6{,}5)^2 = r^2$ subst (5 ; 4): $(5-12{,}5)^2 + (4-6{,}5)^2 = r^2$ $62{,}5 = r^2$ $\therefore \quad (x-12{,}5)^2 + (y-6{,}5)^2 = \frac{250}{4} = \frac{125}{2} = 62{,}5$ **OR/*OF*** By symmetry about LM/*deur simmetrie om LM*: **MK ⊥ KL OR/OF** $\mathbf{M\hat{K}L} = 90°$ (radius ⊥ tangent/*radius ⊥ rkl*) ∴ML is a diameter as it subtends a right angle/*ML is middellyn* ML is a diameter /*ML is 'n middellyn* $r = \frac{ML}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}}$ or /of 7,91 Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML* $x = \frac{5+20}{2} = \frac{25}{2} = 12{,}5$ $\quad y = \frac{4+9}{2} = \frac{13}{2} = 6{,}5$ Centre/*midpt*: (12,5 ; 6,5) Equation of the circle KLM /*Vgl van sirkel KLM*: $\therefore \quad (x-12{,}5)^2 + (y-6{,}5)^2 = \frac{250}{4} = \frac{125}{2} = 62{,}5$	✓ S ✓ $x = 12{,}5$ ✓ $y = 6{,}5$ ✓ value of/*waarde van* r^2 ✓ answer in correct form/*antw in korrekte vorm* (5) ✓ S ✓ value of/*waarde van r* ✓ $x = 12{,}5$ ✓ $y = 6{,}5$ ✓ answer in correct form/*antw in korrekte vorm* (5) **[21]**

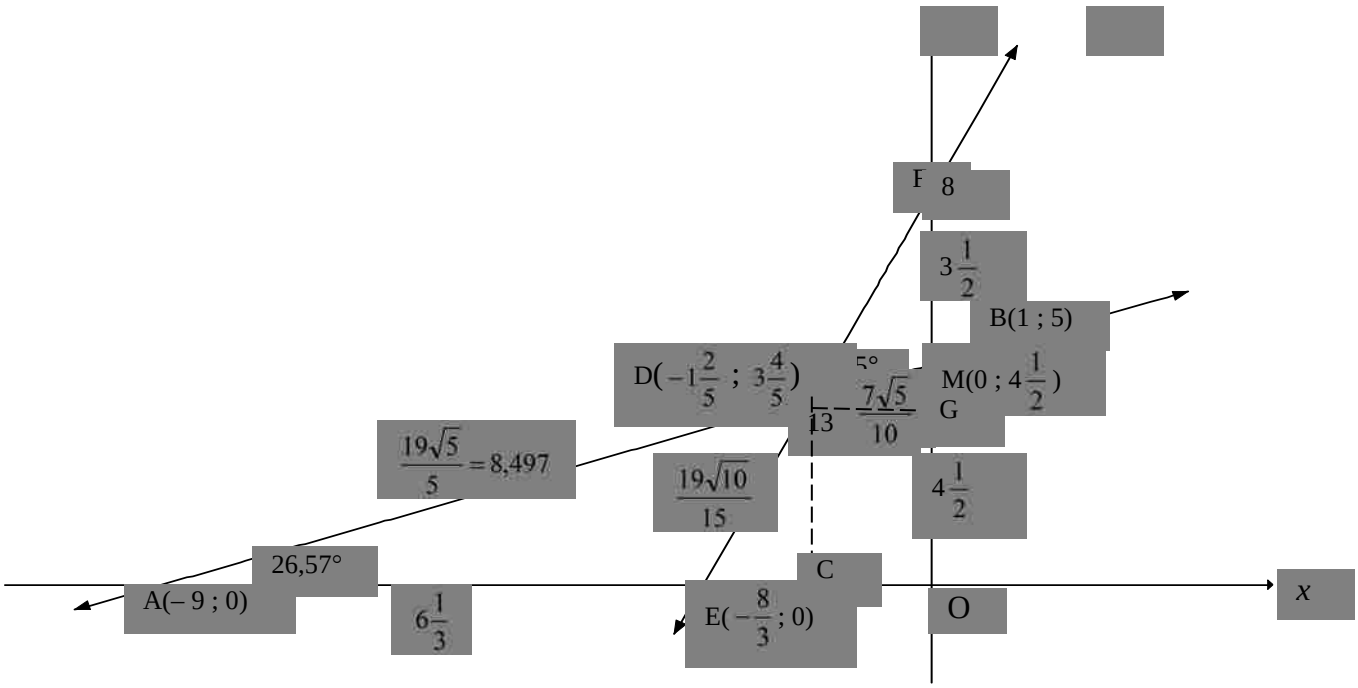
QUESTION/VRAAG 4



4.1	$y = 0$: $3x + 8 = 0$ $x = -\frac{8}{3}$ $\therefore \text{E}\left(-2\frac{2}{3};0\right)$ **OR/*OF*** $\text{E}\left(-\frac{8}{3};0\right)$	✓ y-value/*waarde* ✓ x-value/*waarde* (2)
4.2	$\tan \text{D}\hat{\text{E}}\text{O} = m_{\text{DE}} = 3$ $\therefore \text{D}\hat{\text{E}}\text{O} = 71{,}565... = 71{,}57°$ $\text{D}\hat{\text{A}}\text{E} = 71{,}565...° - 45°$ $= 26{,}57°$	✓ $\tan \text{D}\hat{\text{E}}\text{O} = 3$ ✓ 71,565…° ✓ 26,57° (3)
4.3	$m_{\text{AB}} = \tan 26{,}57°$ $= \frac{1}{2}$ $y = \frac{1}{2}x + c$ **OR/*OF*** $y - y_1 = \frac{1}{2}(x - x_1)$ $5 = \frac{1}{2}(1) + c$ $y - 5 = \frac{1}{2}(x - 1)$ $y = \frac{1}{2}x + 4\frac{1}{2}$ $y = \frac{1}{2}x + \frac{9}{2}$	✓ $m_{\text{AB}} = \tan 26{,}57°$ ✓ $m_{\text{AB}} = \frac{1}{2}$ ✓ subst of m and (1 ; 5)into formula/ *subst m en (1 ; 5) in formule* ✓ equation/*vgl* (4)

4.4	Solve $x - 2y + 9 = 0$ and $y = 3x + 8$ simultaneously: $x - 2(3x+8) + 9 = 0$ $x - 6x - 16 + 9 = 0$ $-5x = 7$ $x = -1\frac{2}{5}$ $\therefore y = 3(-1\frac{2}{5}) + 8$ **OR/*OF*** $-1\frac{2}{5} - 2y + 9 = 0$ $y = 3\frac{4}{5}$ $\quad$ $y = 3\frac{4}{5}$ $\therefore D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$ **OR/OF** $x = 2y - 9$ $y = 3(2y - 9) + 8$ $y = 6y - 27 + 8$ $\therefore y = 3\frac{4}{5}$ $x = 2(3\frac{4}{5}) - 9$ **OR/OF** $3\frac{4}{5} = 3x + 8$ $x = -1\frac{2}{5}$ $\therefore D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$ **OR/*OF*** $3x + 8 = \frac{1}{2}x + 4\frac{1}{2}$ $6x + 16 = x + 9$ $5x = -7$ $\therefore x = -1\frac{2}{5}$ $\therefore y = 3(-1\frac{2}{5}) + 8$ **OR/*OF*** $y = \frac{1}{2}(-1\frac{2}{5}) + 4\frac{1}{2}$ $y = 3\frac{4}{5}$ $\quad$ $y = 3\frac{4}{5}$ $\therefore D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$ **OR/*OF***	✓ subst/*vervang* ✓ *x*-value/*waarde* ✓ subst/*vervang* ✓ *y*-value/*waarde* (4) ✓ subst/*vervang* ✓ *y* value/*waarde* ✓ subst/*vervang* ✓ *x*-value/*waarde* (4) ✓ equating/*gelyk stel* ✓ *x* value/*waarde* ✓ subst/*vervang* ✓ *y*-value/*waarde* (4)

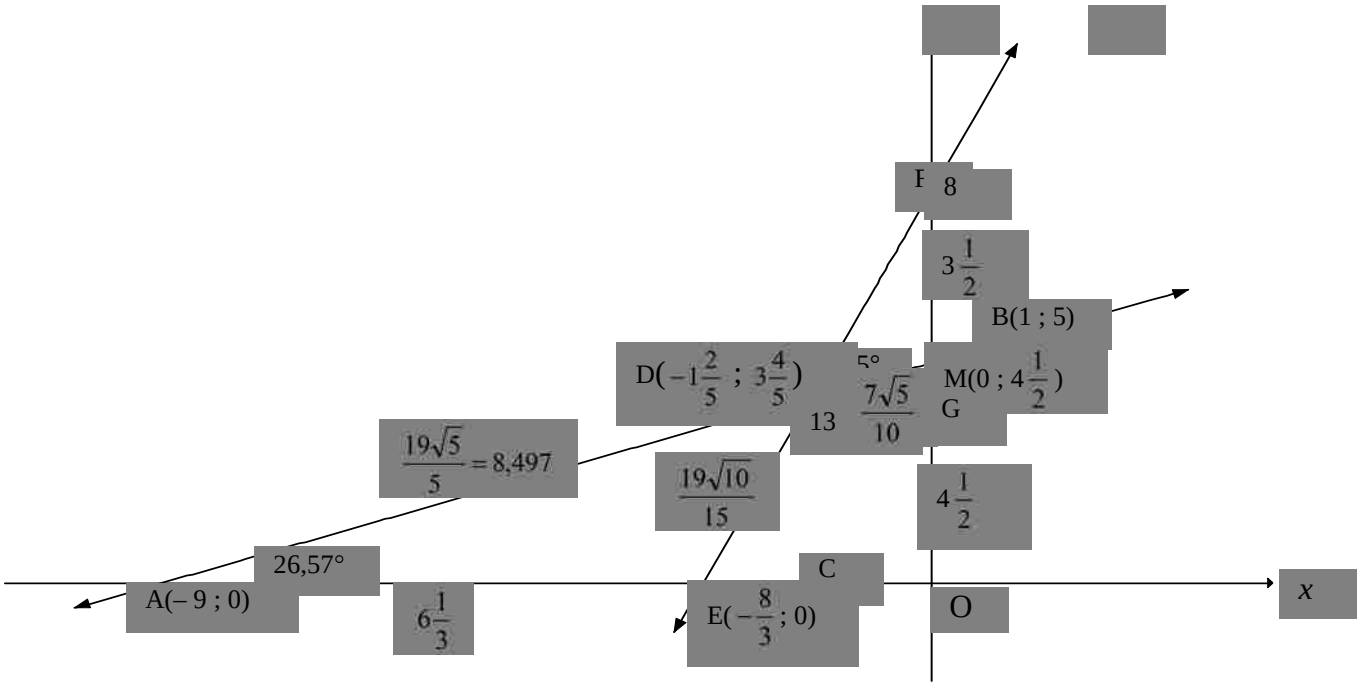
	$x - 2y = -9$ ……(1) $-6x + 2y = 16$ …...(2) (1) + (2): $-5x \quad = 7$ $\therefore\ x = -1\frac{2}{5}$ $\therefore\ -1\frac{2}{5} - 2y = -9$ **OR/*OF*** $y = 3(-1\frac{2}{5}) + 8$ $y = 3\frac{4}{5}$ $\quad$ $y = 3\frac{4}{5}$ $\therefore\ D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$ **OR/*OF*** $y = 3x + 8$ …….(1) $6y = 3x + 27$ …….(2) (1) – (2): $-5y = -19$ $\therefore\ y = 3\frac{4}{5}$ $3\frac{4}{5} = 3x + 8$ $\quad$ $x = 2(3\frac{4}{5}) - 9$ **OR/*OF*** $x = -1\frac{2}{5}$ $\quad$ $x = -1\frac{2}{5}$ $\therefore\ D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$	✓ adding/*optelling* ✓ *x*-value/*waarde* ✓ subst/*vervang* ✓ *y*-value/*waarde* (4) ✓ subtracting/*aftrekking* ✓ *y*-value/*waarde* ✓ subst/*vervang* ✓ *x*-value/*waarde* (4)



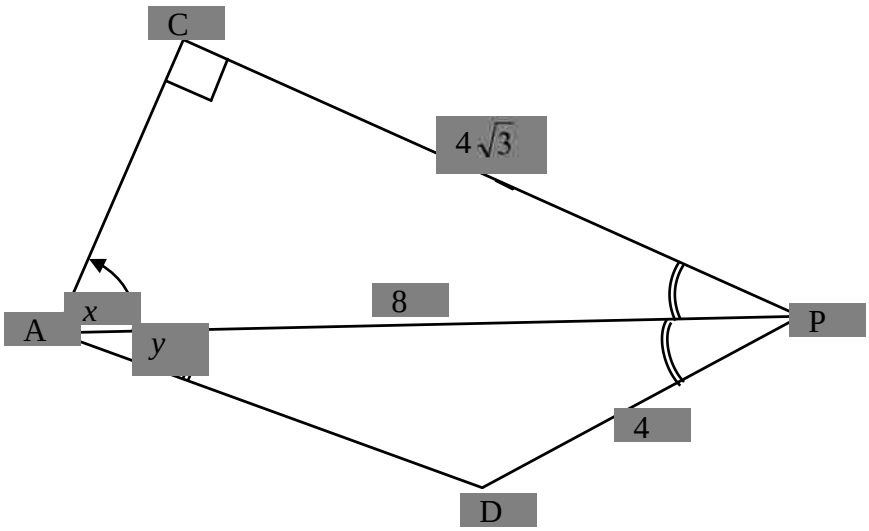
4.5	area DMOE = area ΔAMO – area ΔADE $x_A = 2(0) - 9 \quad \therefore A(-9\ ;\ 0)$ area ΔAMO $= \frac{1}{2}.\ AO.\ OM$ $= \frac{1}{2}(9)(4\frac{1}{2})$ $= 20{,}25$ area ΔADE $= \frac{1}{2}.\ AE.\ y_D$ $= \frac{1}{2}.\ (AO - EO).\ y_D$ $= \frac{1}{2}\left(9 - 2\frac{2}{3}\right)\left(3\frac{4}{5}\right)$ $= 12{,}03$ **OR/OF** area ΔADE $= \frac{1}{2} AD.AE.\sin D\hat{A}E$ $= \frac{1}{2}\ (\frac{19\sqrt{5}}{5}).6\frac{1}{3}.\sin 26{,}57°$ $= 12{,}03$ ∴ area DMOE = 8,22 square units/*vk eenh* **OR/*OF***	✓ correct method/ *korrekte metode* ✓ $x_A = -9$ ✓ $\frac{1}{2}(9)(4\frac{1}{2})$ ✓ $AE = 9 - 2\frac{2}{3} = 6\frac{1}{3}$ ✓ $y_D = 3\frac{4}{5}$ **OR/OF** ✓ $AD = \frac{19\sqrt{5}}{5}$ ✓ $AE = 6\frac{1}{3}$ ✓ answer/*antw* (6)

	area DMOE = area rectangle DCOG + area ΔDMG + area ΔDEC $= (1\frac{2}{5} \times 3\frac{4}{5}) + \frac{1}{2}\left(1\frac{2}{5}\right)(\frac{7}{10}) + \frac{1}{2}(3\frac{4}{5})(\frac{19}{15})$ = 8,22 square units/*vk eenh* **OR/*OF*** area DMOE = area ΔEDO + area ΔODM $= \frac{1}{2}(\text{EO} \times y_{\text{D}}) + \frac{1}{2}(\text{OM} \times -x_D)$ $= \frac{1}{2}\left[\left(\frac{8}{3} \times \frac{19}{5}\right) + \left(\frac{9}{2} \times \frac{7}{5}\right)\right]$ $= \frac{1}{2}\left(\frac{304+189}{30}\right)$ $= \frac{493}{60}$ or/*of* $8\frac{13}{60}$ or/*of* 8,22 square units/*vk eenh* **OR/*OF*** area DMOE = area ΔEOF – area ΔDMF $= \frac{1}{2}(\text{EO} \times \text{OF}) - \frac{1}{2}(\text{OF} - \text{OM})(-x_D)$ $= \frac{1}{2}\left[\left(\frac{8}{3} \times 8\right) + \left(\frac{7}{2} \times \frac{7}{5}\right)\right]$ $= \frac{1}{2}\left(\frac{640-147}{30}\right)$ $= \frac{493}{60}$ or $8\frac{13}{60}$ or 8,22 square units/*vk eenh* **OR/*OF***	✓ correct method/ *korrekte metode* ✓ $3\frac{4}{5}$ ✓ $1\frac{2}{5}$ ✓ 0,7 ✓ $\frac{19}{15}$ ✓ answer (6) ✓ correct method/ *korrekte metode* ✓ $y_{\text{D}} = \frac{19}{5}$ or $3\frac{4}{5}$ ✓ EO = $\frac{8}{3}$ ✓ $-x_{\text{D}} = \frac{7}{5}$ ✓ OM = $\frac{9}{2}$ or $4\frac{1}{2}$ ✓ answer/*antw* (6) ✓ correct method/ *korrekte metode* ✓ $y_{\text{F}} = 8$ ✓ EO = $\frac{8}{3}$ ✓ $-x_{\text{D}} = \frac{7}{5}$ ✓ FM = $3\frac{1}{2}$ ✓ answer/*antw* (6)

	area ΔEOM $= \frac{1}{2}(\text{EO} \times \text{OM})$ $= \frac{1}{2}\left(\frac{8}{3} \times \frac{9}{2}\right)$ $=$ 6 sq units/*vk eenh*	✓ area ΔEOM
	ED $= \sqrt{\left(-\frac{7}{5}+\frac{8}{3}\right)^2+\left(\frac{19}{5}\right)^2}$ and DM $= \sqrt{\left(\frac{7}{5}\right)^2+\left(\frac{9}{2}-\frac{19}{5}\right)^2}$ $= \frac{19\sqrt{10}}{15}$ or 4,005... $= \frac{7\sqrt{5}}{10}$ or 1,565..	✓ ED $= \frac{19\sqrt{10}}{15}$ ✓ DM $= \frac{7\sqrt{5}}{10}$
	area ΔEDM $= \frac{1}{2}(\text{ED} \times \text{DM} \times \sin \text{E}\hat{\text{D}}\text{M})$ $= \frac{1}{2}\left(\frac{19\sqrt{10}}{15}\right)\left(\frac{7\sqrt{5}}{10}\right)\sin 135°$ $= \frac{133}{60}$ or 2,216...	✓ area ΔEDM ✓ correct method/ *korrekte metode*
	$\therefore$ area DMOE = area ΔEOM + area ΔEDM $=$ 6 + 2,216... $= \frac{493}{60}$ or/*of* $8\frac{13}{60}$ or/*of* 8,22 square units/*eenh*^{2}	✓ answer/*antw*
		(6) **[19]**



QUESTION/VRAAG 5



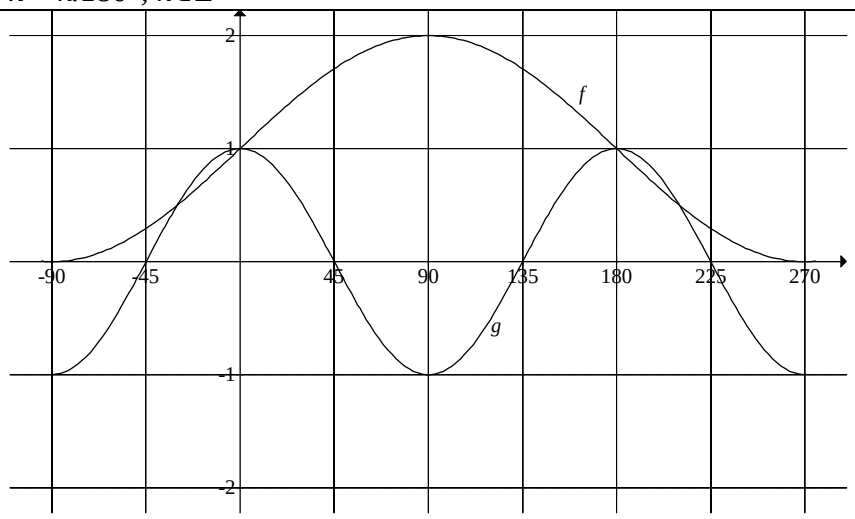
5.1	$\sin C\hat{A}P = \frac{CP}{AP}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60°$ OR/OF $\frac{\sin 90°}{8} = \frac{\sin x}{4\sqrt{3}}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60°$	✓ correct sine ratio/ *korrekte sin-verh* ✓ $\frac{\sqrt{3}}{2}$ (2) ✓ correct sine ratio/ *korrekte sin-verh* ✓ $\frac{\sqrt{3}}{2}$ (2)
5.2	$C\hat{P}A = D\hat{P}A = 30°$ (AP bisects $D\hat{P}C$) $AD^2 = AP^2 + DP^2 - 2.AP.DP.\cos A\hat{P}D$ $= 8^2 + 4^2 - 2(8)(4)\cos 30°$ $= 8^2 + 4^2 - 2(8)(4)(\frac{\sqrt{3}}{2})$ $= 24{,}57...$ $AD = 4{,}96$	✓ $D\hat{P}A = 30°$ ✓ correct subst into cosine rule/ *korrekte subst in cos-reël* ✓ 24,57... ✓ 4,96 (4)

5.3	$\frac{\sin D\hat{A}P}{DP} = \frac{\sin A\hat{P}D}{AD}$ $\frac{\sin y}{4} = \frac{\sin 30°}{4,96}$ $\sin y = \frac{4\sin 30°}{4,96}$ $= 0,403...$ $y = 23,78°$ **OR/*OF*** $AD^2 = AP^2 + DP^2 - 2.AP.DP.\cos D\hat{A}P$ $4^2 = 8^2 + (4,96)^2 - 2(8)(4,96).\cos y$ $\cos y = \frac{8^2 + (4,96)^2 - 4^2}{2(8)(4,96)}$ $\cos y = 0,9148...$ $y = 23,82°$	✓ correct subst into sine rule/ *korrekte subst in sin-reël* ✓ sin y subject ✓ 23,78° (3) ✓ correct subst into cosine rule/ *korrekte subst in cos-reël* ✓ cos y subject ✓ 23,82° (3) **[9]**

QUESTION/VRAAG 6

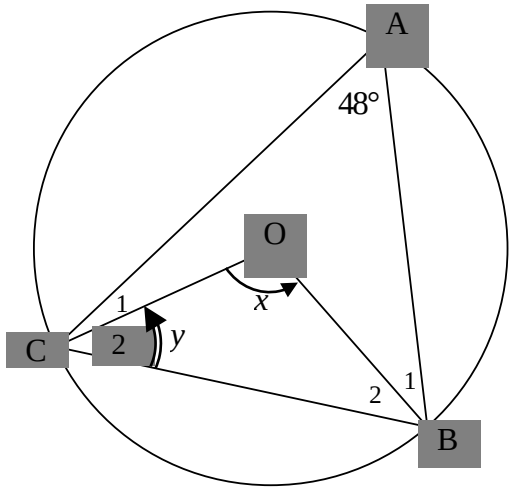
6.1	$\cos^2(180° + x) + \tan(x - 180°)\sin(720° - x)\cos x$ $= (-\cos x)^2 + [-(-\tan x)](-\sin x)(\cos x)$ $= \cos^2 x + \left(\frac{\sin x}{\cos x}\right)(-\sin x)(\cos x)$ $= \cos^2 x - \sin^2 x$ $= \cos 2x$	✓ $(-\cos x)^2$ or $\cos^2 x$ ✓ $\tan x$ or $-(-\tan x)$ ✓ $-\sin x$ ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $\cos^2 x - \sin^2 x$ (5)
6.2	$\sin(\alpha - \beta)$ $= \cos[90° - (\alpha - \beta)]$ $= \cos[(90° - \alpha) + \beta]$ $= \cos(90° - \alpha)\cos\beta - \sin(90° - \alpha)\sin\beta$ $= \sin\alpha\cos\beta - \cos\alpha\sin\beta$ **OR/*OF*** $\sin(\alpha - \beta)$ $= \cos[90° - (\alpha - \beta)]$ $= \cos[(90° + \beta) + (-\alpha)]$ $= \cos(90° + \beta)\cos(-\alpha) - \sin(90° + \beta)\sin(-\alpha)$ $= (-\sin\beta)\cos\alpha - \cos\beta(-\sin\alpha)$ $= \sin\alpha\cos\beta - \cos\alpha\sin\beta$	✓rewrite as/*herskryf* $\cos[(90° - \alpha) + \beta]$ ✓ expansion/ *uitbreiding* ✓simpl/*vereenv* (3) ✓ rewrite as/*herskryf* $\cos[(90° + \beta) + (-\alpha)]$ ✓ expansion/ *uitbreiding* ✓simpl/*vereenv* (3)
6.3	$x^2 - y^2$ $= \sin^2 76° - \cos^2 76°$ $= -(\cos^2 76° - \sin^2 76°)$ $= -\cos 2(76°)$ $= -\cos 152°$ $= -(-\cos 28°)$ **OR/*OF*** $= -\cos(90° + 62°)$ $= \cos 28°$ $= -(-\sin 62°)$ $= \cos(90° - 62°)$ $= \sin 62°$ $= \sin 62°$ **OR/*OF*** $x^2 - y^2$ $= \sin^2 76° - \cos^2 76°$ $= \sin 76° \sin 76° - \cos 76° \cos 76°$ $= \sin 76° \cos 14° - \cos 76° \sin 14°$ $= \sin(76° - 14°)$ $= \sin 62°$ **OR/*OF*** $x^2 - y^2$ $= \sin^2 76° - \cos^2 76°$ $= \cos^2 14° - \sin^2 14°$ $= \cos 2(14°)$ $= \cos 28°$ $= \sin 62°$	✓$-(\cos^2 76° - \sin^2 76°)$ ✓ recognition of cos double angle ✓ $-\cos 152°$ ✓ $\cos 28°$ (4) ✓ $\cos 14°$ ✓ $\sin 14°$ ✓ recognition of sine compound angle ✓ $\sin(76° - 14°)$ (4) ✓ $\cos^2 14°$ ✓ $\sin^2 14°$ ✓ recognition of cos double angle ✓ $\cos 28°$ (4) **[12]**

QUESTION/VRAAG 7

7.1	$0 \leq y \leq 2$ or $y \in [0 ; 2]$	✓ critical values/ *kritieke waardes* ✓ notation/*notasie* (2)
7.2	$\sin x + 1 = \cos 2x$ $\sin x + 1 = 1 - 2\sin^2 x$ $2\sin^2 x + \sin x = 0$ $\sin x(2\sin x + 1) = 0$	✓ $1 - 2\sin^2 x$ ✓ st form/*st vorm* (2)
7.3	$\sin x(2\sin x + 1) = 0$ $\sin x = 0$ *or* $\sin x = -\frac{1}{2}$ $x = 0° + k.360°$ or $x = 210° + k.360°$ or $x = 180° + k.360°$ $x = 330° + k.360°, k \in Z$ **OR/*OF*** $x = k.180°, k \in Z$	✓ $\sin x = 0$ *or* $\sin x = -\frac{1}{2}$ ✓ 0° ; 180° **OR/*OF*** $x = k.180°$ ✓ 210° ; 330° ✓ $k.360°$, $k \in Z$ (4)
7.4		✓ *y*-intercept/*afsnit* ✓ *x*-intercepts/*afsnitte* ✓ min/max points/ *min/maks punte* (3)
7.5	$f(x) = g(x)$ at/*by*: $x = -30° ; 0° ; 180° ; 210°$ $\therefore f(x + 30°) = g(x + 30°)$ at/*by*: $x = -60° ; -30° ; 150° ; 180°$	✓ $-30° ; 0° ; 180° ; 210°$ ✓✓ $-60° ; -30° ; 150° ; 180°$ (3)
7.6	Series will converge if/*Reeks sal konvergeer as:* $-1 < r < 1$ $-1 < 2\cos 2x < 1$ $-\frac{1}{2} < \cos 2x < \frac{1}{2}$ $\therefore 30° < x < 60°$ or $x \in (30° ; 60°)$	✓ $-1 < r < 1$ ✓ $r = 2\cos 2x$ ✓ $-\frac{1}{2} < \cos 2x < \frac{1}{2}$ ✓✓ $30° < x < 60°$ (5) **[19]**

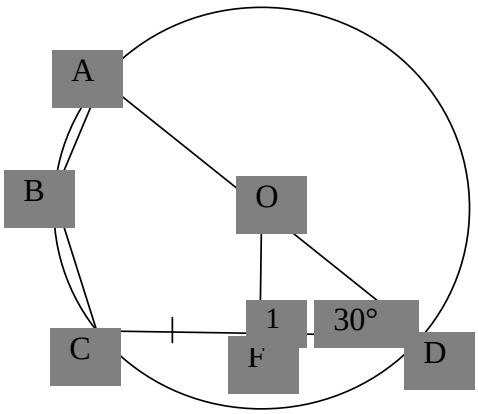
QUESTION/VRAAG 8

8.1



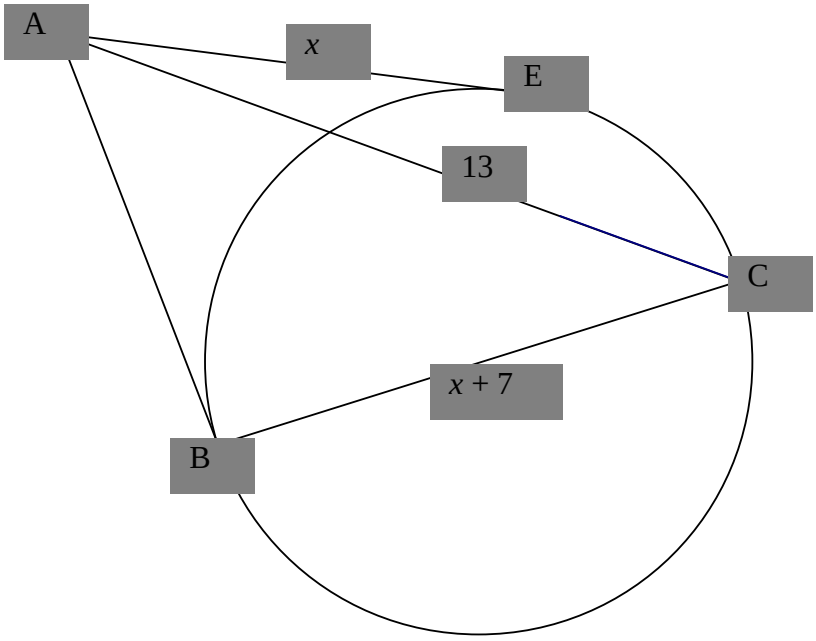
8.1.1	$x = 96°$	(∠ at centre = 2∠ at circumference/ *∠by midpt = 2∠by omtrek*)	✓ S ✓ R (2)
8.1.2	$\hat{C}_2 + \hat{B}_2 = 180° - 96° = 84°$ $y = \hat{B}_2 = 42°$	(sum of ∠s in Δ/ *som v∠e in* Δ) (∠s opp = sides/*∠e teenoor = sye*)	✓ S ✓ S (2)

8.2



8.2.1	$\hat{F}_1 = 90°$	(line from centre to midpt chord/ *lyn vanaf midpt na midpt kd)*	✓ S ✓ R (2)
8.2.2	$A\hat{B}C = 150°$	(opposite ∠s of cyclic quad/ *tos ∠e v koordevh*)	✓ S ✓ R (2)

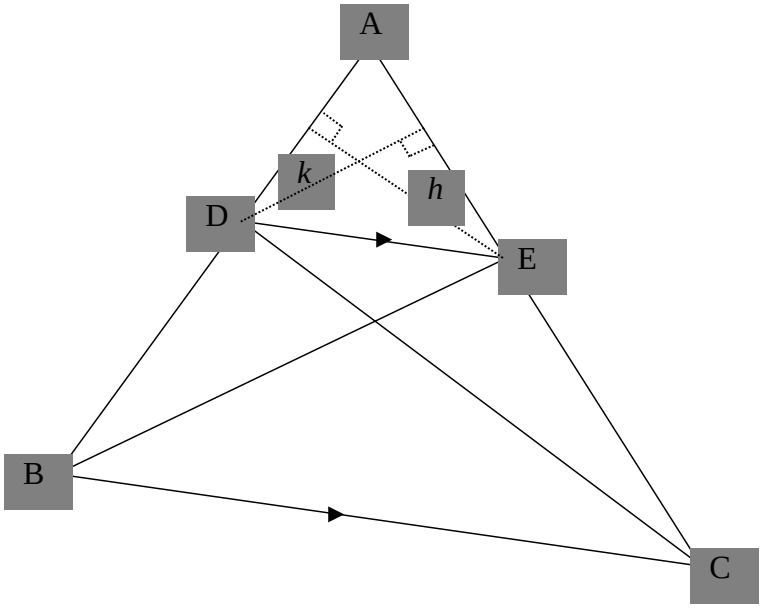
8.3



8.3.1 (a)	tangent ⊥ radius/diameter / *raaklyn ⊥ radius/middellyn*	✓ R (1)
8.3.1 (b)	tangents from common pt **OR** tangents from same pt / *raaklyne v gemeensk pt **OF** raaklyne vanaf dies pt*	✓ R (1)
8.3.2	$AB^2 + BC^2 = AC^2$ $x^2 + (x+7)^2 = 13^2$ (Theorem of/*Stelling van*Pythagoras) $x^2 + x^2 + 14x + 49 = 169$ $2x^2 + 14x - 120 = 0$ $x^2 + 7x - 60 = 0$ $(x-5)(x+12) = 0$ $x = 5 \quad (x \neq -12)$	✓ $AB^2 + BC^2 = AC^2$ ✓ $x^2 + (x+7)^2 = 13^2$ ✓ standard form ✓ answer (4) **[14]**

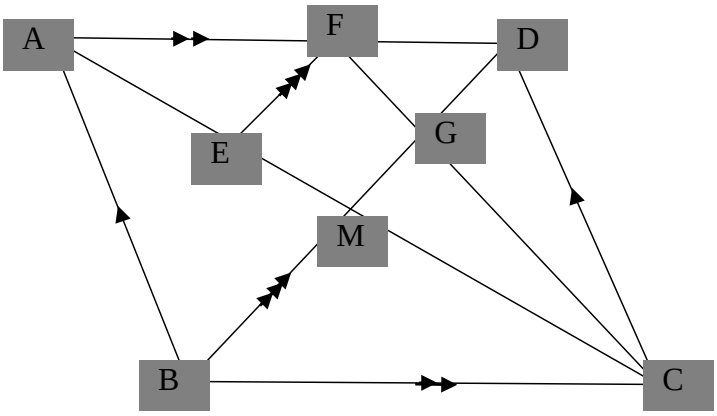
QUESTION/VRAAG 9

9.1



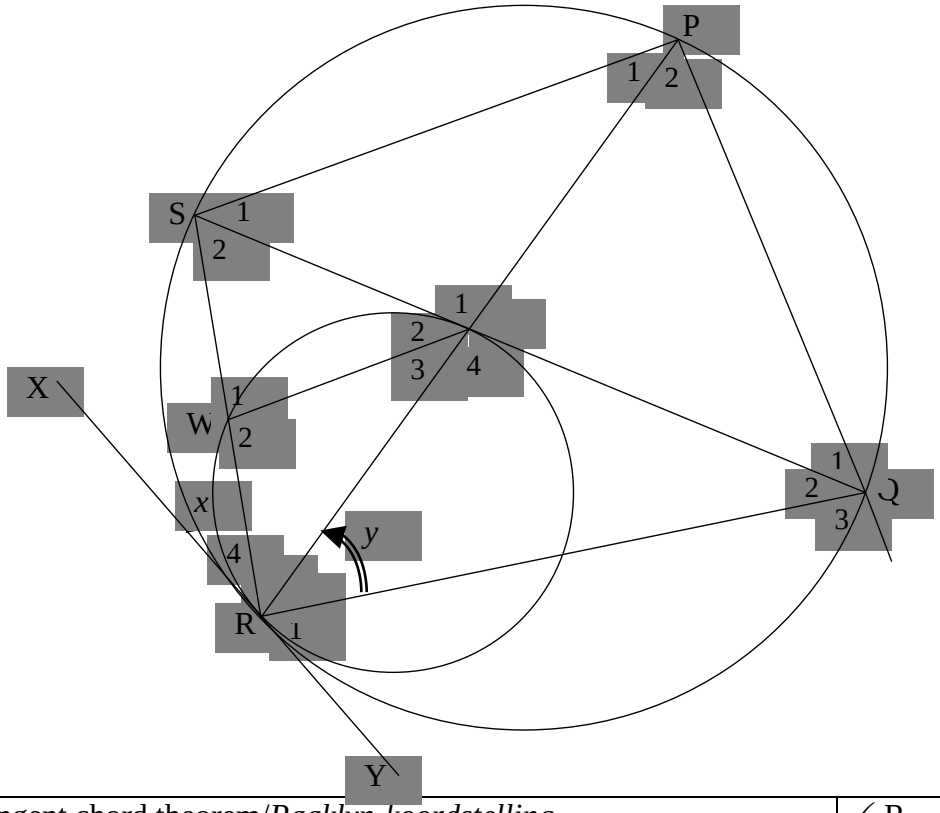
9.1.1	Same base (DE) and same height (between parallel lines) *Dieselfde basis (DE) en dieselfde hoogte (tussen ewewydige lyne)*	✓ same base/*dies basis* between || lines/ *tussen* || *lyne* (1)
9.1.2	$\frac{AD}{DB}$ $\frac{\frac{1}{2}AE \times k}{\frac{1}{2}EC \times k}$ But/*Maar* area ΔDEB = area ΔDEC (Same base and same height/*dieselfde basis en dieselfde hoogte*) $\therefore \frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \frac{\text{area } \Delta ADE}{\text{area } \Delta DEC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$	✓ S ✓ S ✓ S ✓ R ✓ S (5)

9.2

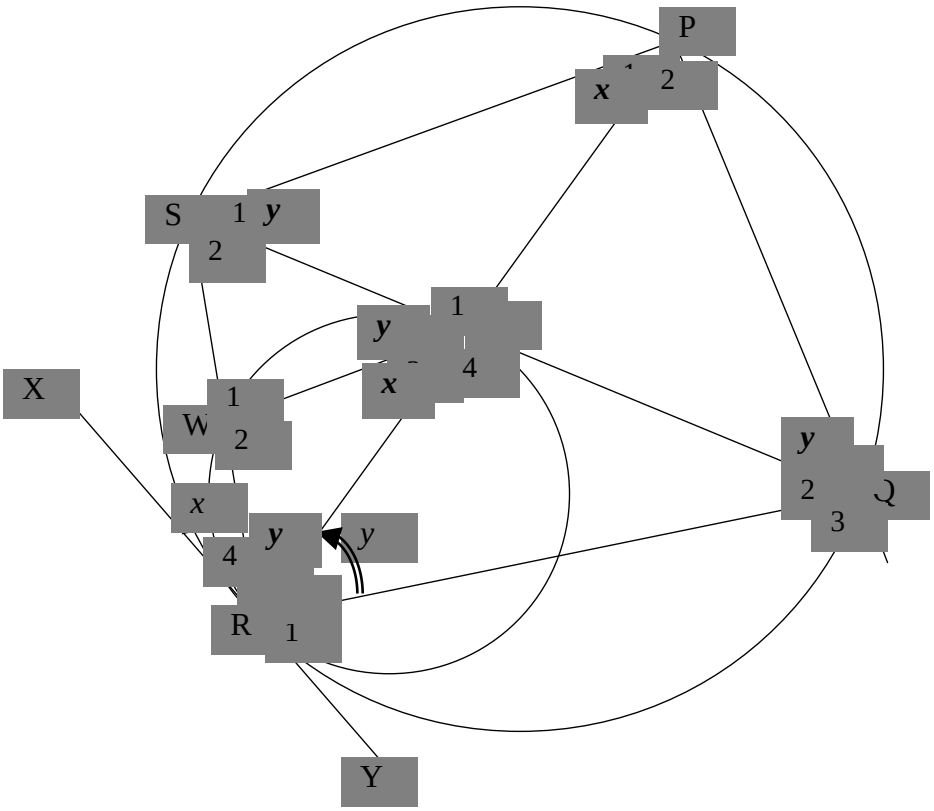


9.2.1	$\frac{EM}{AM} = \frac{FD}{AD}$ (Line parallel one side of Δ **OR** prop th; EF || BD) (*Lyn ewewydig aan sy v Δ* $\frac{EM}{AM} = \frac{3}{7}$ ***OF*** *eweredigst; EF ||BD)*	✓ S ✓R ✓ answer/*antw* (3)
9.2.2	CM = AM (diags of parm bisect/*hoekl parm halv*) $\frac{CM}{ME} = \frac{AM}{ME} = \frac{7}{3}$ (from 9.2.1/*vanaf 9.2.1*)	✓ S ✓R ✓ answer/*antw* (3)
9.2.3	*h* of ΔFDC = *h* of ΔBDC (AD || BC) $\frac{\text{area } \Delta FDC}{\text{area } \Delta BDC} = \frac{\frac{1}{2}FD.h}{\frac{1}{2}BC.h}$ $= \frac{FD}{AD}$ (opp sides of parm =) *(tos sye v parm =)* $= \frac{3}{7}$ **OR/*OF*** $\frac{\text{area } \Delta FDC}{\text{area } \Delta ADC} = \frac{FD}{AD} = \frac{3}{7}$ (same heights) *(dieselfde hoogtes)* But Area ΔADC = Area ΔBDC (diags of parm bisect area) *(hoekl v parm halv opp)* $\frac{\text{area } \Delta FDC}{\text{area } \Delta BDC} = \frac{3}{7}$	✓ AD || BC ✓ subst into area form/ *subst in opp formule* ✓ S ✓ answer/*antw* (4) ✓ S ✓ R ✓ S ✓ answer/*antw* (4) **[16]**

QUESTION/VRAAG 10



10.1.1	Tangent chord theorem/*Raaklyn-koordstelling*	✓ R (1)
10.1.2	Tangent chord theorem/*Raaklyn-koordstelling*	✓ R (1)
10.1.3	Corresponding angles equal/*Ooreenkomstige ∠e gelyk*	✓ R (1)
10.1.4	∠s subtended by chord PQ **OR** ∠s in same segment *∠e onderspan deur dieselfde koord* ***OF*** *∠e in dieselfde segment*	✓ R (1)
10.1.5	alternate ∠s/*verwisselende ∠e* ; WT || SP	✓ R (1)
10.2	$\frac{RW}{RS} = \frac{RT}{RP}$ (Line parallel one side of Δ **OR** prop th; WT || SP) $\therefore RT = \frac{WR.RP}{RS}$ (*Lyn ewewydig aan sy v Δ* ***OF*** *eweredighst: WT || SP)* **OR/*OF*** ΔRTW ||| ΔRPS (∠; ∠; ∠) $\therefore \frac{RW}{RS} = \frac{RT}{RP}$ (ΔRTW ||| ΔRPS) $\therefore RT = \frac{RW.RP}{RS}$	✓ S ✓ R (2) ✓ S ✓ S (2)
10.3	$y = \hat{T}_2 = \hat{R}_3$ (tan chord theorem/*Rkl-koordst*) $y = \hat{R}_3 = \hat{Q}_1$ (∠s in same segment/*∠e in dieselfde segment*)	✓ S ✓ R ✓ S ✓ R (4)



10.4	$\hat{Q}_3 = P\hat{S}R$ $P\hat{S}R = \hat{W}_2$ $\therefore \hat{Q}_3 = \hat{W}_2$ **OR/OF** $\hat{Q}_2 = x$ $\hat{Q}_3 = 180° - (x + y)$ $\hat{W}_2 = 180° - (x + y)$ $\therefore \hat{Q}_3 = \hat{W}_2$	(ext ∠ of cyc quad/*buite∠ v kdvh*) (corresp∠s/*ooreenk ∠e* ; WT || SP) (∠s in same segment/*∠e in dies segment*) (∠s on straight line/*∠e op reguitlyn*) (∠s of ΔWRT/*∠e v ΔWRT*)	✓ S ✓ R ✓ S (3) ✓ R ✓ S ✓ S (3)
10.5	In ΔRTS and ΔRQP: $\hat{R}_3 = \hat{R}_2 = y$ $\hat{S}_2 = \hat{P}_2$ $R\hat{T}S = R\hat{Q}P$ $\therefore \Delta RTS \,\|\|\| \, \Delta RQP$	 (proven above/*hierbo bewys*) (∠s in same segment/*∠e in dies segment*) (3rd angle of Δ) (∠; ∠; ∠)	 ✓ S ✓ S/R ✓ S **OR/OF** (∠; ∠; ∠) (3)

10.6	$\frac{RT}{RQ}=\frac{RS}{RP}$	($\Delta RTS \mid\mid\mid \Delta RQP$)	✓ S
	$\frac{RS}{RP}\times\frac{RS}{RP}=\frac{RT}{RQ}\times\frac{RS}{RP}$		✓ $\times\frac{RS}{RP}$ on both sides
	$\left(\frac{RS}{RP}\right)^2=\left(\frac{RT}{RP}\right)\left(\frac{RS}{RQ}\right)$		
	$=\left(\frac{RW}{RS}\right)\left(\frac{RS}{RQ}\right)$	(proven in 10.2/*bewys in 10.2*)	✓ $\left(\frac{RT}{RP}\right)\left(\frac{RS}{RQ}\right)$
	$=\frac{RW}{RQ}$		(3)
	OR/OF		
	$\frac{RT}{RQ}=\frac{RS}{RP}$	($\Delta RTS \mid\mid\mid \Delta RQP$)	✓ S
	But $RT=\frac{WR.RP}{RS}$	(proven in 10.2/*bewys in 10.2*)	✓ $RT=\frac{WR.RP}{RS}$
	$\therefore\frac{RT}{RQ}=\frac{WR.RP}{RQ.RS}=\frac{RS}{RP}$		
	$WR.RP^2=RQ.RS^2$		✓multiplication/ *vermenigvuldig*
	$\therefore\frac{WR}{RQ}=\frac{RS^2}{RP^2}$		(3)
	OR/OF		
	$\frac{RT}{RS}=\frac{RQ}{RP}$	($\Delta RTS \mid\mid\mid \Delta RQP$)	✓ S
	$RQ=\frac{RT.RP}{RS}$		
	and $WR=\frac{RT.RS}{RP}$	(proven in 10.2/*bewys in 10.2*)	✓ $WR=\frac{RT.RS}{RP}$
	$\frac{WR}{RQ}=\frac{\frac{RT.RS}{RP}}{\frac{RT.RP}{RS}}$		
	$=\frac{RT.RS}{RP}\times\frac{RS}{RT.RP}$		✓ simplification/ *vereenvoudiging*
	$=\frac{RS^2}{RP^2}$		(3)
			[20]

TOTAL/TOTAAL: 150

PolyMathic

Vraestel 6
Okt/Nov
Eksamen

PolyMathic

Graad 12 EindEksamen
Totaal: 150
Tyd: 3ure

VRAAG 1

Die tabel hieronder toon die afstande (in kilometer) wat daagliks deur 'n verkoopsverteenwoordiger op 21 werksdae van 'n sekere maand afgelê is.

131	132	140	140	141	144	146
147	149	150	151	159	167	169
169	172	174	175	178	187	189

1.1 Bereken die gemiddelde afstand wat deur die verkoopsverteenwoordiger afgelê is. (2)

1.2 Skryf die vyfgetal-opsomming vir hierdie stel data neer. (4)

1.3 Gebruik die gekalibreerde lyn op DIAGRAMVEL 1 om 'n mond-en-snordiagram vir hierdie stel data te teken. (2)

1.4 Lewer kommentaar op die skeefheid van die data. (1)

1.5 Bereken die standaardafwyking van die afstand wat afgelê is. (2)

1.6 Die verkoopsverteenwoordiger ontdek dat sy odometer (afstandsmeter) foutief is. Die werklike lesing op elk van die 21 dae is p km meer as wat aangedui is. Skryf neer, in terme van p (waar van toepassing), die:

1.6.1 Werklike gemiddelde (1)

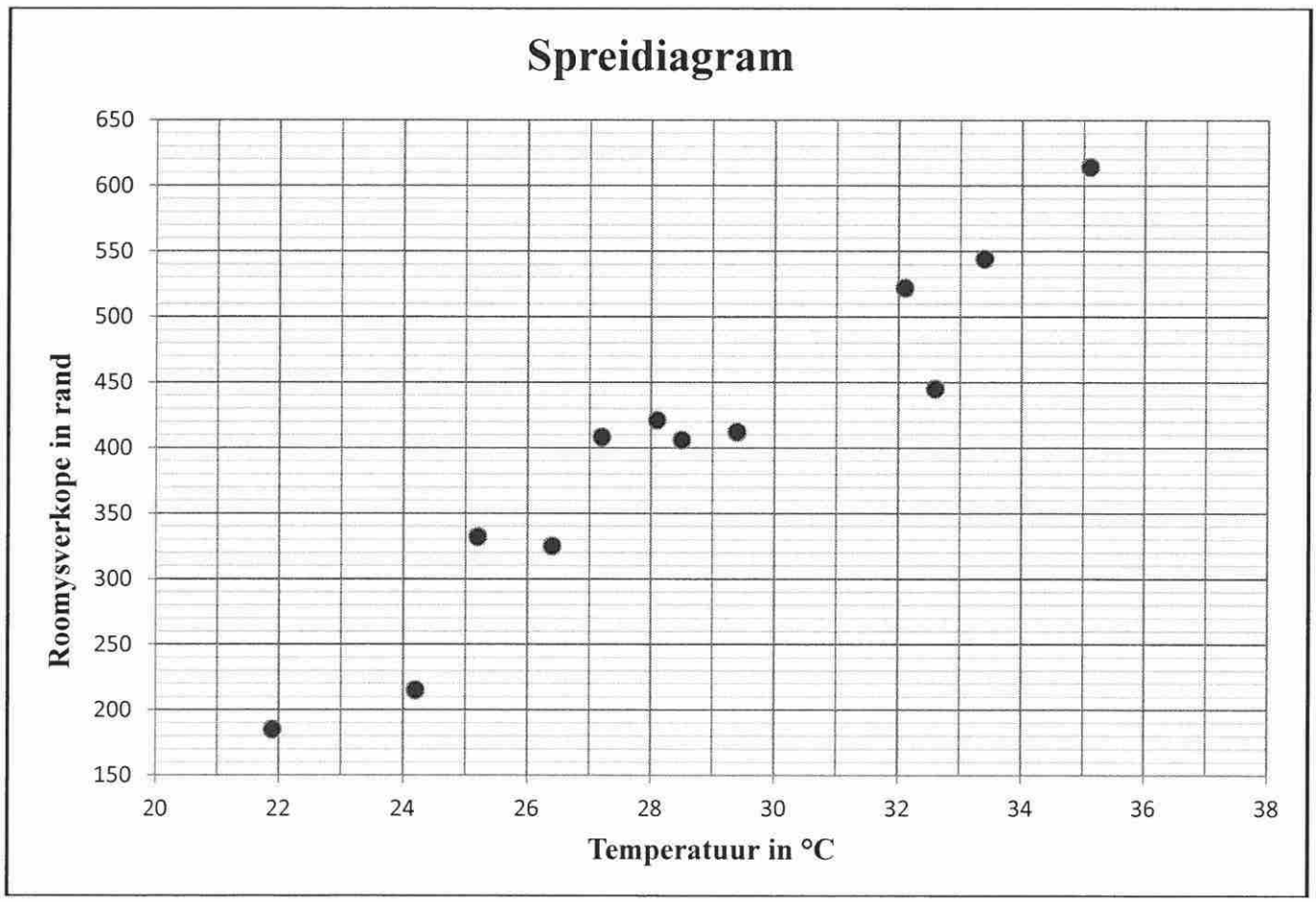
1.6.2 Werklike standaardafwyking (1)

[13]

VRAAG 2

'n Roomyswinkel het vir 12 dae van 'n sekere maand die roomysverkope, in rand, en die maksimum temperatuur, in °C, aangeteken. Die data wat versamel is, word in die tabel en spreidiagram hieronder voorgestel.

Temperatuur in °C	24,2	26,4	21,9	25,2	28,5	32,1	29,4	35,1	33,4	28,1	32,6	27,2
Roomysverkope in rand	215	325	185	332	406	522	412	614	544	421	445	408



2.1 Beskryf die invloed van temperatuur op die roomysverkope in die spreidiagram. (1)

2.2 Gee 'n rede waarom hierdie tendens nie onbepaald kan voortgaan nie. (1)

2.3 Bereken 'n vergelyking vir die kleinstekwadrate-regressielyn (lyn van beste passing). (4)

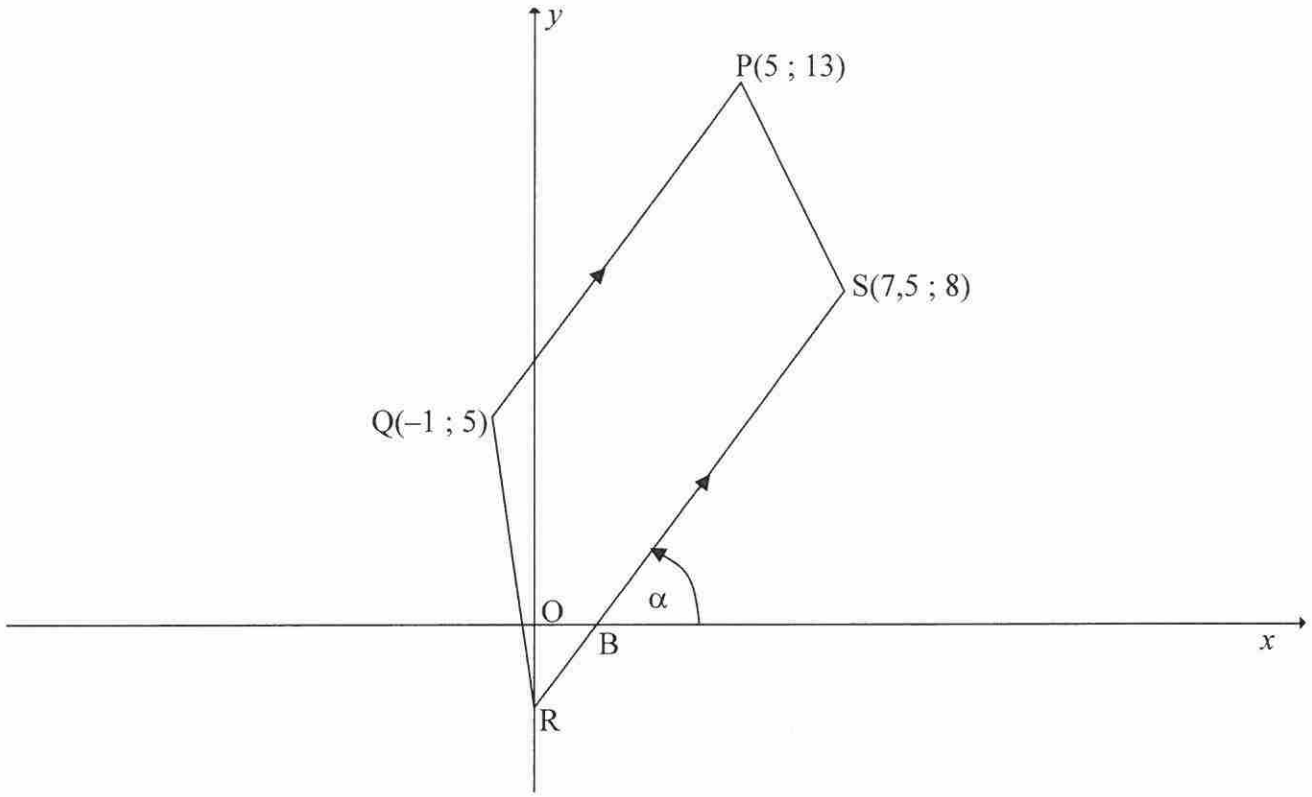
2.4 Bereken die korrelasiekoëffisiënt. (1)

2.5 Lewer kommentaar op die sterkte van die verband tussen die veranderlikes. (1)

[8]

VRAAG 3

In die diagram hieronder is punt P(5 ; 13), Q(–1 ; 5) en S(7,5 ; 8) gegee. SR | | PQ waar R die y-afsnit van SR is. Die x-afsnit van SR is B. QR is verbind.



3.1 Bereken die lengte van PQ. (3)

3.2 Bereken die gradiënt van PQ. (2)

3.3 Bepaal die vergelyking van lyn RS in die vorm $ax + by + c = 0$. (4)

3.4 Bepaal die x-koördinaat van B. (2)

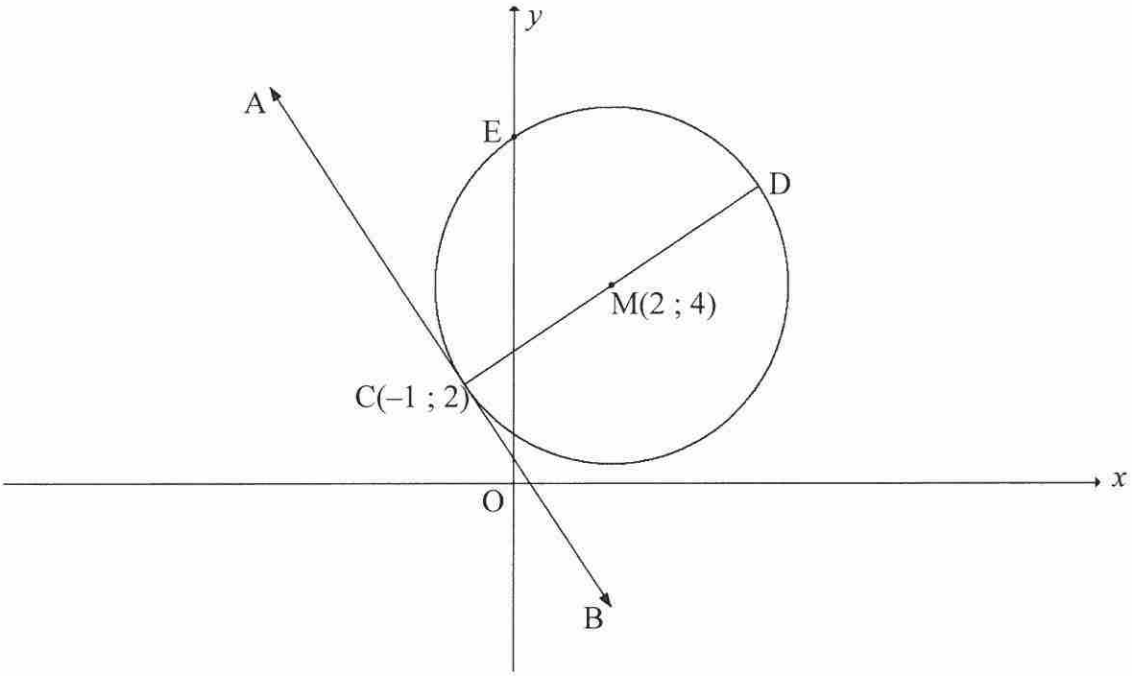
3.5 Bereken die grootte van $O\hat{R}B$. (3)

3.6 Bewys dat QBSP 'n parallelogram is. (4)

[18]

VRAAG 4

4.1 In die diagram hieronder gaan die sirkel met middelpunt M(2 ; 4) deur C(–1 ; 2) en sny die y-as by E. Die middellyn CMD is getrek en ACB is 'n raaklyn aan die sirkel.



4.1.1 Bepaal die vergelyking van die sirkel in die vorm $(x-a)^2+(y-b)^2=r^2$. (3)

4.1.2 Skryf die koördinate van D neer. (2)

4.1.3 Bepaal die vergelyking van AB in die vorm $y = mx + c$. (5)

4.1.4 Bereken die koördinate van E. (4)

4.1.5 Toon aan dat EM ewewydig is aan AB. (2)

4.2 Bepaal of die sirkels met vergelykings $(x+2)^2+(y-4)^2=25$ en $(x-5)^2+(y+1)^2=9$ mekaar sal sny. Toon AL die berekeninge. (6)

[22]

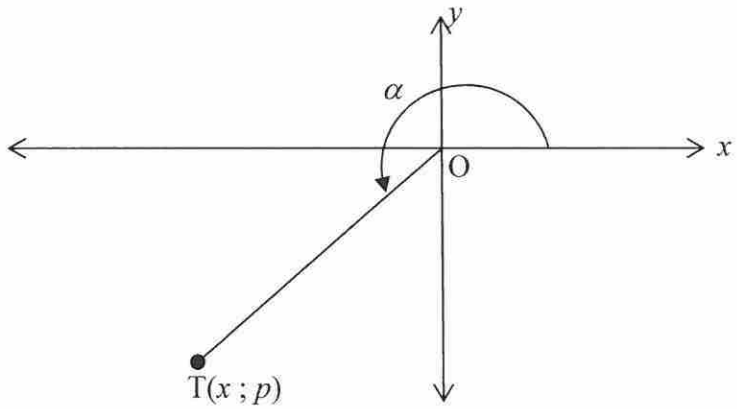
VRAAG 5

5.1 Indien $x = 3\sin\theta$ en $y = 3\cos\theta$, bepaal die waarde van $x^2 + y^2$. (3)

5.2 Vereenvoudig tot 'n enkele term:

$\sin(540° - x).\sin(-x) - \cos(180° - x).\sin(90° + x)$ (6)

5.3 In die diagram hieronder is $T(x\,;p)$ 'n punt in die derde kwadrant en dit word gegee dat $\sin\alpha = \dfrac{p}{\sqrt{1+p^2}}$.



5.3.1 Toon aan dat $x = -1$. (3)

5.3.2 Skryf $\cos(180° + \alpha)$ in terme van p in die eenvoudigste vorm neer. (2)

5.3.3 Toon aan dat $\cos 2\alpha$ as $\dfrac{1-p^2}{1+p^2}$ geskryf kan word. (3)

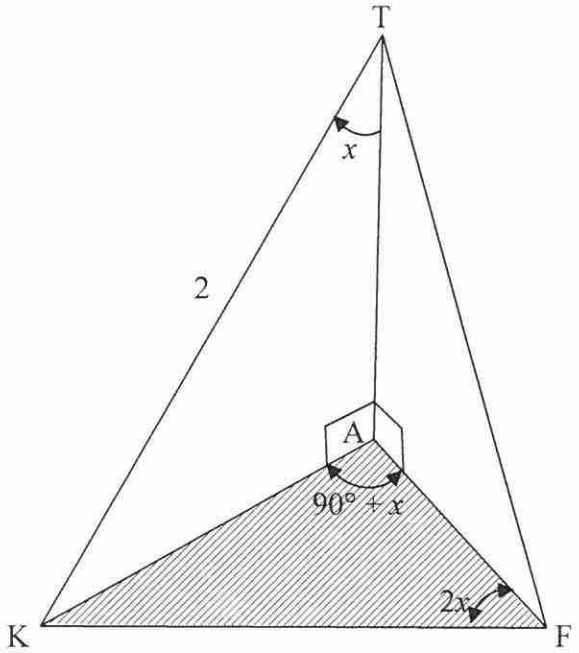
5.4 5.4.1 Vir watter waarde(s) van x sal $\dfrac{2\tan x - \sin 2x}{2\sin^2 x}$ ongedefinieerd wees in die interval $0° \leq x \leq 180°$? (3)

5.4.2 Bewys die identiteit: $\dfrac{2\tan x - \sin 2x}{2\sin^2 x} = \tan x$ (6)

[26]

VRAAG 6

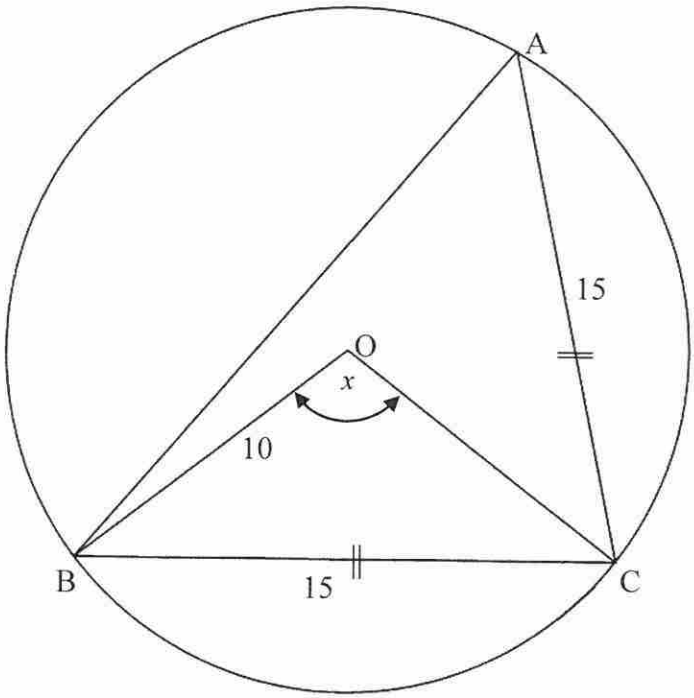
6.1 In die figuur lê punt K, A en F in dieselfde horisontale vlak en TA stel 'n vertikale toring voor. $A\hat{T}K = x$, $K\hat{A}F = 90° + x$ en $K\hat{F}A = 2x$ waar $0° < x < 30°$. TK = 2 eenhede.



6.1.1 Druk AK in terme van $\sin x$ uit. (2)

6.1.2 Bereken die numeriese waarde van KF. (5)

6.2 In die diagram hieronder gaan die sirkel met middelpunt O deur A, B en C. BC = AC = 15 eenhede. BO en OC is verbind. OB = 10 eenhede en $B\hat{O}C = x$.



Bereken:

6.2.1 Die grootte van x (4)

6.2.2 Die grootte van $A\hat{C}B$ (3)

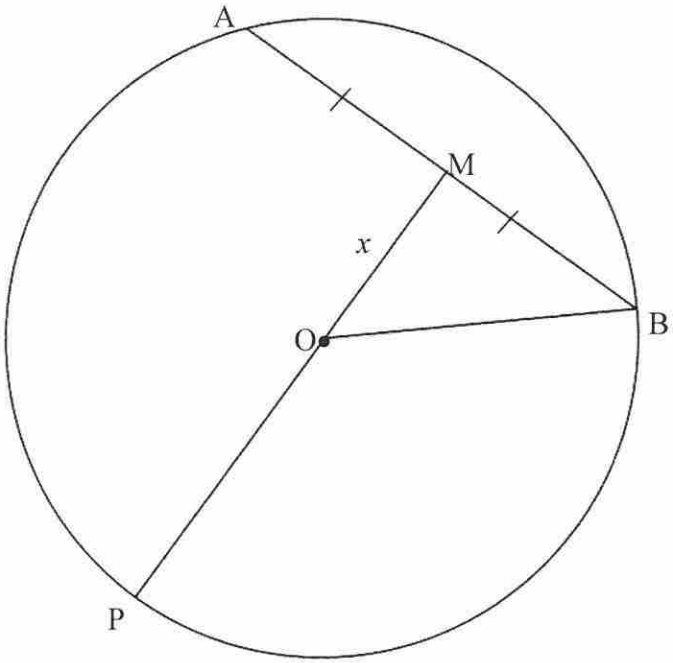
6.2.3 Die oppervlakte van ΔABC (2)

[16]

GEE REDES VIR JOU ANTWOORDE IN VRAAG 7, 8, 9 EN 10.

VRAAG 7

In die diagram is AB 'n koord van die sirkel met middelpunt O. M is die middelpunt van AB. MO word na P verleng, met P 'n punt op die sirkel. OM = x eenhede, AB = 20 eenhede en $\frac{PM}{OM} = \frac{5}{2}$.



7.1 Skryf die lengte van MB neer. (1)

7.2 Gee 'n rede waarom OM $\perp$ AB. (1)

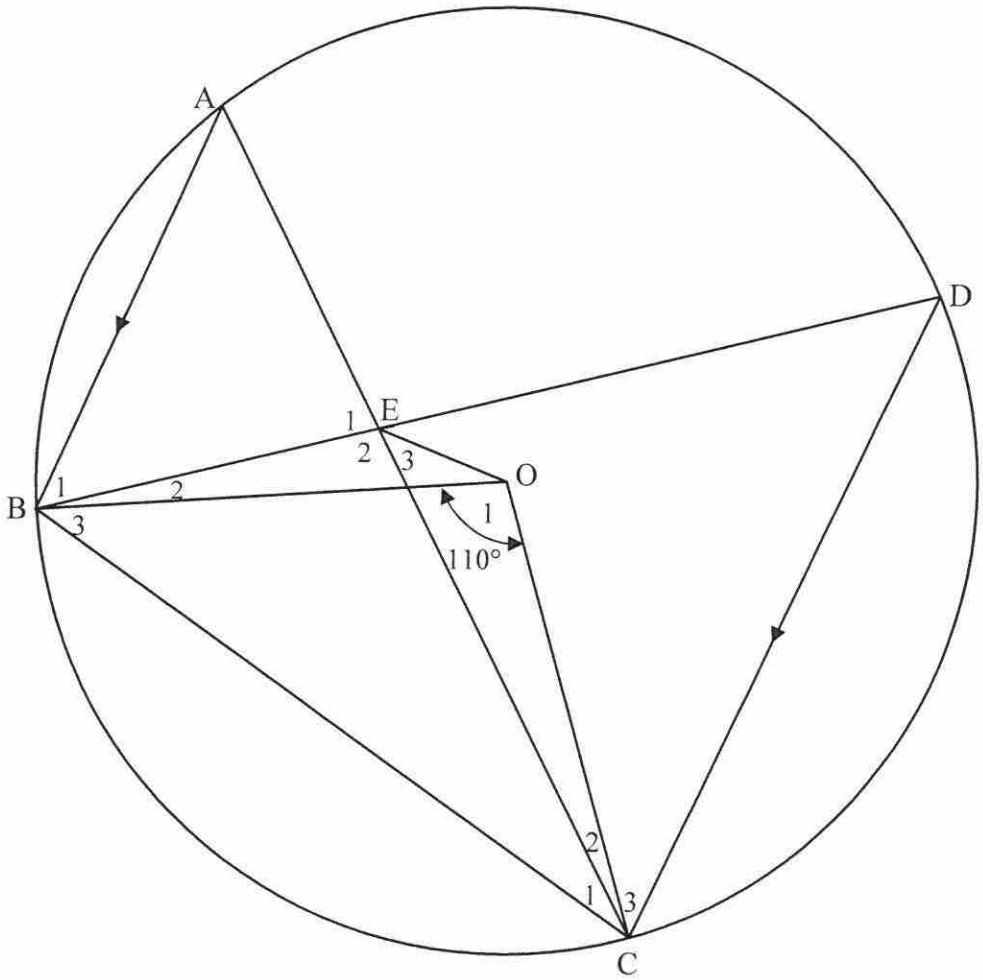
7.3 Toon aan dat $OP = \frac{3x}{2}$ eenhede. (2)

7.4 Bereken die waarde van x. (3)

[7]

VRAAG 8

In die diagram hieronder gaan die sirkel met middelpunt O deur A, B, C en D.
$AB \parallel DC$ en $B\hat{O}C = 110°$.
Die koorde AC en BD sny in E.
EO, BO, CO en BC is verbind.



8.1 Bereken die grootte van die volgende hoeke en gee redes vir jou antwoorde:

8.1.1 $\hat{D}$ (2)

8.1.2 $\hat{A}$ (2)

8.1.3 $\hat{E}_2$ (4)

8.2 Bewys dat BEOC 'n koordevierhoek is. (2)

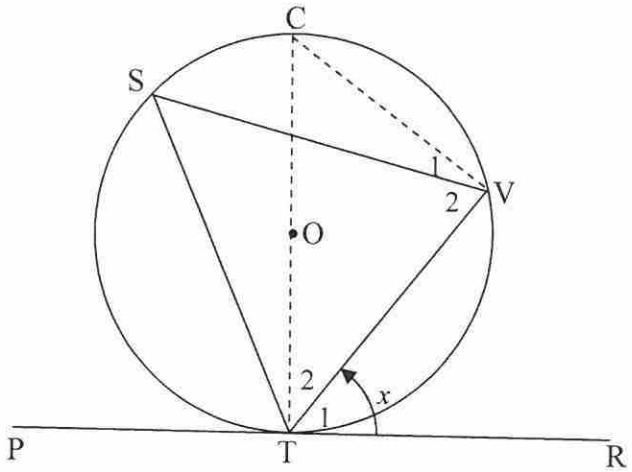
[10]

VRAAG 9

9.1 Voltooi die bewoording van die volgende stelling:

Die buitehoek van 'n koordevierhoek is gelyk aan ... (1)

9.2 In die diagram hieronder gaan die sirkel met middelpunt O deur punt S, T en V. PR is 'n raaklyn aan die sirkel by T. VS, ST en VT is verbind.



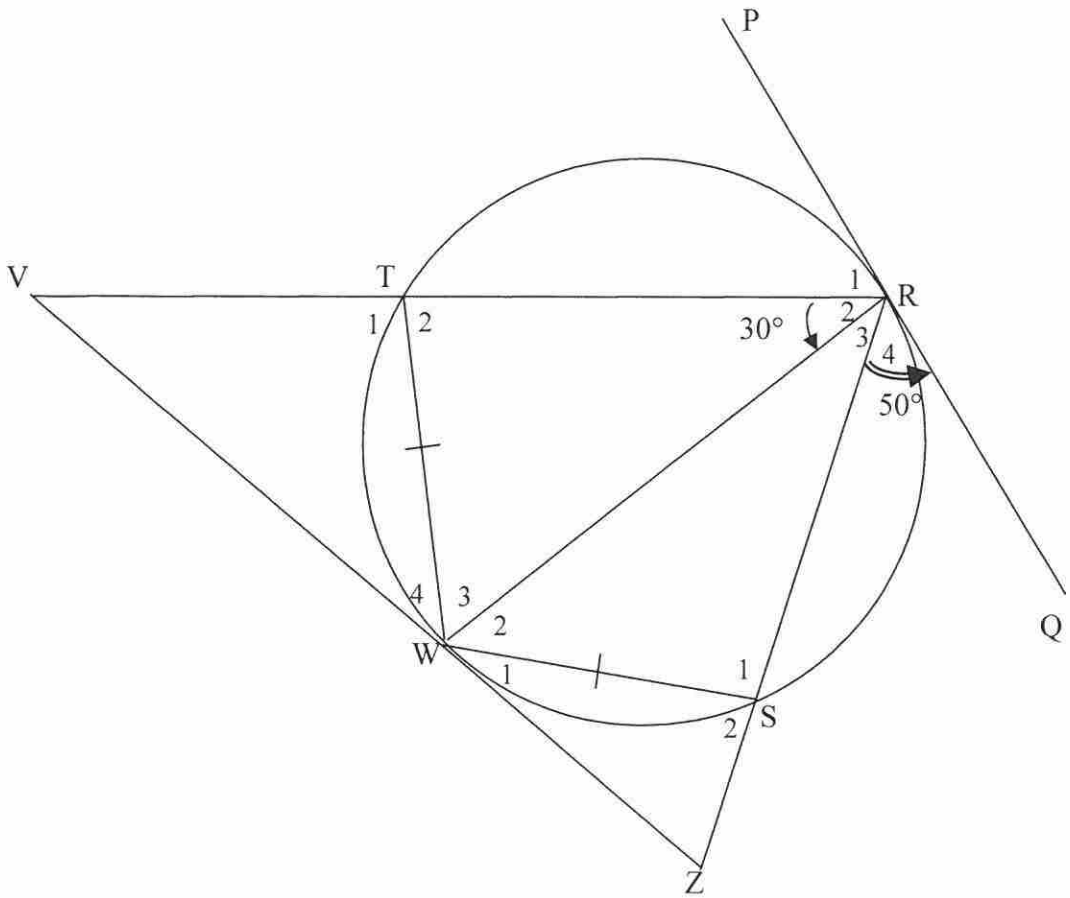
Die gedeeltelik voltooide bewys van die stelling wat beweer dat $V\hat{T}R = \hat{S}$ word hieronder gegee.
Gebruik die diagram hierbo en voltooi die bewys van die stelling op DIAGRAMVEL 3.

Konstruksie: Trek middellyn TC en verbind CV.

Bewering	Rede
Laat: $V\hat{T}R = \hat{T}_1 = x$	
$\hat{V}_1 + \hat{V}_2 =$.....	.....
$\hat{T}_2 = 90° - x$	.....
$\therefore \hat{C} =$	Som van die hoeke van 'n driehoek
$\therefore \hat{S} = x$	.....
$\therefore V\hat{T}R = \hat{S}$	

(5)

9.3 In die figuur is TRSW 'n koordevierhoek met TW = WS. RT en RS word verleng om raaklyn VWZ by V en Z onderskeidelik te ontmoet. PRQ is 'n raaklyn aan die sirkel by R. RW is verbind. $\hat{R}_2 = 30°$ en $\hat{R}_4 = 50°$.



9.3.1 Gee 'n rede waarom $\hat{R}_3 = 30°$. (1)

9.3.2 Noem, met redes, TWEE ander hoeke gelyk aan 30°. (3)

9.3.3 Bepaal, met redes, die grootte van:

(a) $\hat{S}_2$ (3)

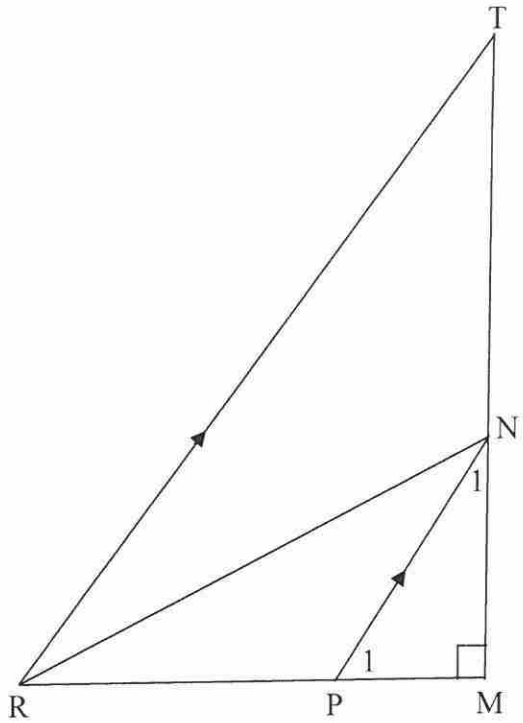
(b) $\hat{V}$ (4)

9.3.4 Bewys dat $WR^2 = RV \times RS$. (5)

[22]

VRAAG 10

In ΔTRM is $\hat{M} = 90°$. NP is ewewydig aan TR getrek met N op TM en P op RM. Dit word verder gegee dat $RT = 3PN$.



10.1 Gee redes vir die bewerings hieronder.
Gebruik DIAGRAMVEL 5.

	Bewering	Rede
	In ΔPNM en ΔRTM :	
10.1.1	$\hat{N}_1 = \hat{T}$	.....
	$\hat{M}$ is gemeenskaplik	
10.1.2	$\therefore \Delta PNM \parallel\parallel\parallel \Delta RTM$	.....

(2)

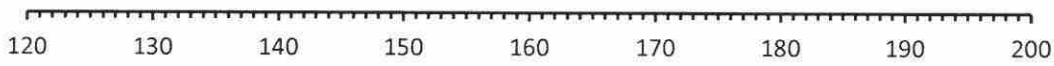
10.2 Bewys dat $\frac{PM}{RM} = \frac{1}{3}$. (2)

10.3 Toon aan dat $RN^2 - PN^2 = 2RP^2$. (4)
[8]

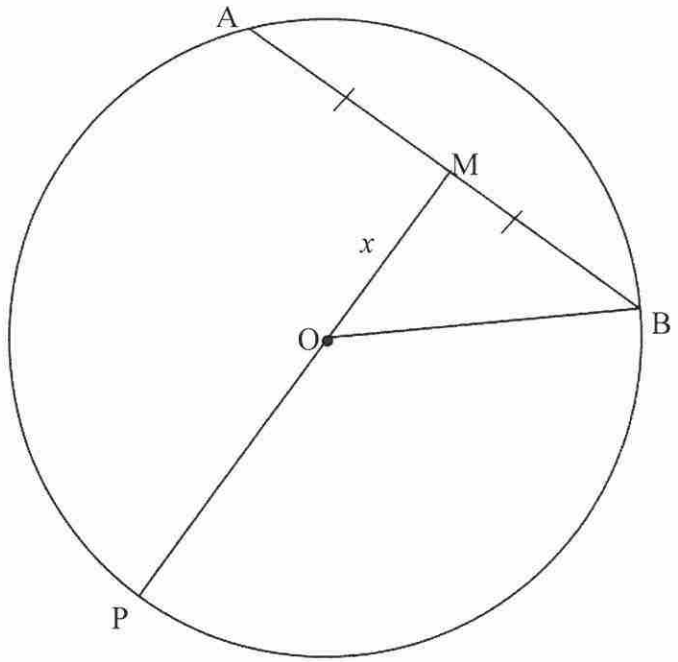
TOTAAL: 150

DIAGRAMVEL 1

VRAAG 1.3

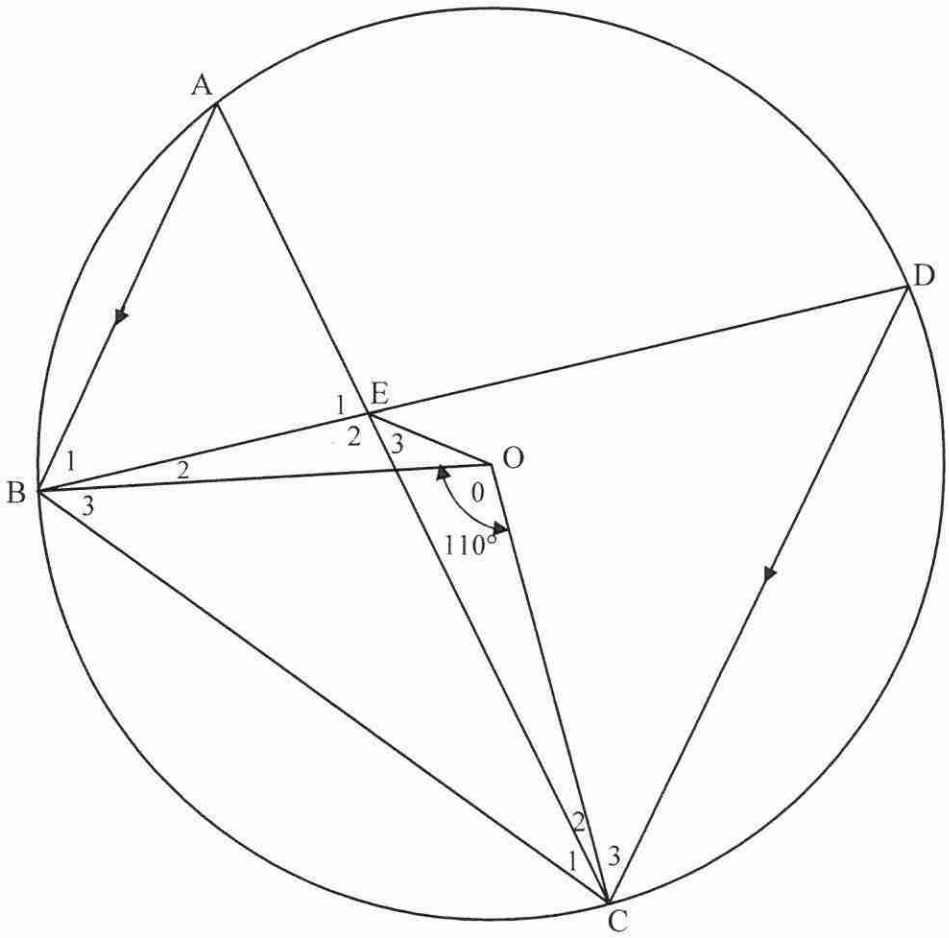


VRAAG 7



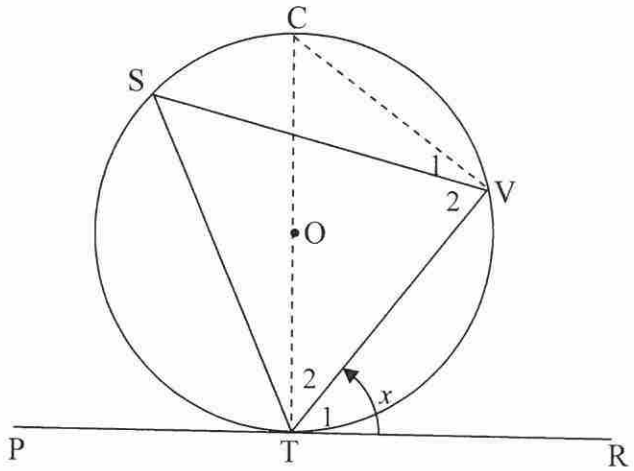
DIAGRAMVEL 2

VRAAG 8



DIAGRAMVEL 3

VRAAG 9.2

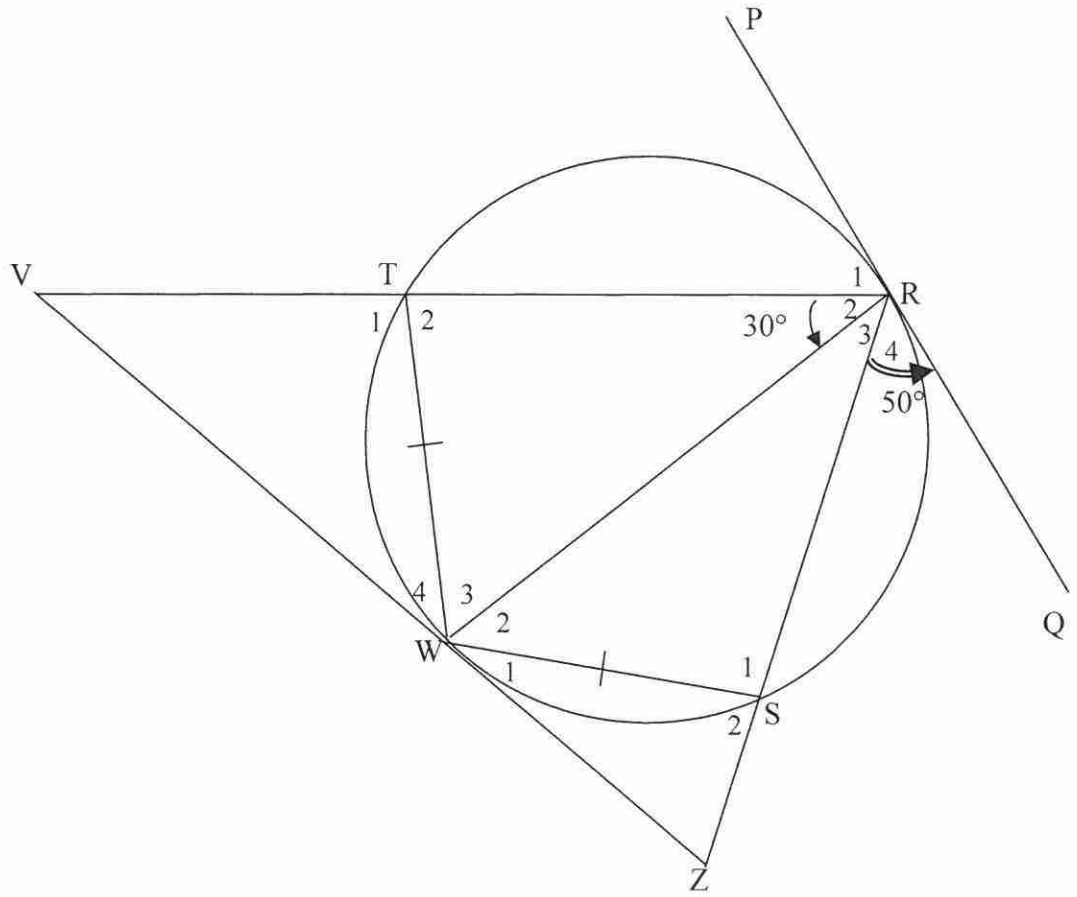


Konstruksie: Trek middellyn CT en verbind CV.

Bewering	Rede
Laat: $V\hat{T}R = \hat{T}_1 = x$	
$\hat{V}_1 + \hat{V}_2 =$.....	.....
$\hat{T}_2 = 90° - x$	.....
$\therefore \hat{C} =$	Som van die hoeke van 'n driehoek
$\therefore \hat{S} = x$	.....
$\therefore V\hat{T}R = \hat{S}$	

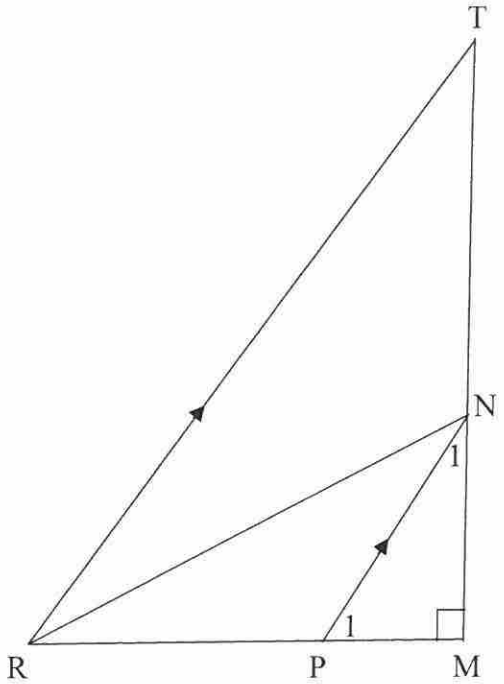
DIAGRAMVEL 4

VRAAG 9.3



DIAGRAMVEL 5

VRAAG 10



10.1

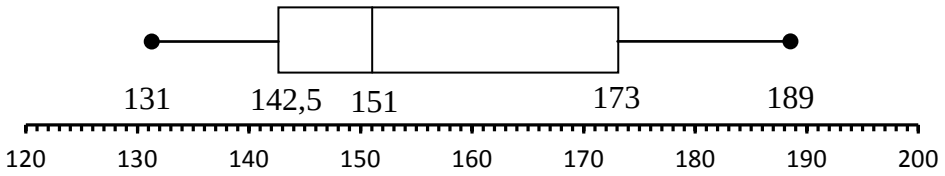
	Bewering	Rede
	In ΔPNM en ΔRTM :	
10.1.1	$\hat{N}_1 = \hat{T}$	.....
	$\hat{M}$ is gemeenskaplik	
10.1.2	$\therefore \Delta PNM \parallel\parallel\parallel \Delta RTM$	.....

Memo

NOTA:

- *As 'n kandidaat 'n vraag TWEEKEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n poging om die vraag te beantwoord, doodgetrek het en nie dit oorgedoen het nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienmemorandum toegepas.*
- *Aanvaarding van antwoorde/waardes om 'n probleem op te los, is ONaanvaarbaar.*

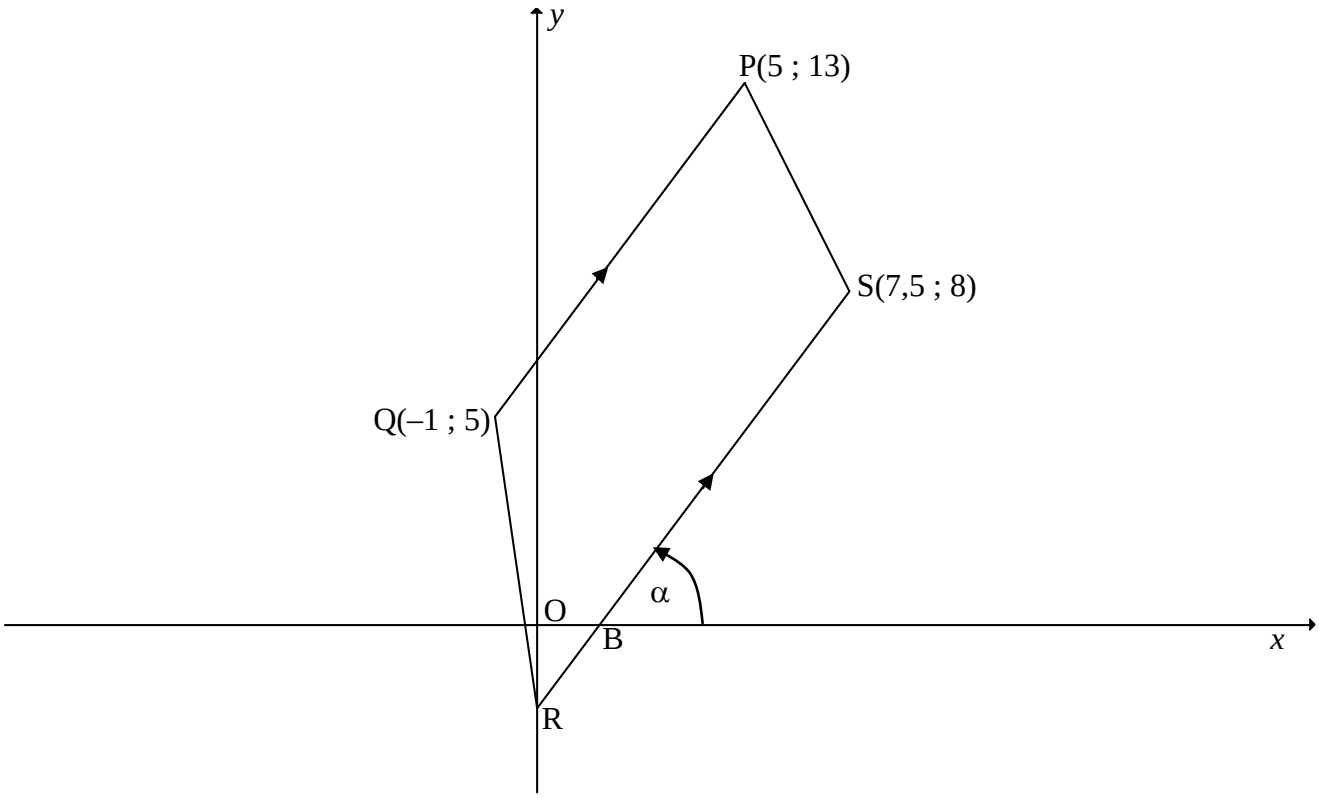
QUESTION/*VRAAG* 1

1.1	$\bar{x} = \frac{3310}{21}$ $= 157{,}62$ Answer only: Full marks *slegs antw: volpunte*	✓ $\frac{3310}{21}$ ✓157,62 (2)
1.2	(131 ; 142,5 ; 151 ; 173 ; 189)	✓ 131 and/ *en* 189 ✓ 142,5 ✓ 173 ✓ 151 (4)
1.3		✓box/*mond* ✓ whiskers/ *snor* (2)
1.4	positively skewed/*positief skeef* **OR/*OF*** skewed to the right/*skeef na regs*	✓ answer/ *antwoord* (1)
1.5	$\sigma = 17{,}27$	✓✓answer/ *antwoord* (2)
1.6.1	$\bar{x} = 157{,}62 + p$	✓ answer (1)
1.6.2	$\sigma = 17{,}27$	✓ answer/ *antwoord* (1) **[13]**

QUESTION/*VRAAG* 2

2.1	As the temperature increases, the sales of ice-creams increase/*Soos die temperatuur styg, neem die verkope toe.* **OR/*OF*** As the temperature decreases, the sales of ice-creams decrease/*Soos die temperatuur daal, neem die verkope af.*	✓ reason/*rede* (1) ✓ reason/*rede* (1)
2.2	The liveable temperature cannot keep on increasing/*Die leefbare temperatuur kan nie aanhou styg nie.*	✓ reason/*rede* (1)
2.3	$a = -460{,}35$ $b = 30{,}09$ $\hat{y} = 30{,}09x - 460{,}35$ **OR/*OF*** $\hat{y} = -460{,}35 + 30{,}09x$ Answer only: Full marks *slegs antw: volpunte*	✓✓ –460,35 ✓ 30,09 ✓ equation/*vgl* (4)
2.4	$r = 0{,}96$	✓ 0,96 (1)
2.5	There is a <u>very strong</u> positive relationship (correlation)/*Daar is 'n <u>baie sterk</u> positiewe verband (korrelasie).*	✓ very strong/*baie sterk* (1) **[8]**

QUESTION/VRAAG 3

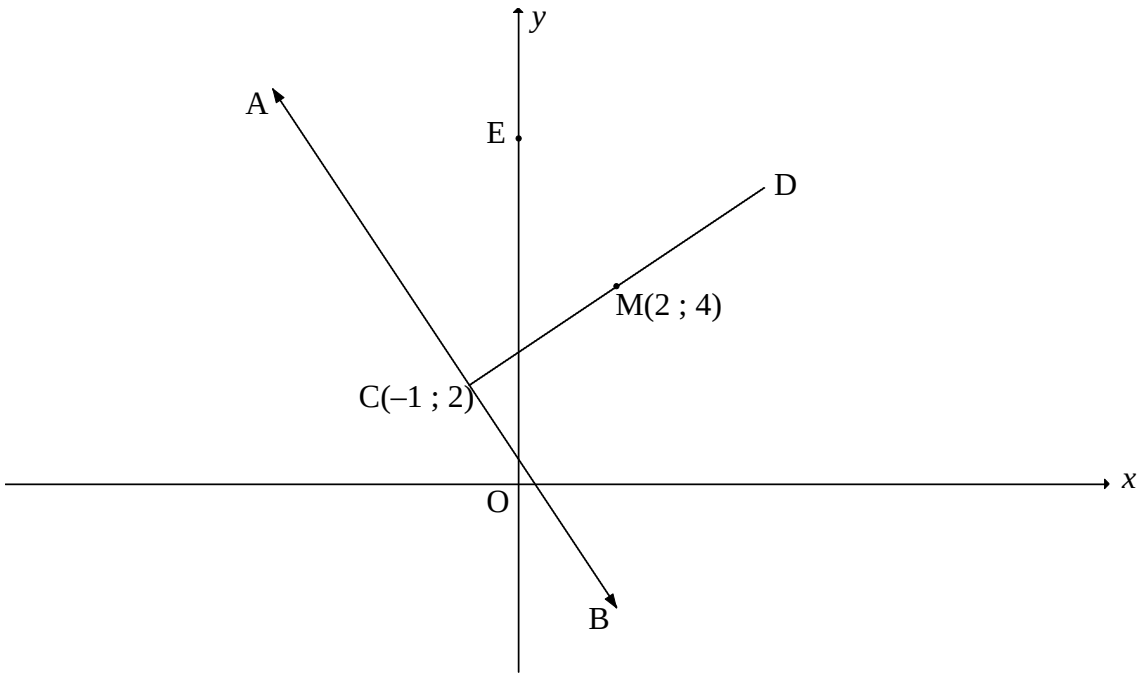


3.1	$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5+1)^2 + (13-5)^2}$ $= 10$	✓ use of distance formula/*gebruik afstandformule* ✓ correct subst into form/*korrekte subst in formule* ✓ 10 (3)
3.2	$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{13-5}{5-(-1)}$ $= \frac{8}{6} = \frac{4}{3}$ Answer only: Full marks *slegs antw: volpunte*	✓ correct subst into gradient formula/ *korrekte subst in gradiëntformule* ✓ gradient/*gradiënt* (2)

3.3	Equation of line RS/*Vgl van lyn RS*: $m_{RS} = m_{PQ} = \frac{4}{3}$ (= gradients, || lines/=*gradiënte*, || *lyne*) $y = mx + c$ $8 = \frac{4}{3}\left(\frac{15}{2}\right) + c$ $c = -2$ $y = \frac{4}{3}x - 2$ $\therefore 4x - 3y - 6 = 0$ **OR/*OF*** $y - y_1 = m(x - x_1)$ $y - 8 = \frac{4}{3}\left(x - \frac{15}{2}\right)$ $y = \frac{4}{3}x - 2$ $\therefore 4x - 3y - 6 = 0$	✓ $m_{RS} = \frac{4}{3}$ ✓ subst of S(7,5 ; 8) and *m* into eq /*subst van S(7,5 ; 8) en m in vgl* ✓ value of *c* /*waarde van c* or/of st form/*st vorm* ✓ equation/*vgl* (4)
3.4	B is the *x*-intercept of/*is die x-afsnit van* $y = \frac{4}{3}x - 2$ $0 = \frac{4}{3}x - 2$ $4x - 6 = 0$ $x = \frac{3}{2}$ **OR/*OF*** $4x - 3(0) - 6 = 0$ $4x - 6 = 0$ $x = \frac{3}{2}$	✓ $y = 0$ ✓ $x = \frac{3}{2}$ (2)
3.5	$\tan\ \alpha = \frac{4}{3}$ $\alpha = 53{,}13° = O\hat{B}R$ (vert opp ∠s/*regoorst ∠e*) $O\hat{R}B = 180° - (90° + 53{,}13°)$ (∠s of Δ/*∠e van Δ*) $= 36{,}87°$	✓ $\tan\ \alpha = \frac{4}{3}$ ✓ 53,13° ✓ 36,87° (3)
3.6	$BS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{\left(\frac{15}{2} - \frac{3}{2}\right)^2 + (8 - 0)^2}$ $= 10$ PQ || BS and/*en* PQ = BS PQBS = parallelogram (1 pair opp sides = and ||/*1 pr tos sye =en* ||) **OR/*OF*** midpoint of/*midpt van* QS: $\left(\frac{-1+7.5}{2}; \frac{5+8}{2}\right) = \left(\frac{13}{4}; \frac{13}{2}\right)$ midpoint of/*midpt van* PB: $\left(\frac{5+1.5}{2}; \frac{13+0}{2}\right) = \left(\frac{13}{4}; \frac{13}{2}\right)$ PQBS = parallelogram (diags bisect each other/*hoekl halv mekaar)* **OR/*OF***	✓ correct subst into form/*korrekte subst in formule* ✓ BS = 10 ✓ BS = PQ ✓ reason/*rede* (4) ✓ $\left(\frac{-1+7.5}{2}; \frac{5+8}{2}\right)$ ✓ $\left(\frac{5+1.5}{2}; \frac{13+0}{2}\right)$ ✓ $\left(\frac{13}{4}; \frac{13}{2}\right)$ ✓ reason/*rede* (4)

	$m_{QB} = \frac{5-0}{-1-1,5} = \frac{5}{-2,5} = -2$ $m_{PS} = \frac{13-8}{5-7,5} = \frac{5}{-2,5} = -2$ $m_{QB} = m_{PS}$ $\therefore$ QB || PS PQ || BS PQBS = parallelogram (both pairs opp sides ||/*beide pr tos sye* ||)	✓ m_{QB} ✓ m_{PS} ✓ QB || PS ✓ reason/*rede* (4)
	OR/*OF* $BS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{\left(\frac{15}{2} - \frac{3}{2}\right)^2 + (8-0)^2}$ $\therefore$ PQ = BS $= 10$ $QB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(-1-1,5)^2 + (5-0)^2} = \sqrt{(2,5)^2 + (5)^2} = \frac{5\sqrt{5}}{2}$ or 5,59 $PS = \sqrt{(5-7,5)^2 + (13-8)^2} = \sqrt{(2,5)^2 + (5)^2} = \frac{\sqrt{125}}{2}$ or 5,59 QB = PS PQBS = parallelogram (both pairs opp sides =/ *beide pr tos sye* =)	✓ correct subst into form/*korrekte subst in formule* ✓ PQ = 10 ✓ QB = PS ✓ reason/*rede* (4) **[18]**

QUESTION/VRAAG 4



4.1.1	Radius = $\sqrt{(2+1)^2+(4-2)^2}$ $r = \sqrt{13}$ Equation of circle/*vgl van sirkel*: $(x-2)^2+(y-4)^2=13$ **OR/*OF*** $(x-2)^2+(y-4)^2=r^2$ $(-1-2)^2+(2-4)^2=r^2$ $r^2=13$ $\therefore\ (x-2)^2+(y-4)^2=13$	✓ $\sqrt{(2+1)^2+(4-2)^2}$ or/*of* $\sqrt{13}$ ✓ $(x-2)^2+(y-4)^2$ ✓ 13 (3) ✓ $(x-2)^2+(y-4)^2$ ✓ $(-1-2)^2+(2-4)^2$ ✓ 13 (3)
4.1.2	At/*by* D: $\frac{-1+x_D}{2}=2$ and/*en* $\frac{2+y_D}{2}=4$ $-1+x_D=4$ and/*en* $2+y_D=8$ $x_D=5$ $y_D=6$ D(5 ; 6) **OR/*OF*** By inspection/*deur inspeksie*: D(5 ; 6)	✓ x – value/*waarde* ✓ y - value/*waarde* (2) ✓ x - value/*waarde* ✓ y - value/*waarde* (2)

4.1.3	$m_{MC} = \frac{4-2}{2+1} = \frac{2}{3}$ $m_{AB} \times m_{MC} = -1$ (Tangent ⊥ radius/*raaklyn ⊥ radius*) $m_{AB} = -\frac{3}{2}$ $y - y_1 = m(x - x_1)$ **OR/OF** $y = mx + c$ $y - 2 = -\frac{3}{2}(x+1)$ $2 = -\frac{3}{2}(-1) + c$ $y = -\frac{3}{2}x + \frac{1}{2}$ $y = -\frac{3}{2}x + \frac{1}{2}$	✓ $m_{MC} = \frac{4-2}{2+1} = \frac{2}{3}$ ✓ $m_{AB} \times m_{MC} = -1$ ✓ $m_{AB} = -\frac{3}{2}$ ✓subst *m* and (–1 ; 2) into eq /*subst m en (–1 ; 2) in vgl* ✓ eq in standard form/ *vgl in st vorm* (5)
4.1.4	At/*by* E: $(0-2)^2 + (y-4)^2 = 13$ $(y-4)^2 = 9$ $y - 4 = \pm 3$ $y = 7$ or $y = 1$ E(0 ; 7) **OR/*OF*** At/*by* E: $(0-2)^2 + (y-4)^2 = 13$ $4 + y^2 - 8y + 16 = 13$ $y^2 - 8y + 7 = 0$ $(y-7)(y-1) = 0$ $y = 7$ or $y = 1$ E(0 ; 7)	✓ $x = 0$ ✓ simplification/ *vereenvoudiging* ✓ *y* - values/*waardes* ✓ E(0 ; 7) (4) ✓ $x = 0$ ✓ simplification/ *vereenvoudiging* ✓ *y* - values/*waardes* ✓ E(0 ; 7) (4)
4.1.5	$m_{EM} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{4-7}{2-0}$ $= -\frac{3}{2}$ $m_{AB} = -\frac{3}{2}$ $\therefore$ EM || AB ($m_{EM} = m_{AB}$)	✓ $m_{EM} = -\frac{3}{2}$ ✓ reason/*rede* (2)

4.2	The centres of the circles are / *Die middelpunte van die sirkels is* P(–2 ; 4) and / *en* Q(5 ; –1) $QP^2 = (-2-5)^2 + (4-(-1))^2$ $QP = \sqrt{74} \approx 8{,}60$ units $r_M + r_P = 5 + 3$ $= 8$ $\therefore\ r_M + r_P < QP$ $\therefore$ The two circles do not intersect/*Die twee sirkels sny nie*	✓ both centres/*albei Midpte* ✓ QP ✓ correct subst into form/*korrekte subst in formule* ✓ distance between 2 centres/*afstand tussen 2 midpte* ✓✓ $r_M + r_P < QP$ (6) **[22]**

QUESTION/VRAAG 5

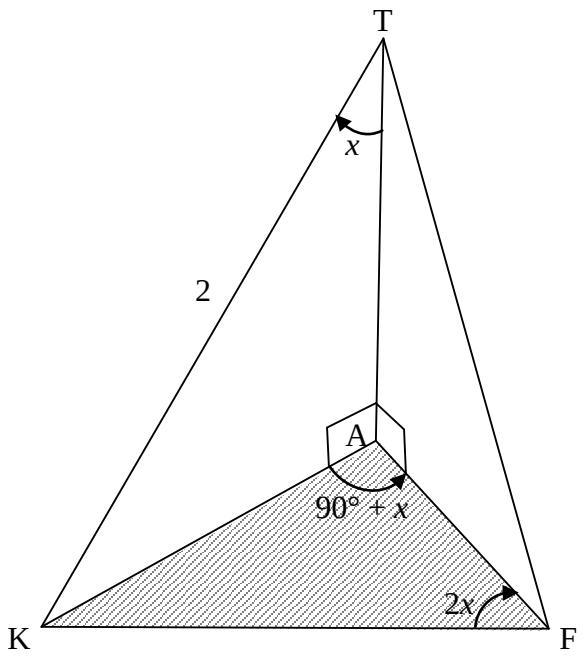
5.1	$x^2 + y^2$ $=(3\sin\theta)^2 + (3\cos\theta)^2$ $=9\sin^2\theta + 9\cos^2\theta$ $=9(\sin^2\theta + \cos^2\theta)$ $=9(1)$ $=9$	✓ simpl/*vereenv* ✓ CF/*GF* = 9 ✓ answer/*antw* (3)
5.2	$\sin(540° - x).\sin(-x) - \cos(180° - x).\sin(90° + x)$ $\sin(180° - x).\sin(-x) - \cos(180° - x).\sin(90° + x)$ $= (\sin x)(-\sin x) - (-\cos x)(\cos x)$ $= -\sin^2 x + \cos^2 x$ $= \cos 2x$	✓ $\sin(540° - x) = \sin x$ ✓ $\sin(-x) = -\sin x$ ✓ $\cos(180° - x) = -\cos x$ ✓ $\sin(90° + x) = \cos x$ ✓ $-\sin^2 x + \cos^2 x$ ✓ $\cos 2x$ (6)
5.3.1	$OT = \sqrt{x^2 + p^2}$ $\sin\alpha = \frac{y_T}{OT}$ $= \frac{p}{\sqrt{x^2 + p^2}}$ $\frac{p}{\sqrt{x^2 + p^2}} = \frac{p}{\sqrt{1 + p^2}}$ $x^2 = 1$ $x = -1$ **OR/*OF*** (P lies in 3rd quadrant) $x^2 + y^2 = r^2$ $x^2 + p^2 = \left(\sqrt{1 + p^2}\right)^2$ $x^2 + p^2 = 1 + p^2$ $x^2 = 1$ $x = -1$ (P lies in 3rd quadrant)	✓ $OT = \sqrt{x^2 + p^2}$ ✓ $\sin\alpha = \frac{y_T}{OT}$ ✓ $x^2 = 1$ (3) ✓ $x^2 + y^2 = r^2$ ✓ subst ✓ $x^2 = 1$ (3)
5.3.2	$\cos(180° + \alpha)$ $= -\cos\alpha$ $= -\left(\frac{-1}{\sqrt{1 + p^2}}\right)$ $= \frac{1}{\sqrt{1 + p^2}}$	✓ $-\cos\alpha$ ✓ answer/*antw* (2)

5.3.3	$\cos 2\alpha$ $= \cos^2\alpha - \sin^2\alpha$ $= \left(\frac{-1}{\sqrt{1+p^2}}\right)^2 - \left(\frac{p}{\sqrt{1+p^2}}\right)^2$ $= \frac{1}{1+p^2} - \frac{p^2}{1+p^2}$ $= \frac{1-p^2}{1+p^2}$	✓ expansion/ *uitbreiding* ✓✓ squaring each term/*kwadreer elke term* (3)
	OR/*OF*	
	$\cos 2\alpha$ $= 1 - 2\sin^2\alpha$ $= 1 - 2\left(\frac{p}{\sqrt{1+p^2}}\right)^2$ $= 1 - 2\left(\frac{p^2}{1+p^2}\right)$ $= 1 - \frac{2p^2}{1+p^2}$ $= \frac{1+p^2-2p^2}{1+p^2}$ $= \frac{1-p^2}{1+p^2}$	✓ expansion/ *uitbreiding* ✓squaring/*kwadrering* ✓ writing as single fraction/*skryf as enkelterm* (3)
	OR/*OF*	
	$\cos 2\alpha$ $= 2\cos^2\alpha - 1$ $= 2\left(\frac{-1}{\sqrt{1+p^2}}\right)^2 - 1$ $= 2\left(\frac{1}{1+p^2}\right) - 1$ $= \frac{2}{1+p^2} - 1$ $= \frac{2-1-p^2}{1+p^2}$ $= \frac{1-p^2}{1+p^2}$	✓ expansion/ *uitbreiding* ✓squaring/*kwadrering* ✓ writing as single fraction/*skryf as enkelterm* (3)

5.4.1	The identity is undefined for/*die identiteit is ongedefinieerd as*: $2\sin^2 x = 0$ $\therefore$ <u>$\sin x = 0$:</u> $x = 0°$; $180°$ or/*of* <u>$\tan x = \infty$:</u> $x = 90°$ $\therefore$ $x = 0°$; $90°$; $180°$	✓ $x = 0°$ ✓ $x = 90°$ ✓ $x = 180°$ (3)
5.4.2	LHS/LK $= \dfrac{2\tan x - \sin 2x}{2\sin^2 x}$ $= \dfrac{2\left(\dfrac{\sin x}{\cos x}\right) - 2\sin x\cos x}{2\sin^2 x}$ $= \left(\dfrac{2\sin x - 2\sin x\cos^2 x}{\cos x}\right) \times \dfrac{1}{2\sin^2 x}$ $= \dfrac{2\sin x(1-\cos^2 x)}{\cos x} \times \dfrac{1}{2\sin^2 x}$ $= \dfrac{2\sin x(\sin^2 x)}{\cos x} \times \dfrac{1}{2\sin^2 x}$ $= \dfrac{\sin x}{\cos x}$ $= \tan x$ = RHS/RK **OR/*OF*** LHS/LK $= \dfrac{2\tan x - \sin 2x}{2\sin^2 x}$ $= \dfrac{2\left(\dfrac{\sin x}{\cos x}\right) - 2\sin x\cos x}{2\sin^2 x} \times \dfrac{\cos x}{\cos x}$ $= \dfrac{2\sin x - 2\sin x\cos^2 x}{2\sin^2 x\cos x}$ $= \dfrac{2\sin x(1-\cos^2 x)}{2\sin^2 x\cos x}$ $= \dfrac{2\sin x.\sin^2 x}{2\sin^2 x\cos x}$ $= \dfrac{\sin x}{\cos x}$ $= \tan x$ = RHS/RK	✓ $\dfrac{\sin x}{\cos x}$ ✓ $2\sin x.\cos x$ ✓ simplify numerator/ *vereenv teller* ✓ factorising/*fakt* ✓ $1 - \cos^2 x = \sin^2 x$ ✓ simplify to/*vereenv na* $\dfrac{\sin x}{\cos x}$ (6) ✓ $\dfrac{\sin x}{\cos x}$ ✓ $2\sin x.\cos x$ ✓ simpl/*vereenv* ✓ factorising/*fakt* ✓ $1 - \cos^2 x = \sin^2 x$ ✓ simplify to /*vereenv na* $\dfrac{\sin x}{\cos x}$ (6) **[26]**

QUESTION/VRAAG 6

6.1

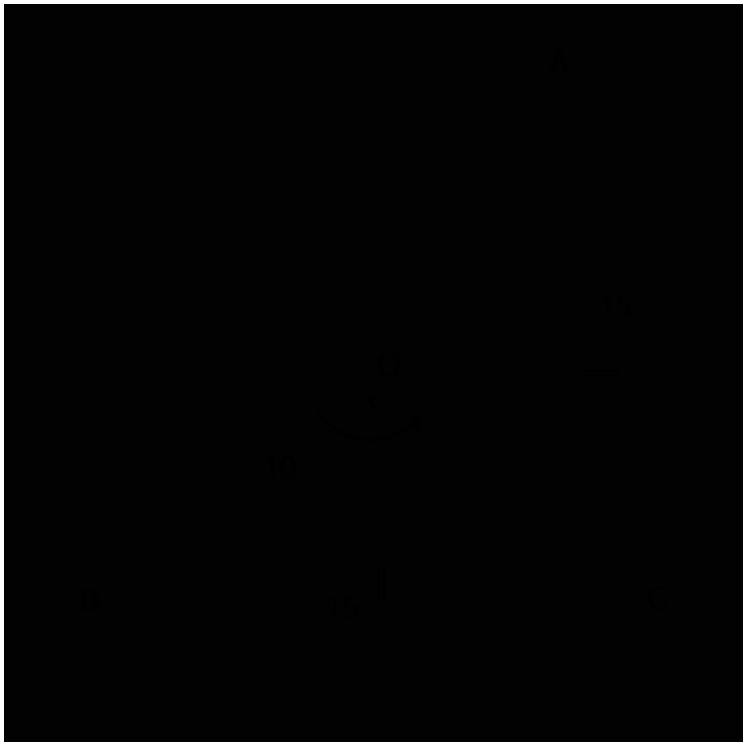


6.1.1	In ΔTAK: $\frac{AK}{KT} = \sin K\hat{T}A$ $AK = KT.\sin x$ $= 2\sin x$ **OR/*OF*** $\frac{\sin K\hat{T}A}{AK} = \frac{\sin K\hat{A}T}{KT}$ $\frac{\sin 90^\circ}{2} = \frac{\sin x}{AK}$ $AK = 2\sin x$	✓ correct trig ratio/ *korrekte trigverh.* ✓answer/*antw* (2) ✓ correct subst into sine rule/*korrekte subst in sin-reël* ✓answer/*antw* (2)

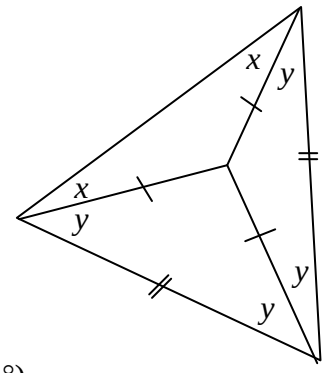
6.1.2	In ΔAKF: $\frac{KF}{\sin K\hat{A}F} = \frac{AK}{\sin A\hat{F}K}$ $\frac{KF}{\sin(90° + x)} = \frac{AK}{\sin 2x}$ $KF = \frac{AK.\sin(90° + x)}{\sin 2x}$ $= \frac{2\sin x.\cos x}{2\sin x.\cos x}$ $= 1$ **OR/*OF*** In ΔAKF: $\frac{KF}{\sin K\hat{A}F} = \frac{AK}{\sin A\hat{F}K}$ $\frac{KF}{\sin(90° + x)} = \frac{AK}{\sin 2x}$ $KF = \frac{AK.\sin(90° + x)}{\sin 2x}$ $= \frac{AT.\tan x.\cos x}{2\sin x.\cos x}$ $= \frac{2\cos x.\frac{\sin x}{\cos x}.\cos x}{2\sin x.\cos x}$ $= 1$	✓ using sine rule/ *gebruik sin-reël* ✓ correct subst into sine rule/*korrekte subst in sin-reël* ✓ sin(90° +x) = cos x ✓ 2sin$x$.cos$x$ ✓ 1 (5) ✓ using sine rule/ *gebruik sin-reël* ✓ correct subst into sine rule/*korrekte subst in sin-reël* ✓ sin(90° +x) = cos x ✓ 2sin$x$.cos$x$ ✓ 1 (5)

$$\boxed{\begin{array}{l} \cos x = \frac{AT}{2} \\ \therefore AT = 2\cos x \end{array}}$$

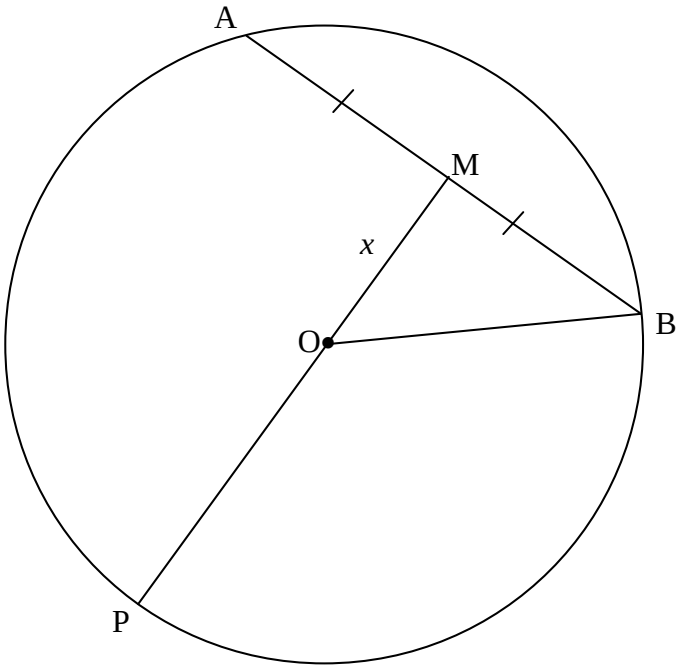
6.2



6.2.1	In ΔBOC: $BC^2 = BO^2 + CO^2 - 2.BO.\ CO\ .\cos x$ $15^2 = 10^2 + 10^2 - 2(10)(10).\ \cos x$ $200 \cos x = -25$ $\cos x = -0{,}125$ $x = 180° - 82{,}82°$ $= 97{,}18°$ **OR/*OF*** Draw a line OD⊥ BC: BD = DC (line from centre ⊥ on chord) ΔOBD ≡ ΔOCD (90°; h; s) $\sin \frac{x}{2} = \frac{7{,}5}{10}$ $\frac{x}{2} = 48{,}59°$ $\therefore\ x = 97{,}18°$	✓ using cosine rule/ *gebruik cos-reël* ✓correct subst/ *korrekte subst* ✓ $\cos x = -0{,}125$ ✓ 97,18° (4) ✓ S/R ✓correct ratio/ *korrekte verh* ✓ value of/*waarde van* $\frac{x}{2}$ ✓ 97,18° (4)

6.2.2	$B\hat{A}C = 48{,}59°$ (∠ at centre=2×∠at circ/∠ *by midpt=2×∠omt*) $A\hat{B}C = B\hat{A}C = 48{,}59°$ (∠'s opp equal sides/*∠e teenoor = sye*) $\therefore A\hat{C}B = 82{,}82°$ (sum of ∠s of Δ/*som van ∠e van Δ*) **OR/*OF*** $A\hat{C}B = \frac{1}{2}A\hat{O}B$ (∠at centre = 2 × ∠at circle) (*∠ by midpt=2×∠omt*) $= \frac{1}{2}[360° - 2(97{,}18°)]$ $= 82{,}82°$ **OR/*OF*** $O\hat{C}B = \frac{1}{2}(180° - 97{,}18°)$ (∠'s opp equal sides; sum of ∠s of Δ) $= 41{,}41°$ (*∠e teenoor = sye; som van ∠e van Δ*)  $A\hat{C}B = 2(41{,}41°)$ $= 82{,}82°$	✓ S ✓ S ✓82,82° (3) ✓ S ✓ S ✓82,82° (3) ✓ S ✓ S ✓82,82° (3)
6.2.3	Area/*Oppervlakte* ΔABC $= \frac{1}{2}(BC)(AC)\sin A\hat{C}B$ $= \frac{1}{2}(15)(15)(\sin 82{,}82°)$ $= 111{,}62\ cm^2$	✓correct subst into area rule/*korrekte subst in opp-reël* ✓ $111{,}62\ cm^2$ (2) **[16]**

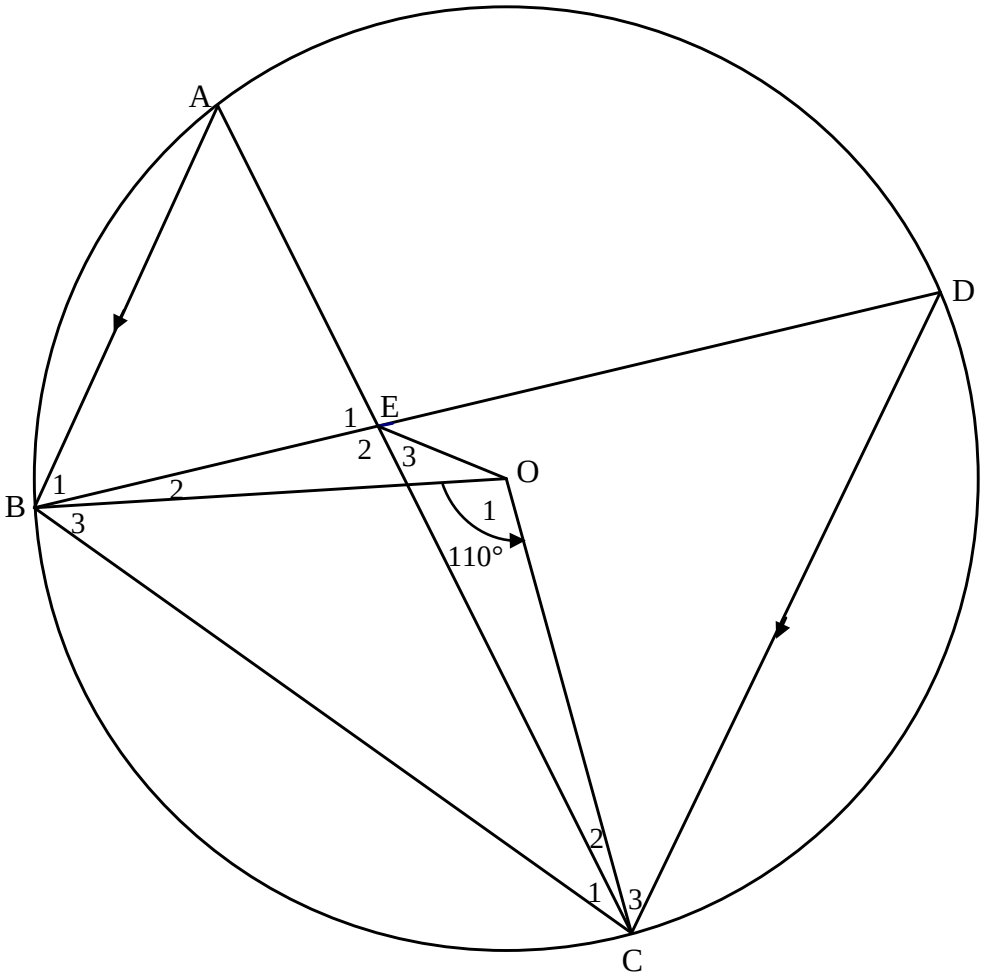
QUESTION/VRAAG 7



7.1	MB = 10 cm	✓ answer/*antw* (1)
7.2	line from centre to midpoint of chord is perpendicular to chord/*lyn vanaf midpt na midpt van koord is loodreg op koord* **OR/*OF*** line from centre bisects chord/*lyn vanaf midpt halveer koord*	✓ answer/*antw* (1) ✓ answer/*antw* (1)
7.3	$\frac{MP}{OM} = \frac{5}{2}$ $\frac{x + OP}{x} = \frac{5}{2}$ $2x + 2OP = 5x$ $OP = \frac{3x}{2}$ **OR/*OF*** $\frac{OP}{OM} = \frac{3}{2}$ $OP = \frac{3x}{2}$	✓ $\frac{x + OP}{x} = \frac{5}{2}$ ✓ $OP = \frac{3x}{2}$ (2) ✓ $\frac{OP}{OM} = \frac{3}{2}$ ✓ $OP = \frac{3x}{2}$ (2)

7.4	$OM^2 + MB^2 = OB^2$ $x^2 + 10^2 = \left(\frac{3x}{2}\right)^2$ $4x^2 + 400 = 9x^2$ $5x^2 = 400$ $x^2 = 80$ $x = 8{,}94$ or $4\sqrt{5}$ or $\sqrt{80}$	✓ subst into/*subst* Pythagoras ✓ $4x^2 + 400 = 9x^2$ ✓ answer/*antw* (3) **[7]**

QUESTION/VRAAG 8

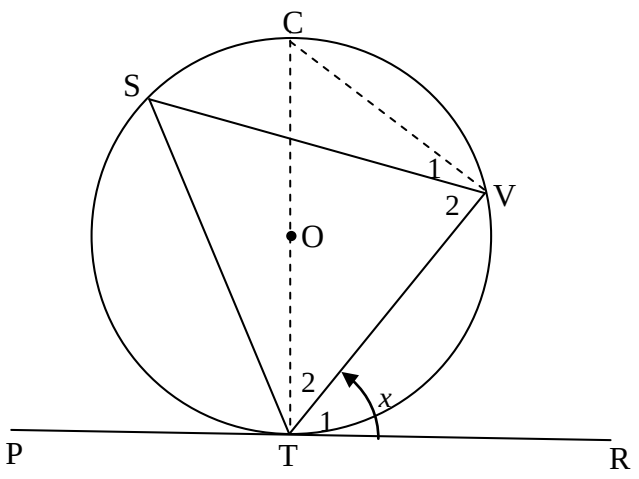


8.1.1	$\hat{D} = \frac{1}{2}\hat{O}_1 = 55°$ (∠ at centre=2×∠at circ/∠ *by midpt=2×∠by omt*)	✓S ✓R (2)
8.1.2	$\hat{A} = \frac{1}{2}\hat{O}_1 = 55°$ (∠ at centre=2×∠at circ/∠ *by midpt=2×∠by omt*) **OR/*OF*** $\hat{A} = \hat{D} = 55°$ (∠s in same segment/*∠e in dieselfde segment*)	✓S ✓R (2) ✓S ✓R (2)
8.1.3	$\hat{B}_1 = \hat{D} = 55°$ (alternate ∠s/*verwiss∠e*; AB || DC) $\hat{E}_2 = \hat{B}_1 + \hat{A}$ (ext∠ of∆ = sum of opp∠s/*buite∠ v∆=som v tos∠e*) $= 55° + 55°$ $\hat{E}_2 = 110°$	✓S ✓R ✓R ✓ answer/*antw* (4)
8.2	$\hat{E}_2 = \hat{O}_1 = 110°$ (proven in/*bewys in* 8.1.3) BEOC is a cyclic quadrilateral (equal ∠s subtended by line/ *gelyke ∠e onderspan deur lyn*)	✓S ✓R (2) **[10]**

QUESTION/VRAAG 9

9.1	the interior opposite angle/*die teenoorstaande binnehoek.*	✓ answer/*antw* (1)

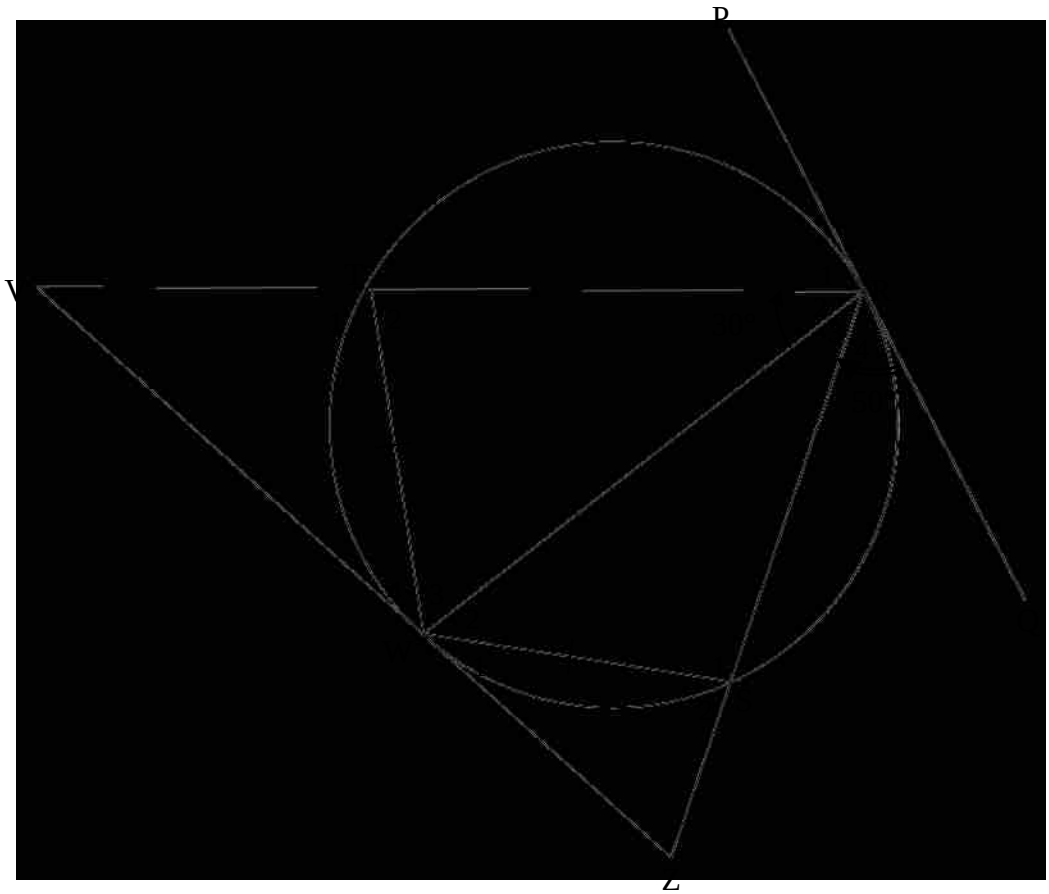
9.2



Construction: Draw diameter CT and join CV.
Konstruksie: Trek middellyn CT en verbind CV.

$\hat{V}_1 + \hat{V}_2 = \mathbf{90°}$	**∠ in semi-circle/*∠ in halfsirkel***	✓ S ✓ R
$\hat{T}_2 = 90° - x$	**Tangent ⊥ diameter/radius/*raaklyn ⊥ middellyn/radius***	✓ R
$\therefore \hat{C} = \boldsymbol{x}$	Sum of the angles of triangle/*Som van die hoeke van 'n driehoek*	✓ S
$\therefore \hat{S} = x$	**∠'s same segment/*∠e in dieselfde segment***	✓ R
$\therefore V\hat{T}R = \hat{S}$		(5)

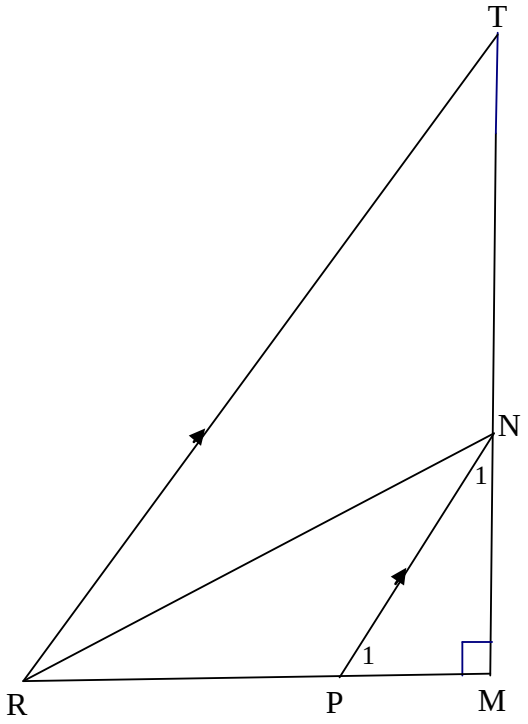
9.3



9.3.1	Equal chords subtend equal ∠s/*Gelyke koorde onderspan gelyke ∠e*	✓ R (1)
9.3.2	$\hat{W}_4 = 30°$ (tan chord theorem/*rkl-koordst*) $\hat{W}_1 = 30°$	✓ answer/*antw* ✓ reason/*rede* ✓ answer/*antw* (3)
9.3.3(a)	$\hat{R}_4 = \hat{W}_2 = 50°$ (tan chord theorem/*rkl-koordst*) $\hat{S}_2 = \hat{R}_3 + \hat{W}_2$ (ext∠ of Δ/*buite ∠ v Δ*) $\therefore \hat{S}_2 = 80°$ **OR/*OF*** $\hat{R}_2 = \hat{R}_3 = 30°$ (= chords subtend =∠s /= *kde onderspan=∠e*) $\hat{R}_4 = \hat{W}_2 = 50°$ (tan chord theorem/*rkl-koordst*) $\therefore \hat{S}_2 = 80°$	✓ S ✓R ✓ S (3) ✓ S ✓R ✓ S (3)

9.3.3(b)	$\hat{T}_2 = \hat{S}_2 = 80°$ (ext $\angle$ of cyclic quad/*buite*$\angle$ *van koordevh*) $V + \hat{W}_4 = \hat{T}_2$ (ext$\angle$ of Δ/*buite*$\angle$ *van* Δ) $\therefore\ \hat{V} = 50°$	✓ S ✓ R ✓ S ✓ S (4)
9.3.4	In ΔRVW and/*en* ΔRWS: $\hat{R}_2 = \hat{R}_3 = 30°$ (proven/*bewys* in 9.3.1) $\hat{V} = \hat{W}_2 = 50°$ (proven/*bewys* in 9.3.3) $V\hat{W}R = \hat{S}_1$ (3rd $\angle$ in Δ) $\therefore \Delta RVW \,\|\|\| \, \Delta RWS$ ($\angle\angle\angle$) $\therefore \frac{WR}{RV} = \frac{RS}{WR}$ ($\Delta RVW \,\|\|\| \, \Delta RWS$) $\therefore WR^2 = RV.RS$	✓ using the correct Δs/ *gebruik korrekte* Δ*e* ✓ S ✓ S ✓ R (3rd $\angle$ in Δ) or ($\angle\angle\angle$) ✓ S (5) **[22]**

QUESTION/VRAAG 10



10.1.1	corresponding ∠s/*ooreenkomstige∠e*; PN || RT	✓ answer/*antw* (1)
10.1.2	∠; ∠; ∠ **OR/*OF*** ∠; ∠	✓ answer/*antw* (1)
10.2	$\frac{PM}{RM} = \frac{PN}{RT}$ ($\Delta PNM \,\|\|\| \Delta RTM$) $= \frac{PN}{3PN}$ $= \frac{1}{3}$	✓ S ✓ S (2)
10.3	$\frac{PM}{RM} = \frac{1}{3}$ $\therefore \frac{RP}{RM} = \frac{2}{3}$ $RN^2 - PN^2 = (RM^2 + NM^2) - (PM^2 + NM^2)$ (Pyth) $= RM^2 - PM^2$ $= \left(\frac{3}{2}RP\right)^2 - \left(\frac{1}{2}RP\right)^2$ $= \frac{9}{4}RP^2 - \frac{1}{4}RP^2$ $= 2RP^2$ **OR/*OF***	✓ Use of Pyth. for RN^2 and PN^2 ✓ $RM = \frac{3}{2}RP$ ✓ $PM = \frac{1}{2}RP$ ✓ $\frac{9}{4}RP^2$ & $\frac{1}{4}RP^2$ (4)

	$RN^2 - PN^2 = (RM^2 + NM^2) - (PM^2 + NM^2)$ (Pyth) $= RM^2 - PM^2$ $= (3PM)^2 - PM^2$ $= 8PM^2$ $= 2(2PM)^2$ $= 2RP^2$ **OR/*OF*** $RN^2 - PN^2 = (RM^2 + NM^2) - (PM^2 + NM^2)$ (Pyth) $= RM^2 - PM^2$ $= (RP + PM)^2 - PM^2$ $= RP^2 + 2RP.PM + PM^2 - PM^2$ $= RP^2 + 2RP.\frac{1}{2}RP$ $= 2RP^2$	✓ Use of Pyth. for RN^2 and PN^2 ✓RM = RP + PM ✓ $(3PM)^2 - PM^2$ ✓ RP = 2PM (4) ✓ Use of Pyth. for RN^2 and PN^2 ✓RM = RP + PM ✓ expansion/ *uitbreiding* ✓ $PM = \frac{1}{2}RP$ (4) **[8]**

TOTAL/*TOTAAL*: 150

PolyMathic

Vraestel 7
Okt/Nov
Eksamen

PolyMathic

Graad 12 EindEksamen
Totaal: 150
Tyd: 3ure

VRAAG 1

Die tabel hieronder toon die totale vet (in gram, tot die naaste telgetal afgerond) en energie (in kilojoule, tot die naaste 100 afgerond) van 10 items wat by 'n kitskosrestaurant verkoop word.

Vet (in gram)	9	14	25	8	12	31	28	14	29	20
Energie (in kilojoule)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

1.1 Stel die inligting hierbo in 'n spreidiagram voor deur die rooster te gebruik wat in die ANTWOORDEBOEK verskaf word. (3)

1.2 Die vergelyking van die kleinstekwadrate-regressielyn is $\hat{y} = 154{,}60 + 77{,}13x$.

1.2.1 'n Item by die restaurant bevat 18 gram vet. Bereken hoeveel (getal) kilojoule energie hierdie item sal verskaf. Gee jou antwoord tot die naaste 100 kJ afgerond. (2)

1.2.2 Trek die kleinstekwadrate-regressielyn op die spreidiagram wat vir VRAAG 1.1 geteken is. (2)

1.3 Identifiseer 'n uitskieter in die datastel. (1)

1.4 Bereken die waarde van die korrelasiekoëffisiënt. (2)

1.5 Lewer kommentaar op die sterkte van die verband tussen die vetinhoud en die hoeveelheid (getal) kilojoule energie. (1)

[11]

VRAAG 2

'n Groep van 30 leerders het elkeen willekeurig twee dobbelstene een keer gerol en die som van die waardes op die boonste vlakke van die dobbelstene is aangeteken. Die data word in die frekwensietabel hieronder getoon.

Som van die waardes op boonste vlakke	**Frekwensie**
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

2.1 Bereken die gemiddelde van die data. (2)

2.2 Bepaal die mediaan van die data. (2)

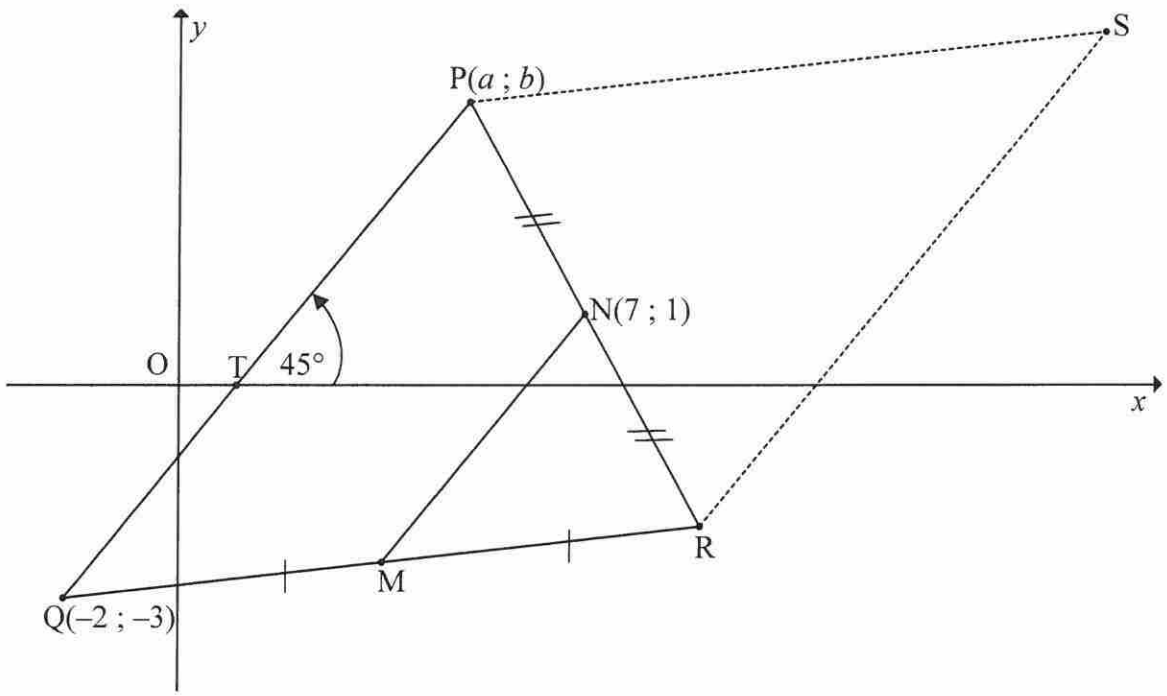
2.3 Bepaal die standaardafwyking van die data. (2)

2.4 Bepaal hoeveel keer die som van die aangetekende waardes van die dobbelstene binne EEN standaardafwyking vanaf die gemiddelde is. Toon jou berekeninge. (3)

[9]

VRAAG 3

In die diagram hieronder vorm die lyn wat Q(–2 ; –3) en P(a ; b), a en $b > 0$, verbind, 'n hoek van 45° met die positiewe x-as. QP = $7\sqrt{2}$ eenhede. N(7 ; 1) is die middelpunt van PR en M is die middelpunt van QR.



Bepaal:

3.1 Die gradiënt van PQ (2)

3.2 Die vergelyking van MN in die vorm $y = mx + c$ en gee redes (4)

3.3 Die lengte van MN (2)

3.4 Die lengte van RS (1)

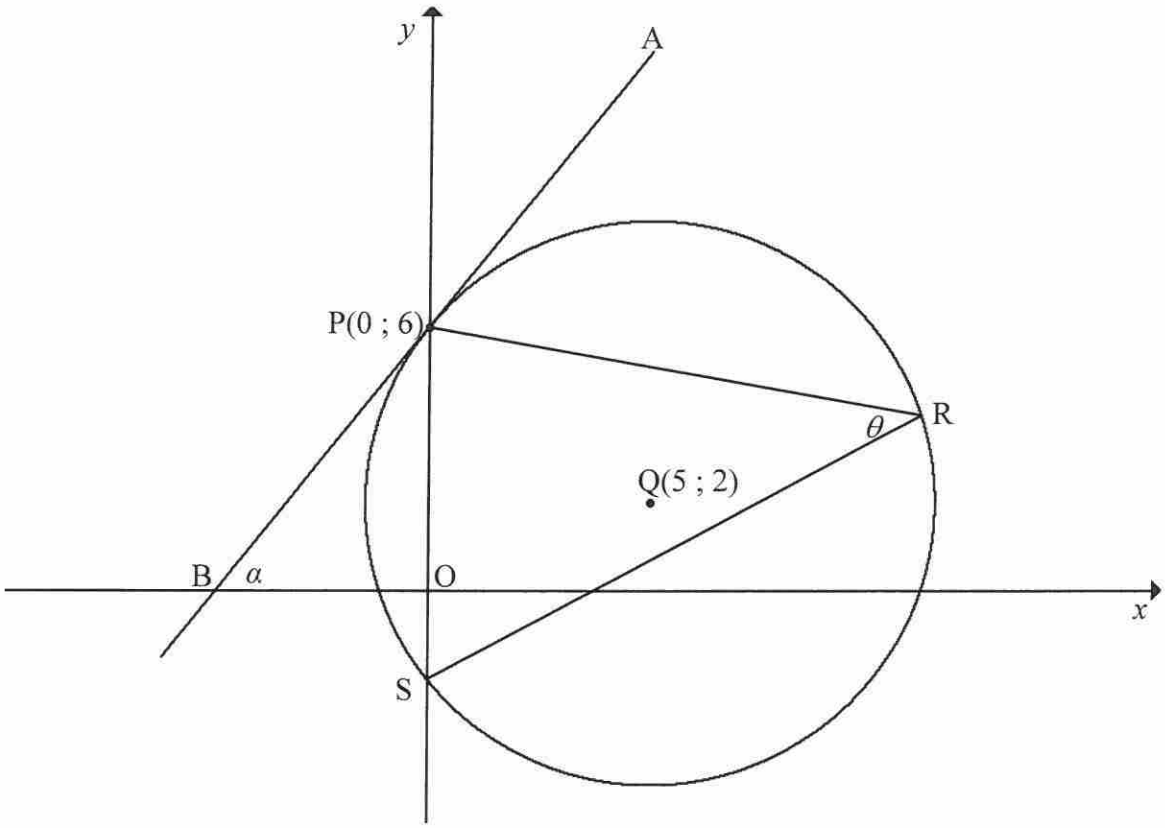
3.5 Die koördinate van S sodat PQRS, in hierdie volgorde, 'n parallelogram is (3)

3.6 Die koördinate van P (6)

[18]

VRAAG 4

In die diagram hieronder is Q(5 ; 2) die middelpunt van 'n sirkel wat die y-as by P(0 ; 6) en S sny. Die raaklyn APB by P sny die x-as by B en vorm die hoek α met die positiewe x-as. R is 'n punt op die sirkel en $P\hat{R}S = \theta$.



4.1 Bepaal die vergelyking van die sirkel in die vorm $(x-a)^2 + (y-b)^2 = r^2$. (3)

4.2 Bereken die koördinate van S. (3)

4.3 Bepaal die vergelyking van die raaklyn APB in die vorm $y = mx + c$. (4)

4.4 Bereken die grootte van α. (2)

4.5 Bereken, met redes, die grootte van θ. (4)

4.6 Bereken die oppervlakte van ΔPQS. (4)

[20]

VRAAG 5

5.1 Gegee dat $\sin 23° = \sqrt{k}$, bepaal, in die eenvoudigste vorm, die waarde van elk van die volgende in terme van k, SONDER om 'n sakrekenaar te gebruik:

5.1.1 $\sin 203°$ (2)

5.1.2 $\cos 23°$ (3)

5.1.3 $\tan(-23°)$ (2)

5.2 Vereenvoudig die volgende uitdrukking tot 'n enkele trigonometriese funksie:

$$\frac{4\cos(-x).\cos(90° + x)}{\sin(30° - x).\cos x + \cos(30° - x).\sin x}$$ (6)

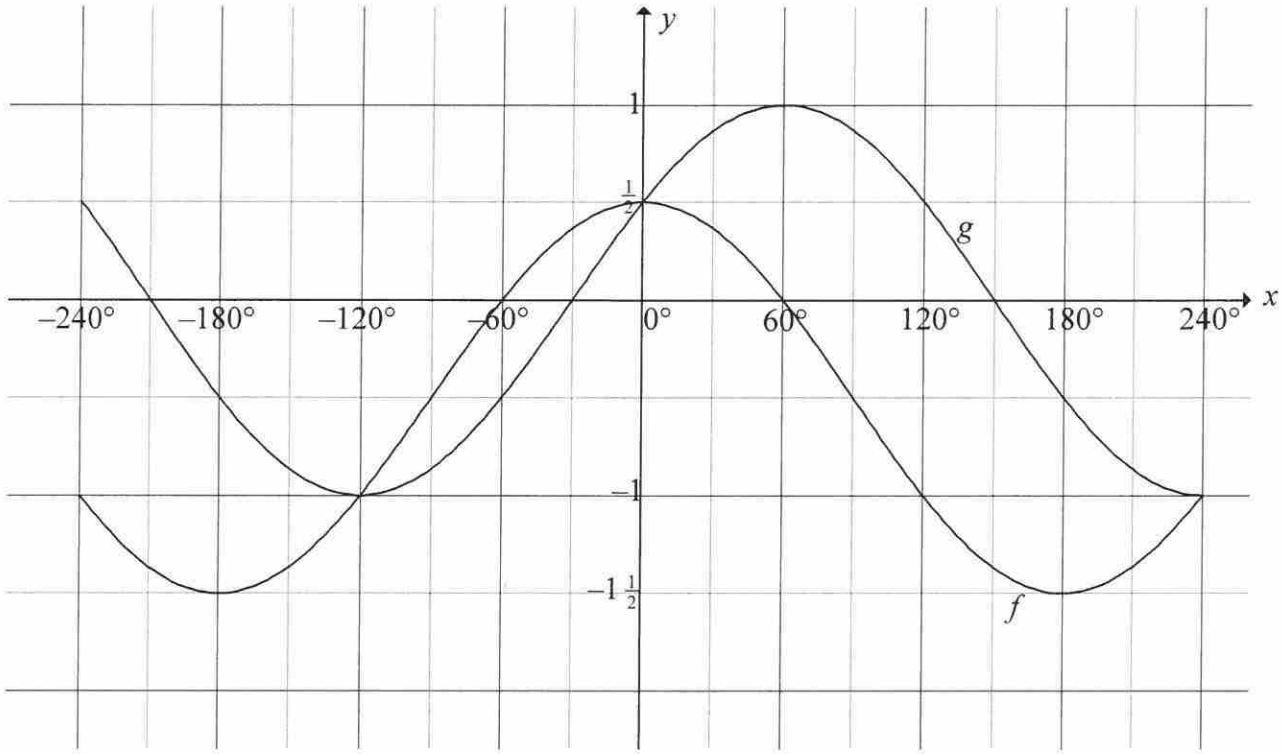
5.3 Bepaal die algemene oplossing van $\cos 2x - 7\cos x - 3 = 0$. (6)

5.4 Gegee dat $\sin\theta = \frac{1}{3}$, bereken die numeriese waarde van $\sin 3\theta$, SONDER om 'n sakrekenaar te gebruik. (5)

[24]

VRAAG 6

In die diagram hieronder is die grafieke van $f(x) = \cos x + q$ en $g(x) = \sin(x + p)$ op dieselfde assestelsel vir $-240° \leq x \leq 240°$ geskets. Die grafieke sny by $\left(0° ; \frac{1}{2}\right)$, $(-120° ; -1)$ en $(240° ; -1)$.



6.1 Bepaal die waardes van p en q. (4)

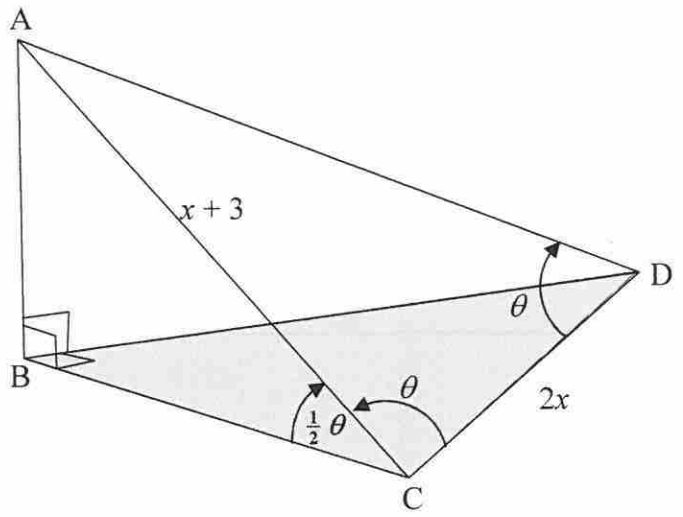
6.2 Bepaal die waardes van x, in die interval $-240° \leq x \leq 240°$, waarvoor $f(x) > g(x)$. (2)

6.3 Beskryf 'n transformasie wat die grafiek van g moet ondergaan om die grafiek van h, waar $h(x) = -\cos x$ te vorm. (2)

[8]

VRAAG 7

'n Hoek van 'n reghoekige blok hout word afgesny en in die diagram hieronder getoon. Die skuinsvlak, dit is ΔACD, is 'n gelykbenige driehoek met $A\hat{D}C = A\hat{C}D = \theta$. Verder is $A\hat{C}B = \frac{1}{2}\theta$, $AC = x + 3$ en $CD = 2x$.



7.1 Bepaal 'n uitdrukking vir $C\hat{A}D$ in terme van θ. (1)

7.2 Bewys dat $\cos\theta = \frac{x}{x+3}$. (4)

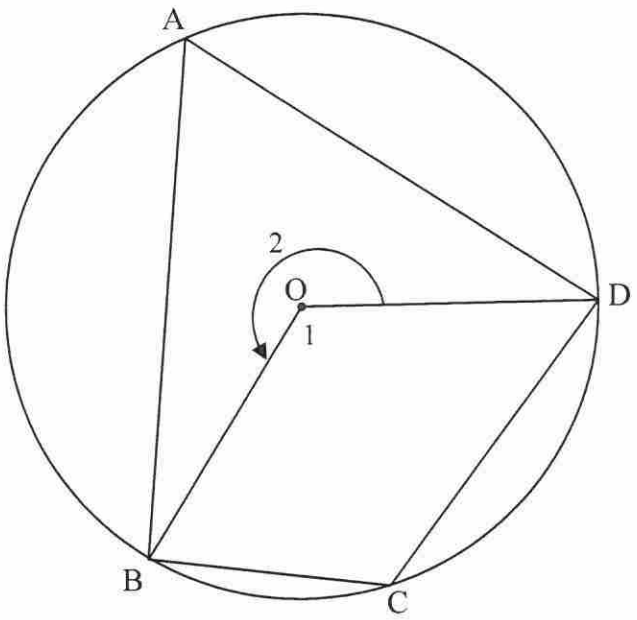
7.3 As verder gegee word dat $x = 2$, bereken AB, die hoogte van die stuk hout. (5)

[10]

Gee redes vir ALLE bewerings in VRAAG 8, 9, 10 en 11.

VRAAG 8

8.1 In die diagram hieronder is koordevierhoek ABCD in die sirkel met middelpunt O getrek.

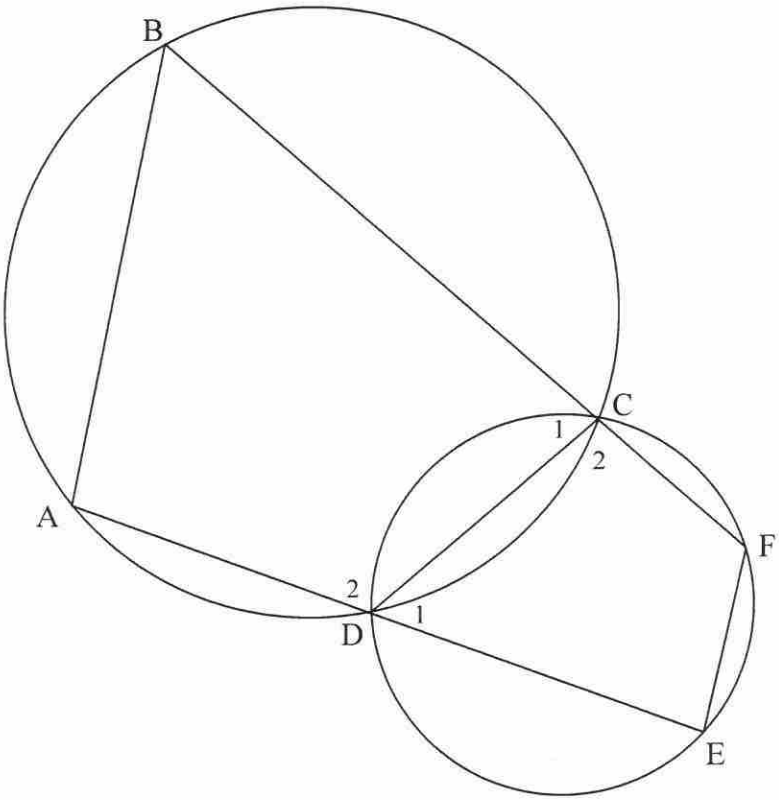


8.1.1 Voltooi die volgende stelling:

Die hoek wat deur 'n koord by die middelpunt van 'n sirkel onderspan word, is … die hoek wat deur dieselfde koord op die omtrek van 'n sirkel onderspan word. (1)

8.1.2 Gebruik VRAAG 8.1.1 om te bewys dat $\hat{A} + \hat{C} = 180°$. (3)

8.2 In die diagram hieronder is CD 'n gemeenskaplike koord van die twee sirkels. Reguitlyne ADE en BCF is getrek. Koorde AB en EF is getrek.

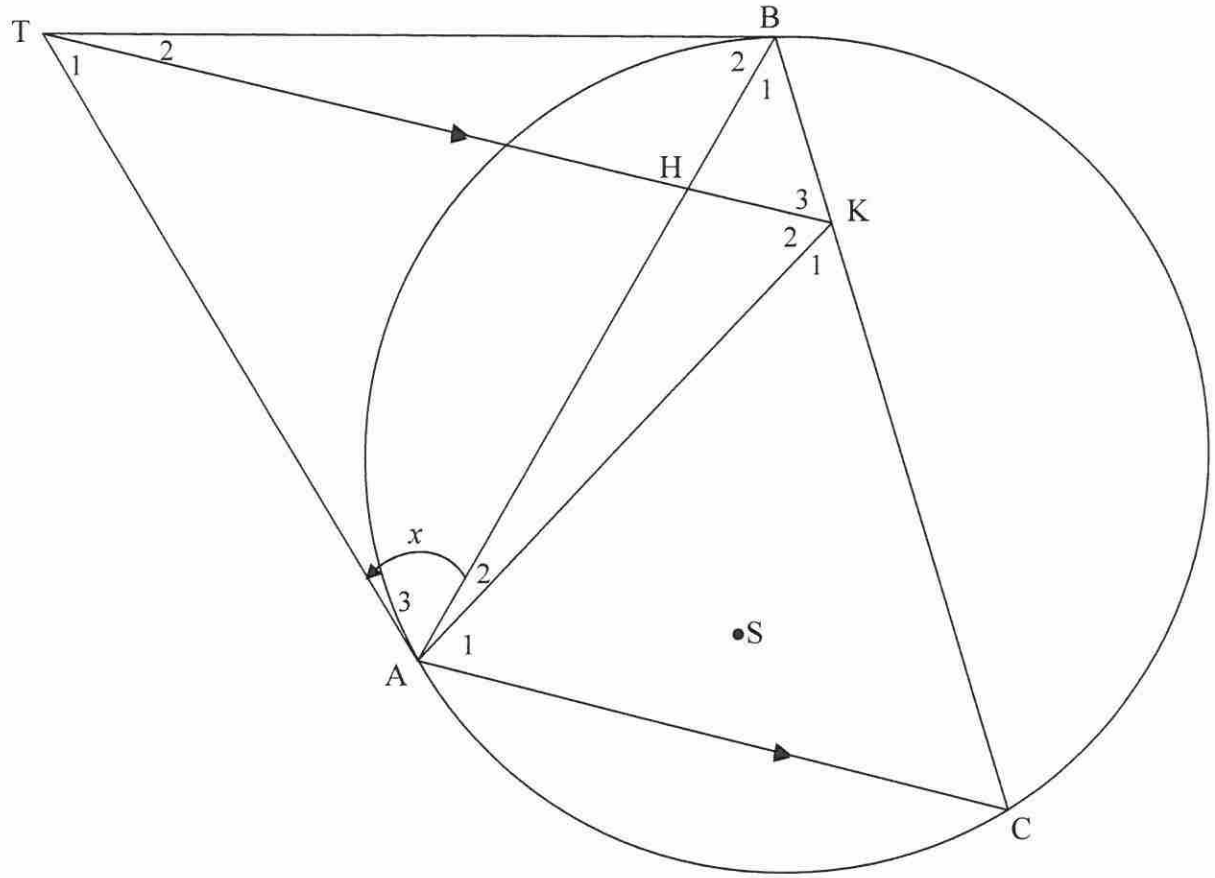


Bewys dat EF | | AB. (5)

[9]

VRAAG 9

In die diagram hieronder is ΔABC in die sirkel getrek. TA en TB is raaklyne aan die sirkel. Die reguitlyn THK is ewewydig aan AC met H op BA en K op BC. AK is getrek. Gestel $\hat{A}_3 = x$.



9.1 Bewys dat $\hat{K}_3 = x$. (4)

9.2 Bewys dat AKBT 'n koordevierhoek is. (2)

9.3 Bewys dat TK vir $A\hat{K}B$ halveer. (4)

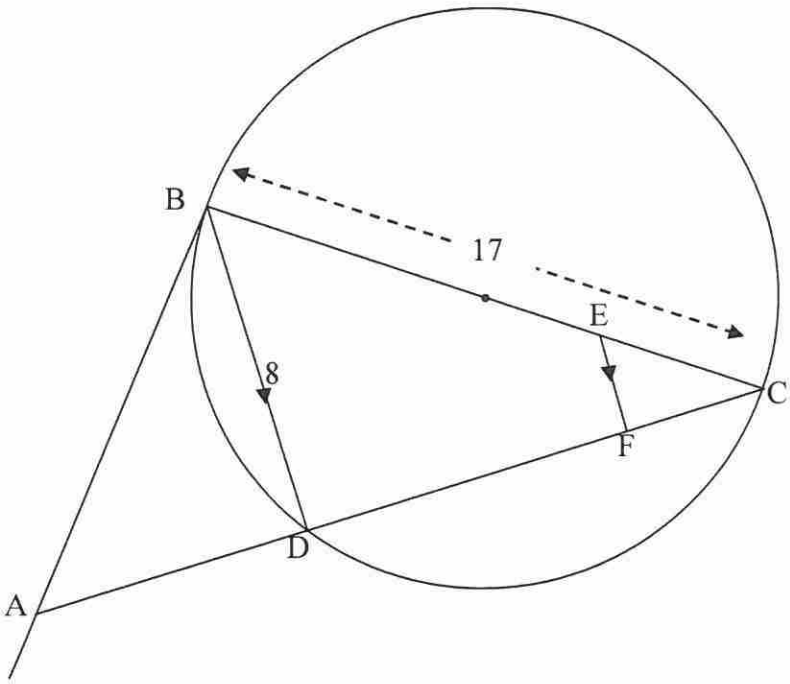
9.4 Bewys dat TA 'n raaklyn aan die sirkel is wat deur die punte A, K en H gaan. (2)

9.5 S is 'n punt binne die sirkel sodat die punte A, S, K en B konsiklies is. Verduidelik waarom A, S, B en T ook konsiklies is. (2)

[14]

VRAAG 10

In die diagram hieronder is BC = 17 eenhede, waar BC 'n middellyn van die sirkel is. Die lengte van koord BD is 8 eenhede. Die raaklyn by B ontmoet CD verleng by A.



10.1 Bereken, met redes, die lengte van DC. (3)

10.2 E is 'n punt op BC sodat BE : EC = 3 : 1. EF is ewewydig aan BD met F op DC.

10.2.1 Bereken, met redes, die lengte van CF. (3)

10.2.2 Bewys dat $\Delta BAC \,|||\, \Delta FEC$. (5)

10.2.3 Bereken die lengte van AC. (4)

10.2.4 Skryf neer, met redes, die radius van die sirkel wat deur punt A, B en C gaan. (2)

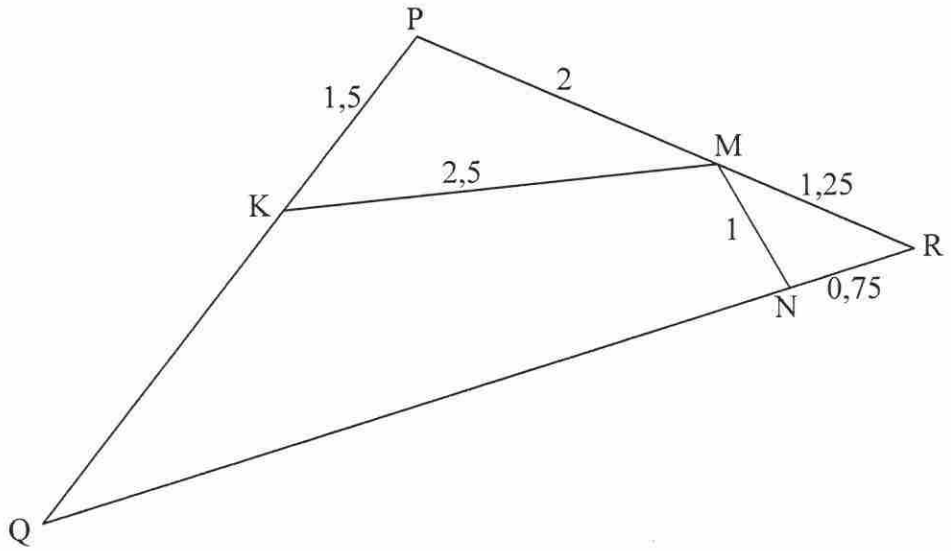
[17]

VRAAG 11

11.1 Voltooi die volgende stelling:

As die sye van twee driehoeke in dieselfde verhouding is, dan is die driehoeke ... (1)

11.2 In die diagram hieronder is K, M en N onderskeidelik punte op sye PQ, PR en QR van ΔPQR. KP = 1,5; PM = 2; KM = 2,5; MN = 1; MR = 1,25 en NR = 0,75.



11.2.1 Bewys dat $\Delta KPM \mid\mid\mid \Delta RNM$. (3)

11.2.2 Bepaal die lengte van NQ. (6)
[10]

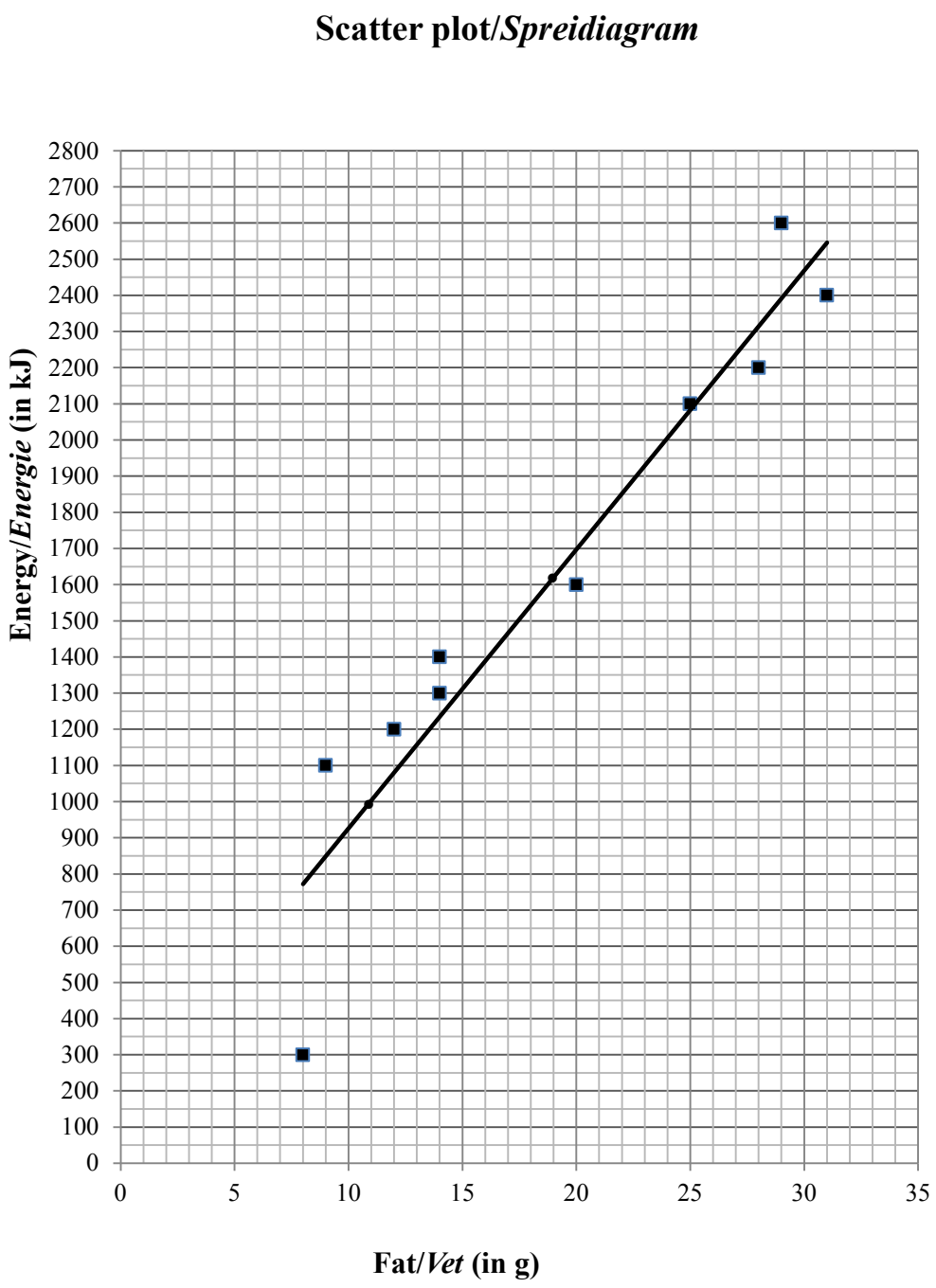
TOTAAL: 150

Memo

QUESTION/*VRAAG* 1

Fat/*Vet* (in g)	9	14	25	8	12	31	28	14	29	20
Energy/*Energie* (in kJ)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

1.1



1.2.2

1.1
no marks:
0 – 2 points correctly

✓plotting 3 – 5 points correctly

✓✓plotting 6 – 9points correctly

✓✓✓plotting all 10 points correctly

geen punte: 0 – 2 punte korrek

✓ *stip 3 – 5 pte korrek*

✓✓*stip 6 –9 pte korrek*

✓✓✓ *stip al 10 pte korrek*

(3)

1.2.2
✓ *y* – int close to (0 ; 150)
✓ one pt close to (25 ; 2100) or (20 ; 1700)
(2)

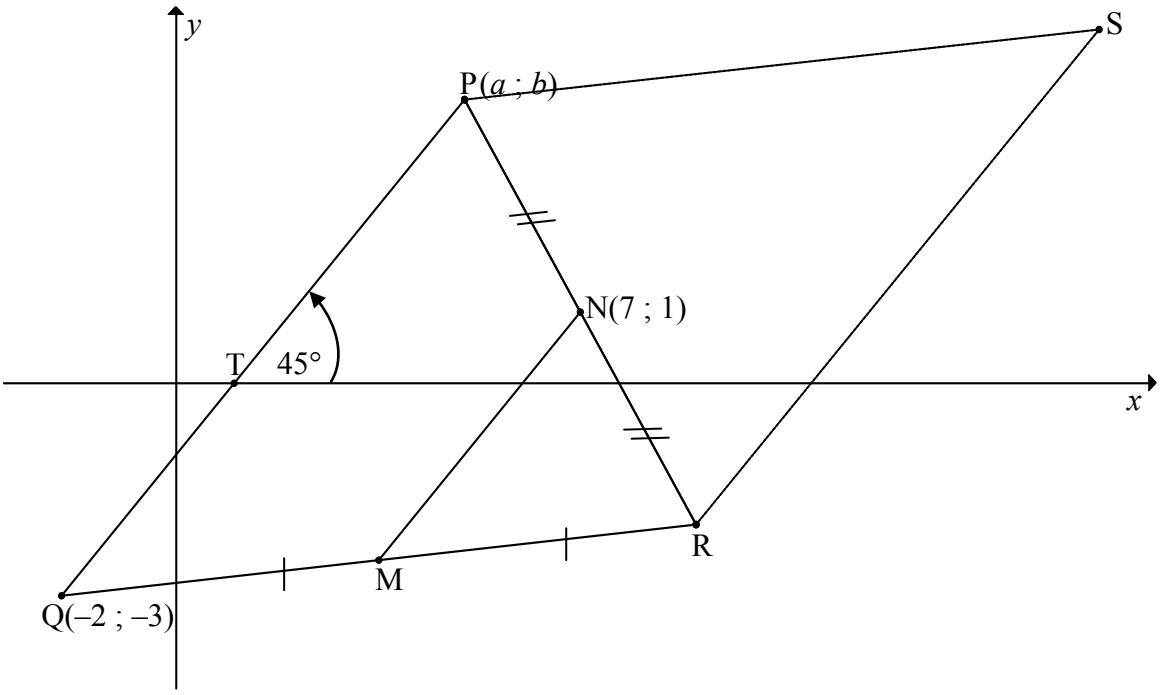
1.2.1	$\hat{y} = 154{,}60 + 77{,}13(18)$ $= 1\ 542{,}94 \approx 1\ 500$ kJ	✓ subst ✓ answ rounded off correctly/ *antw korrek afgerond* (2)
1.3	(8 ; 300)	✓ answ/*antw* (1)
1.4	$r = 0{,}9520... \approx 0{,}95$	✓✓ answ/*antw* (2)
1.5	very strong positive relationship/ *baie sterk positiewe verband*	✓ strong/ *sterk* (1) **[11]**

QUESTION/*VRAAG* 2

Sum of the values on uppermost faces/ *Som van die waardes op boonste vlakke*	**Frequency/ *Frekwensie***
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

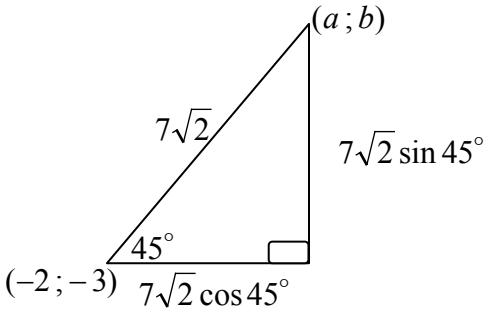
2.1	mean/*gemiddelde* $= \frac{2(0)+3(3)+4(2)+...12(1)}{30} = \frac{202}{30}$ $= 6,73$	✓202 ✓ answ/*antw* (2)
2.2	median/*mediaan* $= \frac{T_{15}+T_{16}}{2} = \frac{7+7}{2} = 7$	✓✓ answ/*antw* (2)
2.3	SD/*SA* = 2,264... ≈ 2,26	✓✓ answ/*antw* (2)
2.4	(6,73 – 2,26 ; 6,73 + 2,26) = (4,47 ; 8,99) ∴4 + 4 + 8 + 3 = 19 times/*keer*	✓lower boundary ✓upper boundary ✓ answ/*antw* (3) **[9]**

QUESTION/*VRAAG* 3

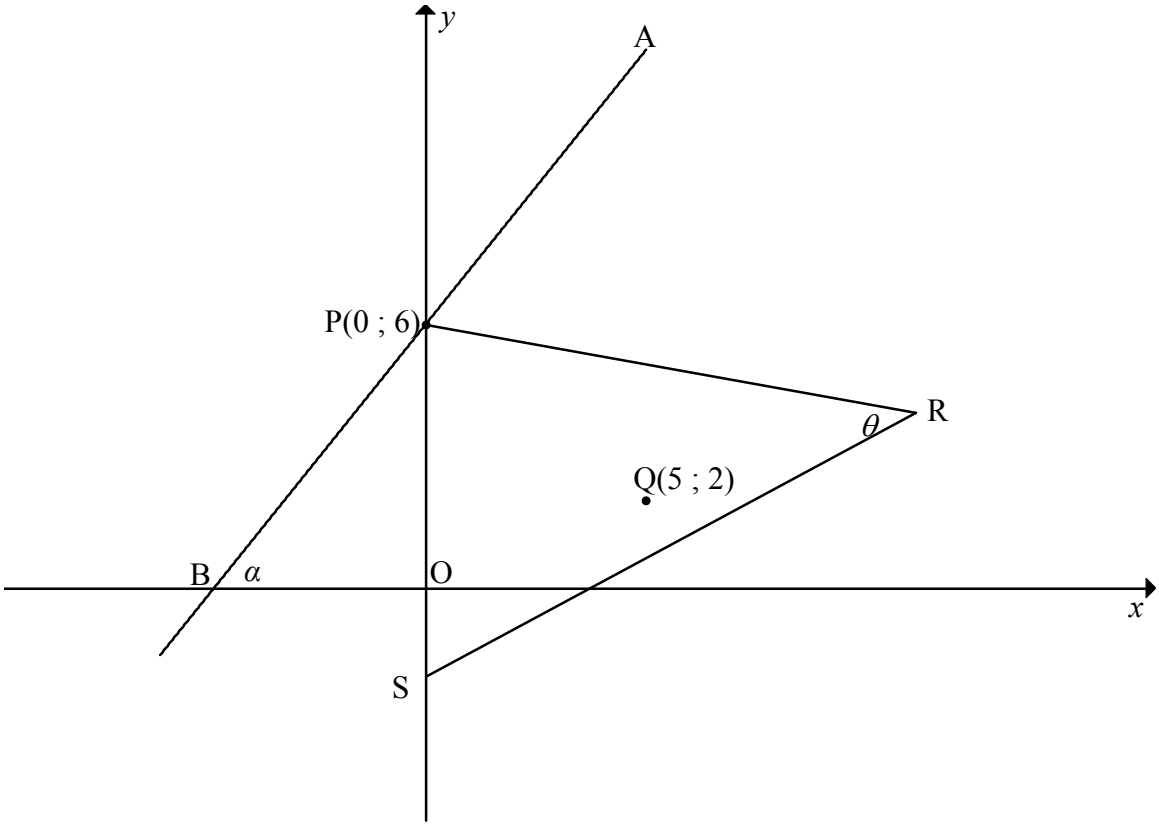


3.1	$m_{PQ} = \tan 45°$ $= 1$	✓ $m = \tan 45°$ ✓ answ/*antw* (2)
3.2	MN || QP [midpt theorem/*midpt-stelling*] $\therefore m_{MN} = 1$ $\therefore y - y_1 = m(x - x_1)$ $\therefore y - 1 = 1(x - 7)$ $\therefore \quad y = x - 6$ **OR/*OF*** MN || PQ [midpt theorem/*midpt-stelling*] $\therefore m_{MN} = 1$ $\therefore y = mx + c$ $\therefore 1 = 1(7) + c$ $-6 = c$ $\therefore \quad y = x - 6$	✓ S **OR** R ✓ m_{MN} ✓ subst *m* and/*en* N(7 ; 1) ✓ equation/*vgl* (4) ✓ S **OR** R ✓ m_{MN} ✓ subst *m* and/*en* N(7 ; 1) ✓ equation/*vgl* (4)
3.3	$MN = \frac{1}{2}PQ$ [midpoint theorem/midp stelling] $\therefore MN = \frac{7\sqrt{2}}{2} \approx 4{,}95$	✓ S ✓ answ/antw (2)

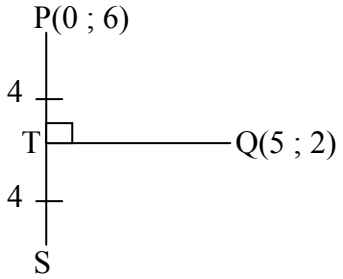
3.5	QN = NS [diag of ‖m/hoekl van ‖m] $\frac{-2+x_S}{2} = 7$ and/en $\frac{-3+y_S}{2} = 1$ $\therefore\ x_S = 16$ $\therefore\ y_S = 5$ **OR/*OF*** QN = NS [diag of ‖m/hoekl van ‖m] $\therefore$ by inspection/deur inspeksie: S(16 ; 5)	✓ method/metode ✓ x-value/waarde ✓ y-value/waarde (3) ✓ method/metode ✓ x-value/waarde ✓ y-value/waarde (3)
3.6	Equation of/*Vgl van PQ*: $y = x + c$ $-3 = -2 + c$ $y = x - 1$ $\therefore a = b + 1$ .....(1) From distance formula/*Van afstandsformule*: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $7\sqrt{2} = \sqrt{(a-(-2))^2 + (b-(-3))^2}$ $\therefore 98 = (a+2)^2 + (b+3)^2$ .....(2) Subst (1) into (2): $98 = (b+1+2)^2 + (b+3)^2$ $98 = b^2 + 6b + 9 + b^2 + 6b + 9$ $0 = 2b^2 + 12b - 80$ $0 = b^2 + 6b - 40$ $\therefore 0 = (b + 10)(b - 4)$ $\therefore\ b = 4$ (since $b > 0$) Subst $b = 4$ into (1): $\therefore a = 4 + 1 = 5$ $\therefore$ P(5 ; 4) **OR/*OF*** Equation of/*Vgl van PQ*: $y = x + c$ $-3 = -2 + c$ $y = x - 1$ $\therefore a = b + 1$ .....(1) From distance formula/*Van afstandsformule*: $7\sqrt{2} = \sqrt{(a-(-2))^2 + (b-(-3))^2}$ $\therefore 98 = (a+2)^2 + (b+3)^2$ .....(2) Subst (1) into (2): $98 = (b+1+2)^2 + (b+3)^2$ $98 = 2(b+3)^2$ $49 = (b+3)^2$ $\pm 7 = b + 3$ $\pm 7 - 3 = b$ $\therefore\ b = 4$ (since $b > 0$) Subst $b = 4$ into (1): $\therefore a = 4 + 1 = 5$ $\therefore$ P(5 ; 4)	✓ eq of/*vgl van* PQ ✓ subst Q & $7\sqrt{2}$ into/*in* distance formula/ *afstandsformule* ✓ subst eq of/*vgl v.* PQ ✓ st form/*st vorm* ✓ value of/*waarde van b* ✓ value of/*waarde van a* (6) ✓ eq of/*vgl van* PQ ✓ subst Q & $7\sqrt{2}$ into/*in* distance formula/ *afstandsformule* ✓ subst eq of/*vgl v.* PQ ✓ simplification/ *vereenvoudig* ✓ value of/*waarde van b* ✓ value of/*waarde van a* (6)

	OR/*OF* Equation of/*Vgl van PQ*: $y = x + c$ $-3 = -2 + c$ $y = x - 1$ $\quad \therefore a = b + 1$ .....(1) From distance formula/*Van afstandsformule*: $7\sqrt{2} = \sqrt{(a-(-2))^2 + (b-(-3))^2}$ $98 = (a+2)^2 + (a-1+3)^2$ $= 2(a+2)^2$ $\therefore\ a + 2 = 7$ (since/*aangesien* $a > 0$) $\therefore\ a = 5$ Subst $a = 4$ into (1): $\therefore b = 5 - 1 = 4$ $\therefore$ P(5 ; 4)	✓ eq of/*vgl van* PQ ✓ subst Q & $7\sqrt{2}$ into/*in* distance formula/ *afstandsformule* ✓ subst eq of/*vgl v.* PQ ✓ simplification/ *vereenvoudig* ✓ value of/*waarde van a* ✓ value of/*waarde van b* (6)
	OR/*OF*  $a = -2 + 7\sqrt{2}\cos 45^\circ = 5$ $b = -3 + 7\sqrt{2}\sin 45^\circ = 4$	✓✓✓✓ ✓ ✓ (6) **[17]**

QUESTION/*VRAAG* 4

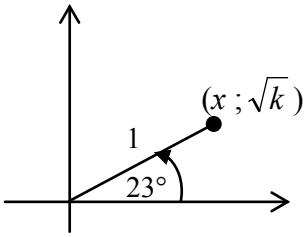


4.1	$(x-5)^2+(y-2)^2=r^2$ $(0-5)^2+(6-2)^2=r^2$ $25+16=r^2$ $41=r^2$ $\therefore\ (x-5)^2+(y-2)^2=41$ **OR/*OF*** $PQ=\sqrt{(0-5)^2+(6-2)^2}$ $=\sqrt{25+16}$ $r=\sqrt{41}$ $\therefore\ (x-5)^2+(y-2)^2=41$	✓ subst (5 ; 2) into circle eq/*in sirkelvgl* ✓ value of/*waarde van* r^2 ✓ equation/*vgl* (3) ✓ subst (5 ; 2) & (0 ; 6) into dist. form/*in afst. form* ✓ value of/*waarde van r* ✓ equation/*vgl* (3)
4.2	$(0-5)^2+(y-2)^2=41$ $25+(y-2)^2=41$ $25+y^2-4y+4=41$ $y^2-4y-12=0$ $(y-6)(y+2)=0$ $y\neq 6 \quad or/of \quad y=-2$ $\therefore$ S(0 ; –2) or $y=-2$	✓ $x=0$ ✓ st form/*st. vorm* ✓ answ/*antw* (neg value) (3)

	OR/*OF* $(0-5)^2+(y-2)^2 = 41$ $25+(y-2)^2 = 41$ $(y-2)^2 = 16$ $y-2 = \pm 4$ $y = 2 \pm 4$ $y \neq 6$ or/*of* $y = -2$ $\therefore$ S(0 ; –2) **OR/*OF*** Draw/*Trek* QT ⊥ PS PT = TS [line from centre ⊥ to chord/ *lyn van midpt ⊥ koord*] PT = $y_P - y_Q = 6-2 = 4$ $y_Q - y_S = 4$ $y_S = 2-4 = -2$ $\therefore$ S(0 ; –2) 	✓ $x = 0$ ✓ square form/ *kwadraatvorm* ✓ answ/*antw* (neg value) (3) ✓ $x = 0$ ✓✓ $y = -2$ (3)
4.3	$m_{PQ} = \frac{6-2}{0-5}$ $= -\frac{4}{5}$ $m_{PQ} \times m_{APB} = -1$ [tan/*raakl* ⊥ radius] $\therefore\ m_{APB} = \frac{5}{4}$ $\therefore y = \frac{5}{4}x + 6$	✓ subst (0 ; 6) & (5 ; 2) into grad form/*in grad. formule* ✓ m_{PQ} ✓ m_{APB} ✓ equation/*vgl* (4)
4.4	$\tan\alpha = \frac{5}{4}$ $\therefore\ \alpha = 51{,}34°$ **OR/*OF*** B(4,8 ; 0) $\therefore \tan\alpha = \frac{6}{4{,}8}$ $\therefore\ \alpha = 51{,}34°$	✓ $\tan\alpha = m_{APB}$ ✓ answ/*antw* (2) ✓ $\tan\alpha = \frac{6}{4{,}8}$ ✓ answ/*antw* (2)

4.5	$\theta = B\hat{P}S$ [tan-chord th/*raakl-koordst.*] $= 90° - \alpha$ [∠ sum in Δ/∠ *som van* Δ] $= 90° - 51{,}34°$ $= 38{,}66°$ **OR/*OF*** PS = 8 PQ = SQ = $\sqrt{41}$ $PS^2 = PQ^2 + SQ^2 - 2.PQ.SQ.\cos P\hat{Q}S$ $64 = 41 + 41 - 2.41.\cos P\hat{Q}S$ $\cos P\hat{Q}S = \frac{18}{82}$ $P\hat{Q}S = 77{,}32°$ $\theta = \frac{1}{2}P\hat{Q}S$ [∠ at centre = 2 × ∠ circumf] $= 38{,}66°$	✓ S ✓ R ✓ $90° - \alpha$ ✓ answ/*antw* (4) ✓ correct subst into cosine rule ✓ $P\hat{Q}S = 77{,}32°$ ✓ R ✓ answ/*antw* (4)
4.6	Area ΔPQS = $\frac{1}{2}$PS ×height/*hoogte* $= \frac{1}{2}(8)(5)$ = 20 sq units/*vk eenh* **OR/*OF*** $P\hat{Q}S = 2 \times 38{,}66°$ [∠at centre = 2×∠ at circum/ *midpts∠=2omtreks∠*] $= 77{,}32°$ Area ΔPQS = $\frac{1}{2}PQ.QS.\sin P\hat{Q}S$ $= \frac{1}{2}.\sqrt{41}.\sqrt{41}.\sin 77{,}32°$ = 20 sq units/*vk eenh*	✓ area formula/*e*: ΔPQS ✓ PS = 8 ✓ $\perp h = 5$ ✓ answ/*antw* (4) ✓ size of/*grootte v* $P\hat{Q}S$ ✓ area rule/*reël*: ΔPQS ✓ subst correctly/ *subst korrek* ✓ answ/*antw* (4) **[20]**

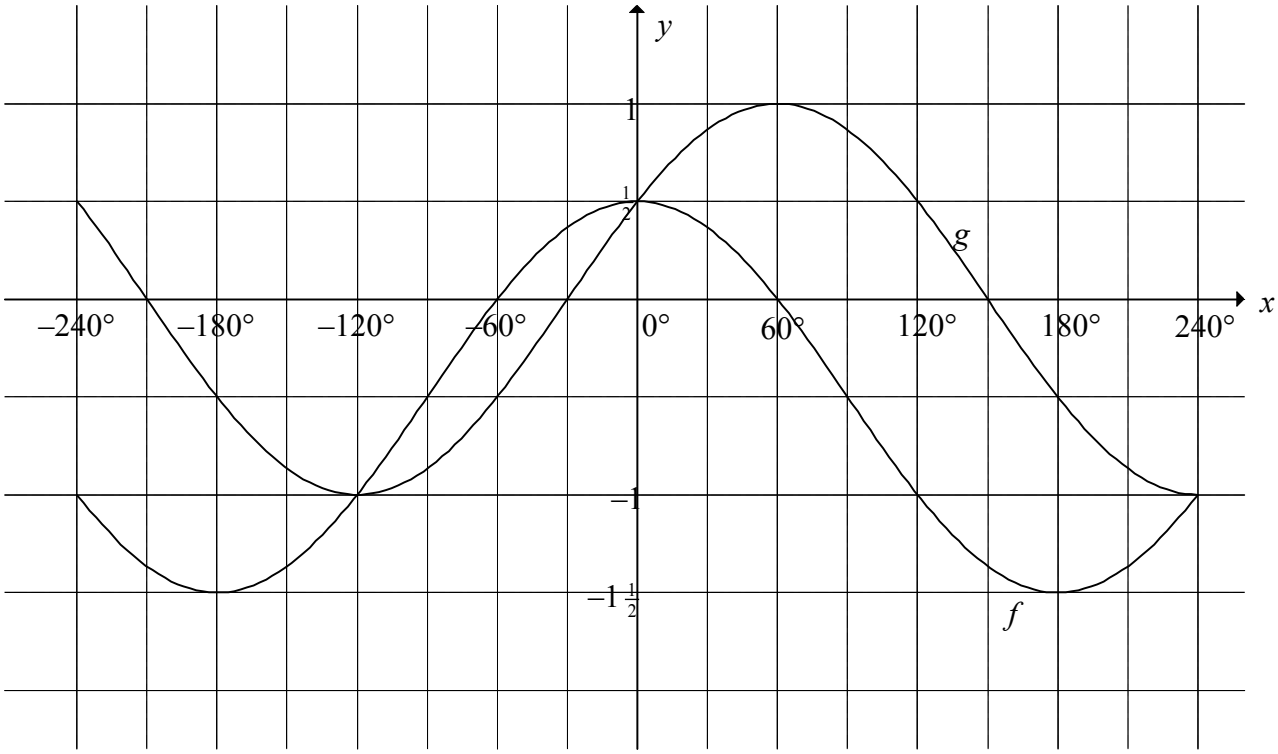
QUESTION/*VRAAG* 5

5.1.1	$\sin 203°$ $= -\sin 23°$ $= -\sqrt{k}$	✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2)
5.1.2	$\cos^2 23° = 1 - \sin^2 23°$ $= 1 - k$ $\cos 23° = \sqrt{1-k}$ **OR/*OF*** $x^2 + (\sqrt{k})^2 = 1$ $x^2 = 1 - k$ $x = \sqrt{1-k}$ $\cos 23° = \dfrac{\sqrt{1-k}}{1} = \sqrt{1-k}$ 	✓identity/*identiteit* ✓ $\cos^2 23°$ ito/*itv k* ✓ answ/*antw* (3) ✓ $x^2 = 1 - k$ ✓ x ito/*itv k* ✓ answ/*antw* (3)
5.1.3	$\tan(-23°) = -\tan 23°$ $= -\dfrac{\sin 23°}{\cos 23°}$ $= -\dfrac{\sqrt{k}}{\sqrt{1-k}} = -\sqrt{\dfrac{k}{1-k}}$ **OR/*OF*** $\tan(-23°) = -\tan 23°$ $= -\dfrac{\sqrt{k}}{\sqrt{1-k}} = -\sqrt{\dfrac{k}{1-k}}$	✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2) ✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2)
5.2	$\dfrac{4\cos x.(-\sin x)}{\sin(30° - x + x)}$ $= \dfrac{-4\sin x.\cos x}{\sin 30°}$ $= \dfrac{-4\sin x.\cos x}{\frac{1}{2}}$ $= -8\sin x.\cos x$ $= -4(2\sin x.\cos x)$ $= -4\sin 2x$	✓ $\cos x$ ✓ $-\sin x$ ✓ $\sin(\alpha + \beta)$ ✓ $\dfrac{1}{2}$ ✓ double sine form / *dubbel sin form* ✓ answ/*antw* (6)

	OR/OF $\dfrac{4\cos x.(-\sin x)}{(\sin 30°\cos x-\cos 30°\sin x)\cos x+(\cos 30°\cos x+\sin 30°\sin x)\sin x}$ $=\dfrac{-4\sin x.\cos x}{(\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x)\cos x+(\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x)\sin x}$ $=\dfrac{-2(2\sin x.\cos x)}{\frac{1}{2}\cos^2 x+\frac{1}{2}\sin^2 x}$ $=\dfrac{-2(2\sin x.\cos x)}{\frac{1}{2}(\cos^2 x+\sin^2 x)}$ $=\dfrac{-2(2\sin x.\cos x)}{\frac{1}{2}(1)}$ $=-8\cos x\sin x$ $=-4(2\sin x\cos x)$ $=-4\sin 2x$	✓ $\cos x$ ✓ $-\sin x$ ✓ $\frac{1}{2}\cos^2 x+\frac{1}{2}\sin^2 x$ ✓ $\frac{1}{2}$ ✓ double sine form / *dubbel sin form* ✓ answ/*antw* (6)

5.3	$\cos 2x - 7\cos x - 3 = 0$ $2\cos^2 x - 1 - 7\cos x - 3 = 0$ $2\cos^2 x - 7\cos x - 4 = 0$ $(2\cos x + 1)(\cos x - 4) = 0$ $\therefore \cos x = -\frac{1}{2}$ or/*of* $\cos x = 4$ (no solution) $\therefore x = 120° + n.360°$ or/*of* $x = 240° + n.360°$; $n \in Z$ **OR/OF** $\therefore x = \pm 120° + n.360°$; $n \in Z$	✓ expansion/ *uitbreiding* ✓ $2\cos^2 x - 7\cos x - 4 = 0$ ✓ factors/*faktore* ✓ $\cos x = -\frac{1}{2}$ ✓ 120° & 240° ✓ $+ n.360°$ **OR/OF** ✓ $\pm 120°$ ✓ $+ n.360°$ (6)
5.4	$\sin 3\theta = \sin(2\theta + \theta)$ $= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$ $= 2\sin\theta\cos\theta\cos\theta + (1 - 2\sin^2\theta)\sin\theta$ $= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$ $= 3\sin\theta - 4\sin^3\theta$ $= 3(\frac{1}{3}) - 4(\frac{1}{3})^3$ $= 1 - \frac{4}{27}$ $= \frac{23}{27}$	✓ expansion of/ *uitbreiding van* $\sin(2\theta + \theta)$ ✓ expansions of $\sin 2\theta$ AND $\cos 2\theta$ ✓ $1 - \sin^2\theta$ ✓ subst ✓ answ/*antw* (5) **[24]**

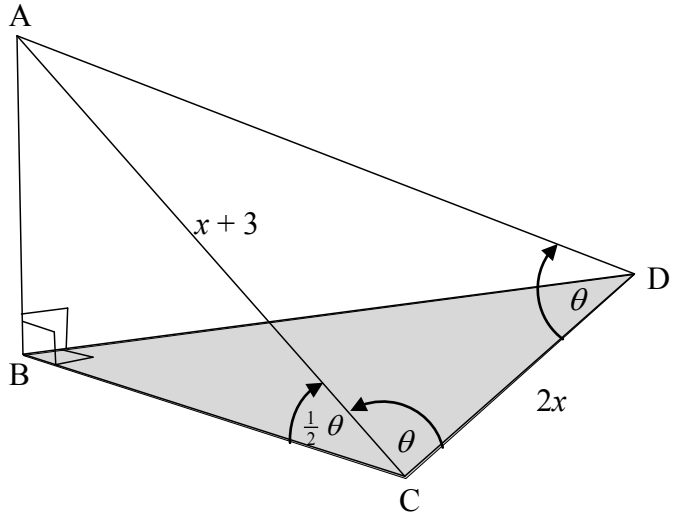
QUESTION/*VRAAG* 6

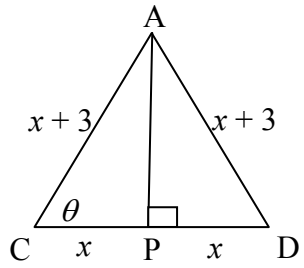


6.1	$f(x) = \cos x - \frac{1}{2}$ and/*en* $g(x) = \sin(x + 30°)$ $\therefore p = 30°$ and/*en* $q = -\frac{1}{2}$ **OR/*OF*** $\sin(60° + p) = 1$ and/*en* $\cos 0° + q = \frac{1}{2}$ $\therefore p = 30°$ $\therefore q = -\frac{1}{2}$	✓ $f(x) = \cos x - \frac{1}{2}$ ✓ $g(x) = \sin(x + 30°)$ ✓ value of/*waarde v p* ✓ value of/*waarde v q* (4) ✓ $\sin(60° + p) = 1$ ✓ $\cos 0° + q = \frac{1}{2}$ ✓ value of/*waarde v p* ✓ value of/*waarde v q* (4)
6.2	$x \in (-120° ; 0°)$ **OR/*OF*** $-120° < x < 0°$	✓ critical values/ *kritiese waardes* ✓ correct interval/ *korrekte interval* (2)

6.3	The graph of g has to shift 60° to the left and then be reflected about the x-axis./*Die grafiek van g moet 60° na links skuif en dan om die x-as gereflekteer word.*	✓ 60° left/*links* ✓ reflection about x-axis/*refleksie om x-as* (2)
	OR/*OF* The graph of g must be reflected about the x-axis and then be shifted 60° to the left./*Die grafiek van g moet om die x-as gereflekteer word en dan met 60° na links geskuif word.*	✓ reflection about x-axis/*refleksie om x-as* ✓ 60° left/*links* (2)
	OR/*OF* The graph of g has to shift 120° to the right./*Die grafiek van g moet 120° na regs geskuif word.*	✓ ✓ 120° right/*regs* (2)
	OR/*OF* The graph of g has to shift 240° to the left./*Die grafiek van g moet met 240° na links geskuif word*	✓ ✓ 240° left/*links* (2) **[8]**

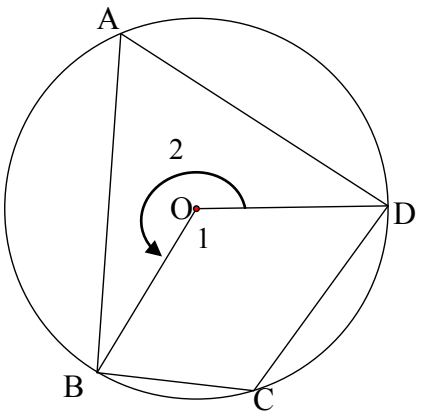
QUESTION/*VRAAG* 7



7.1	$C\hat{A}D = 180° - 2\theta$ [∠s sum of Δ/*∠e som van Δ*]	✓ answ/*antw* (1)
7.2	$\frac{\sin\theta}{x+3} = \frac{\sin(180° - 2\theta)}{2x}$ $\frac{\sin\theta}{x+3} = \frac{\sin 2\theta}{2x}$ $\frac{\sin\theta}{x+3} = \frac{2\sin\theta.\cos\theta}{2x}$ $\cos\theta = \frac{2x\sin\theta}{2(x+3)\sin\theta}$ $\cos\theta = \frac{x}{x+3}$	✓ correct subst into sine rule/*korrekte subst in sin-reël* ✓ $\sin 2\theta$ ✓ $2\sin\theta.\cos\theta$ ✓ $\cos\theta$ as subject/ *as onderwerp* (4)
	OR/*OF* AD = $x + 3$ [sides opp = ∠s/*sye to = ∠e*] $AC^2 = AD^2 + CD^2 - 2AD.CD.\cos\theta$ $(x+3)^2 = (x+3)^2 + (2x)^2 - 2(2x)(x+3).\cos\theta$ $0 = 4x^2 - 4x(x+3)\cos\theta$ $\cos\theta = \frac{4x^2}{4x(x+3)}$ $= \frac{x}{x+3}$	✓ AD = $x + 3$ ✓ correct subst into cosine rule/*korrekte subst in cos-reël* ✓ simplification/ *vereenvoudiging* ✓ $\cos\theta$ as subject/ *as onderwerp* (4)
	OR/*OF* Draw/*Trek* AP ⊥ CD $\cos\theta = \frac{x}{x+3}$  	✓ ✓ constr/*konstr* ✓ ✓ sketch shown/ *toon skets* (4)

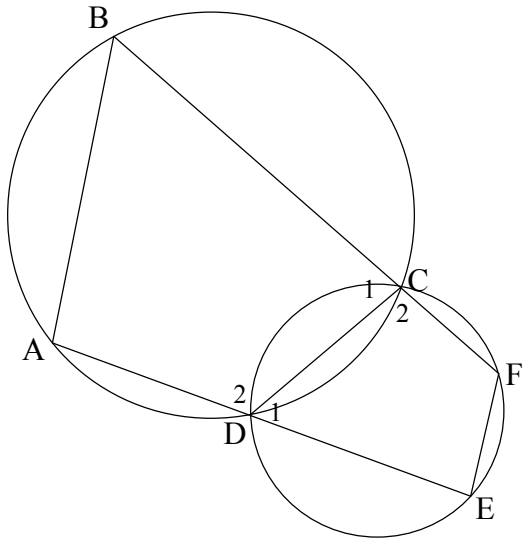
7.3	$\cos\theta = \frac{2}{5}$ $\therefore\ \theta = 66{,}42°$ In ΔABC: $\sin\frac{1}{2}\theta = \frac{AB}{AC}$ $\sin 33{,}21° = \frac{AB}{5}$ $\therefore\ AB = 5\sin 33{,}21°$ $= 2{,}74$ **OR/*OF*** $\sin\frac{\theta}{2} = \frac{AB}{5}$ $\therefore AB = 5\sin\frac{\theta}{2}$ but/*maar*: $\cos\theta = \frac{2}{5}$ $1 - 2\sin^2\frac{\theta}{2} = \frac{2}{5}$ $\sin^2\frac{\theta}{2} = \frac{3}{10}$ $\sin\frac{\theta}{2} = \sqrt{\frac{3}{10}}$ $\therefore AB = 5\sqrt{\frac{3}{10}} = \sqrt{\frac{15}{2}} = 2{,}74$	✓ $\cos\theta = \frac{2}{5}$ ✓ size of/*grootte v* θ ✓ correct ratio/ *korrekte verh* ✓ subst correctly/ *korrek* ✓ answ/*antw* (5) ✓ $AB = 5\sin\frac{\theta}{2}$ ✓ equation/*vgl* ✓ simplification/ *vereenvoudiging* ✓ value of/*waarde v* $\sin\frac{\theta}{2}$ ✓ answ/*antw* (5) **[10]**

QUESTION/*VRAAG* 8



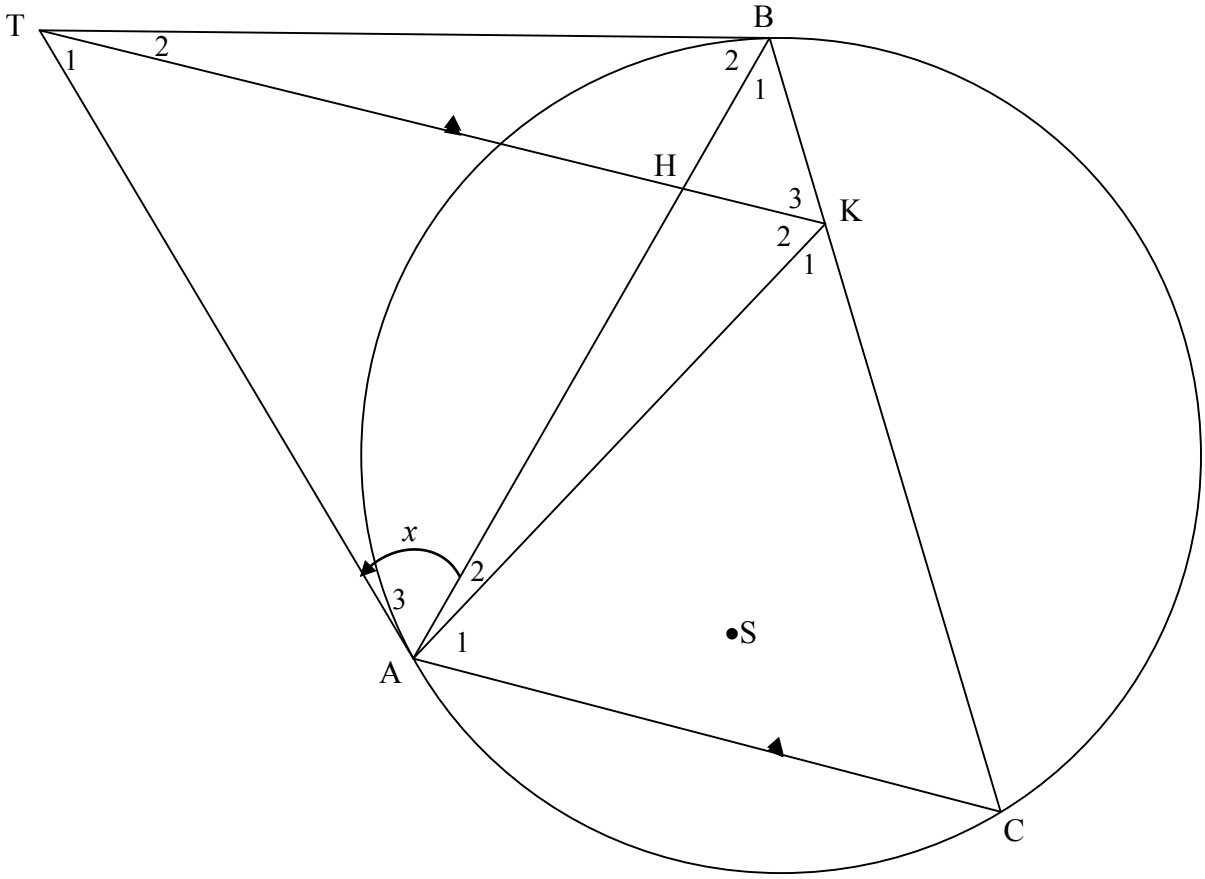
8.1.1	twice or double /*twee keer of dubbel*	✓ R (1)
8.1.2	$\hat{O}_1 = 2\hat{A}$ [∠ at centre = 2×∠ at circ/*midpts∠ =2×omtreks∠*] $\hat{O}_2 = 2\hat{C}$ [∠ at centre = 2×∠ at circ/*midpts∠ =2×omtreks∠*] $\hat{O}_1 + \hat{O}_2 = 360°$ [∠s in a rev/*∠e in omw of om 'n pt*] $2\hat{A} + 2\hat{C} = 360°$ $\therefore \hat{A} + \hat{C} = 180°$ **OR/OF** Let/*Gestel* $\hat{O}_1 = 2x$ $\hat{A} = x$ [∠ at centre = 2×∠ at circ/*midpts∠ =2×omtreks∠*] $\hat{O}_2 = 360° - 2x$ [∠s in a rev/*∠e in omw of om 'n pt*] $\hat{C} = 180° - x$ [∠ at centre = 2×∠ at circ/*midpts∠ =2×omtreks∠*] $\therefore \hat{A} + \hat{C} = 180°$	✓ S ✓ S ✓ S (3) ✓ S ✓ S ✓ S (3)

8.2



8.2	$\hat{A} = \hat{C}_2$	[ext ∠ of cyclic quad/*buite∠ v kdvh*]	✓ S ✓ R
	$\hat{E} = 180° - \hat{C}_2$	[opp ∠s of cyclic quad/*tos∠e v kdvh*]	✓ S ✓ R
	$\therefore\ \hat{E} = 180° - \hat{A}$		
	$\therefore$ EF || AB	[co-interior ∠s 180°/*ko-binne∠e 180°*]	✓ R
			(5)
	OR/*OF*		
	$\hat{B} = \hat{D}_1$	[ext ∠ of cyclic quad/*buite∠ v kdvh*]	✓ S ✓ R
	$\hat{F} = 180° - \hat{D}_1$	[opp ∠s of cyclic quad/*tos∠e v kdvh*]	✓ S ✓ R
	$\therefore\ \hat{F} = 180° - \hat{B}$		
	$\therefore$ EF || AB	[co-interior ∠s 180°/*ko-binne∠e 180°*]	✓ R
			(5)
			[9]

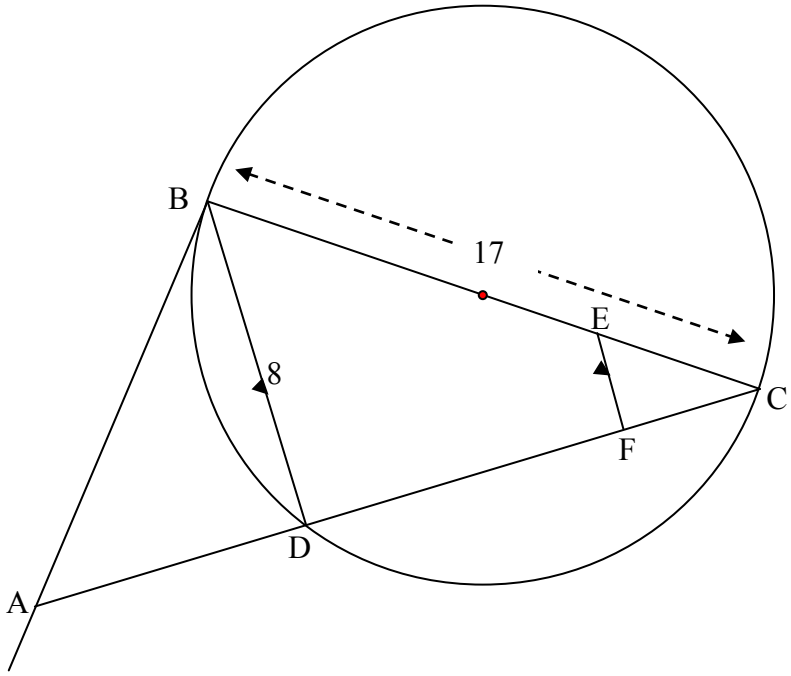
QUESTION/*VRAAG* 9



9.1	$\hat{K}_3 = \hat{C}$ $= \hat{A}_3$ $= x$	[corresp ∠s/*ooreenk ∠e* ; CA| |KT] [tan-chord th/*raakl-koordst*]	✓ S ✓ R ✓ S ✓ R (4)
9.2	$\hat{K}_3 = x = \hat{A}_3$ ∴ AKBT is cyc quad	[proved/*bewys* in 9.1] [line (BT) subtends equal ∠s/ *lyn (BT) onderspan gelyke ∠e*] **OR/OF** [converse ∠s in same segment/ *omgek ∠e in dies segment*]	✓ S ✓ R (2)
9.3	$\hat{K}_3 = \hat{C}$ $= \hat{B}_2$ $= \hat{K}_2$ ∴ TK bisects/*halveer* $A\hat{K}B$ **OR/*OF*** $\hat{K}_2 = \hat{B}_2$ $= \hat{A}_3$	[proven in 9.1] [tan-chord th/*raakl-koordst*] [∠s in the same segm/*∠e in dies segm*] [∠s in the same seg/*∠e in dies segm*] [tans from same pt; ∠s opp equal sides/ *rkle v dies pt; ∠e to gelyke sye*]	 ✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ S ✓ R

	$\therefore \quad = \hat{K}_3$ [proven in 9.1] $\therefore$ TK bisects/*halveer* $A\hat{K}B$	(4)
9.4	$\hat{A}_3 = \hat{K}_2 = x$ [proven/*bewys*] $\therefore$ TA tangent [converse tan chord theorem OR $\angle$ between line and chord/ *omgekeerde raakl-kdst* ***OF*** *$\angle$ tussen lyn en koord*]	✓ S ✓ R (2)
9.5	$B\hat{S}A = B\hat{K}A = 2x$ [A,S,K & B concyclic/*konsiklies*] $A\hat{T}B = 180° - 2x$ [A,T,B & K concyclic/*konsiklies*] $\therefore$ points A, S, B and T are also concyclic/*punte A, S, B en T is ook konsiklies* [opp $\angle$s of quad = 180°/*tos $\angle$e van vierhoek=180°*] **OR/*OF*** A, S K and B are concyclic. A, K, B and T are concyclic. $\therefore$ A, S, B and T are concyclic. **OR/*OF*** The circle passing through points A, K and B contains the point S on the circumference (A, ,S, K and B concyclic)./*Die sirkel deur punt A, K en B bevat die punt S op die omtrek (A, S, K en B konsiklies).* The circle passing through A, K and B contains the point T on the circumference (proven in 9.2)./*Die sirkel deur punt A, K en B bevat die punt T op die omtrek (bewys in 9.2).* $\therefore$ points A, S, B and T are also concyclic/*punte A, S, B en T is konsiklies*	✓ S (both/*beide* statements/ *bewerings*) ✓ R (2) ✓ S ✓ S (2) ✓ S ✓ S (2) **[14]**

QUESTION/*VRAAG* 10



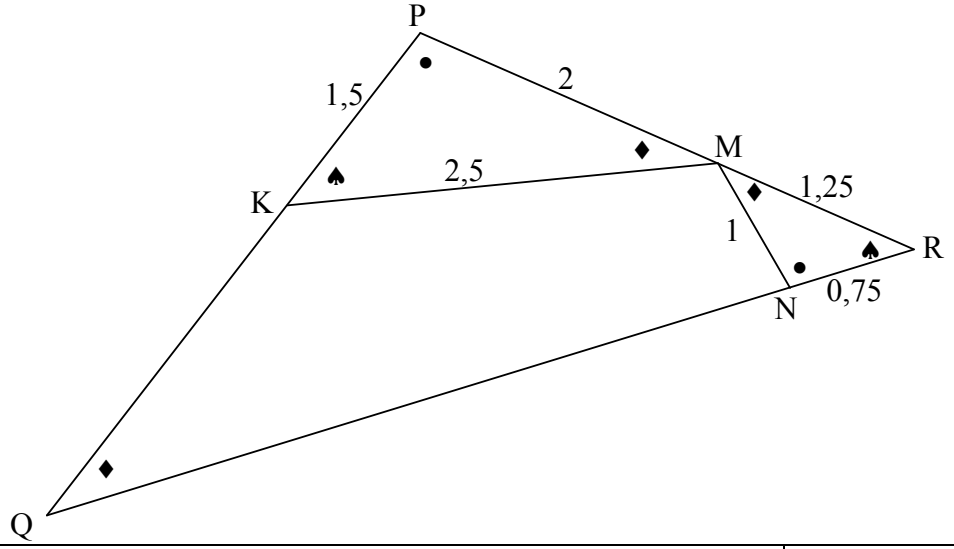
10.1	$B\hat{D}C = 90°$ [∠ in semi circle/∠ *in halfsirkel*] $DC^2 = 17^2 - 8^2$ [Th of/*stelling v* Pythagoras] $= 225$ $\therefore DC = 15$	✓ S ✓ using/*gebruik* Pyth *korrek*/ correctly ✓ answ/*antw* (3)
10.2.1	$\frac{CF}{CD} = \frac{CE}{CB}$ [line | | one side of Δ/*lyn* | | *een sy van* Δ] **OR/OF** ΔCEF | | | ΔCBD $\therefore \frac{CF}{15} = \frac{1}{4}$ $\therefore CF = 3{,}75$	✓ S/R ✓ subst correctly/ *korrek* ✓ answ/*antw* (3)
10.2.2	$B\hat{D}C = 90°$ [∠ in semi circle/∠ *in halfsirkel*] $E\hat{F}C = B\hat{D}C$ [corresp ∠s/*ooreenk ∠e*; EF| |BD] $A\hat{B}C = 90°$ [tan ⊥ diameter/*raakl ⊥ middellyn*] In ΔBAC and/*en* Δ FEC: $A\hat{B}C = E\hat{F}C$ [proven/bewys] $\hat{C} = \hat{C}$ [common/*gemeen*] ∴ ΔBAC | | | Δ FEC [∠∠∠] **OR/*OF*** $B\hat{D}C = 90°$ [∠ in semi circle/∠ *in halfsirkel*] $E\hat{F}C = B\hat{D}C$ [corresp ∠s/*ooreenk ∠e*; EF| |BD] $A\hat{B}C = 90°$ [tan ⊥ diameter/*raakl ⊥ middellyn*] In ΔBAC and/*en* Δ FEC: $A\hat{B}C = E\hat{F}C$ [proven/bewys] $\hat{C} = \hat{C}$ [common/*gemeen*]	✓ S/R ✓ S ✓ R ✓ S ✓ R (5) ✓ S/R ✓ S ✓ R ✓ S

	$B\hat{A}C = F\hat{E}C$ [∠ sum in Δ/∠ *som van* Δ] $\therefore \Delta BAC \mid\mid\mid \Delta FEC$	✓ S (5)
10.2.3	$EC = \frac{1}{4} \times 17 = 4{,}25$ $\frac{AC}{EC} = \frac{BC}{FC}$ [ΔBAC ||| Δ FEC] $\frac{AC}{4{,}25} = \frac{17}{3{,}75}$ $\therefore AC = 19{,}27$ or/*of* $19\frac{4}{15}$ **OR/*OF*** $\cos \hat{C} = \frac{CF}{CE} = \frac{BC}{AC}$ $\therefore \frac{3{,}75}{4{,}25} = \frac{17}{AC}$ $\therefore AC = 19{,}27$ or/*of* $19\frac{4}{15}$ **OR/*OF*** $\Delta BCA \mid\mid\mid \Delta DBC$ $CB^2 = CD \cdot AC$ $AC = \frac{BC^2}{DC}$ $= \frac{17^2}{15}$ $= 19{,}27$ or/*of* $19\frac{4}{15}$ **OR/*OF*** $\hat{C} = A\hat{B}D$ [tan-chord theorem/*rkl-kdstelling*] $\frac{AD}{8} = \tan A\hat{B}D$ $= \tan\hat{C}$ $= \frac{8}{15}$ $\therefore AD = \frac{64}{15}$ $\therefore AC = 19{,}27$ or/*of* $19\frac{4}{15}$	✓ length of/*lengte v* EC ✓ S ✓ subst correctly/ *korrek* ✓ answ/*antw* (4) ✓✓ correct ratios/ *korrekte verh's* ✓ subst correctly/ *korrek* ✓ answ/*antw* (4) ✓ S **OR** Pyth th ✓ correct ratio ✓ subst ✓ answ/*antw* (4) ✓ S ✓ correct ratio ✓ subst ✓ answ/*antw* (4)

10.2.4	AC is diameter of the circle passing through A, B and C [chord subtends 90° **OR** converse ∠ in semi circle] *AC is middellyn van die sirkel wat deur die punte A, B en C gaan* [*koord onderspan 90°* ***OF*** *omgek ∠ in halfsirkel*] $\therefore \text{radius} = \frac{1}{2} \times 19{,}27 = 9{,}63$ or/*of* $9\frac{19}{30}$ or/*of* $\frac{1}{2}$AC	✓ S/R ✓ answ/*antw* (2) **[17]**

QUESTION/*VRAAG* 11

11.1	equiangular or similar/*gelykhoekig of gelykvormig*	✓ answ/*antw* (1)



11.2.1	$\frac{KP}{RN} = \frac{1,5}{0,75} = 2$; $\frac{PM}{NM} = \frac{2}{1} = 2$; $\frac{KM}{RM} = \frac{2,5}{1.25} = 2$ $\therefore \frac{KP}{RN} = \frac{PM}{NM} = \frac{KM}{RM}$ $\therefore \Delta KPM \mid\mid\mid \Delta RNM$ [Sides of Δ in prop/*sye v Δ eweredig*] **OR/*OF*** $\frac{RN}{KP} = \frac{0,75}{1,5} = \frac{1}{2}$; $\frac{NM}{PM} = \frac{1}{2}$; $\frac{RM}{KM} = \frac{1,25}{2,5} = \frac{1}{2}$ $\therefore \frac{RN}{KP} = \frac{NM}{PM} = \frac{RM}{KM}$ $\therefore \Delta KPM \mid\mid\mid \Delta RNM$ [Sides of Δ in prop/*sye v Δ eweredig*] **OR/*OF*** In ΔMNR: $1,25^2 = 1^2 + 0,75^2 = 1,5625$ $\therefore M\hat{N}R = 90°$ [converse Pyth theorem] In ΔPKM: $2,5^2 = 1,5^2 + 2^2 = 6,25$ $\therefore \hat{P} = 90°$ [converse Pyth theorem] $\cos P\hat{K}M = \frac{1,5}{2,5} = \frac{3}{5}$ and $\cos \hat{R} = \frac{0,75}{1,25} = \frac{3}{5}$ $\therefore P\hat{K}M = \hat{R}$ In ΔKPM and ΔRNM $P\hat{K}M = \hat{R}$ [proved] $\hat{P} = M\hat{N}R$ [proved] $\therefore \Delta KPM \mid\mid\mid \Delta RNM$ [∠;∠;∠ OR 3rd ∠]	✓✓✓ all 3 statements/ *al 3 bewerings* (3) ✓✓✓ all 3 statements/ *al 3 bewerings* (3) ✓ $\hat{P} = M\hat{N}R$ ✓ $P\hat{K}M = \hat{R}$ ✓[∠;∠;∠ OR 3rd ∠] (3)

11.2.2	$P\hat{K}M = \hat{R}$ [ΔKPM ||| ΔRNM] ∴ $\hat{P}$ is common/*gemeen* ∴ ΔRPQ ||| ΔKPM [∠∠∠] $\frac{RP}{KP} = \frac{RQ}{KM}$ [ΔRPQ ||| ΔKPM] ∴ $\frac{3,25}{1,5} = \frac{RQ}{2,5}$ ∴ RQ = $\frac{2,5 \times 3,25}{1,5}$ = 5,42 or $5\frac{5}{12}$ ∴ NQ = 5,42 – 0,75 = 4,67 or $4\frac{2}{3}$	✓ S ✓ ΔRPQ|||ΔKPM ✓ S ✓ subst correctly/ *korrek* ✓ RQ = $5\frac{5}{12}$ ✓ NQ = answ/*antw* (6)
	OR/OF	
	$R\hat{N}M = \hat{P}$ [ΔKPM ||| ΔRNM] ∴ $\hat{R}$ is common/*gemeen* ∴ ΔRNM ||| ΔRPQ [∠∠∠] $\frac{RP}{RN} = \frac{RQ}{RM}$ [ΔRNM ||| ΔRPQ] ∴ $\frac{3,25}{0,75} = \frac{RQ}{1,25}$ ∴ RQ = 5,42 or $5\frac{5}{12}$ ∴ NQ = 5,42 – 0,75 = 4,67 or $4\frac{2}{3}$	✓ S ✓ ΔRNM ||| ΔRPQ ✓ S ✓ subst correctly/ *korrek* ✓ RQ = $5\frac{5}{12}$ ✓ NQ = answ/*antw* (6)
	OR/*OF* In ΔMNR: $1,25^2 = 1^2 + 0,75^2 = 1,5625$ ∴ $M\hat{N}R = 90°$ [converse Pyth theorem] In ΔPKM: $2,5^2 = 1,5^2 + 2^2 = 6,25$ ∴ $\hat{P} = 90°$ [converse Pyth theorem] In ΔMNR and ΔQPR ∠R is common $M\hat{N}R = \hat{P} = 90°$ ∴ ΔMNR ||| ΔQPR [∠∠∠] $\frac{RP}{RN} = \frac{RQ}{RM}$ [ΔRNM ||| ΔRPQ] ∴ $\frac{3,25}{0,75} = \frac{RQ}{1,25}$ ∴ RQ = 5,42 or $5\frac{5}{12}$ ∴ NQ = 5,42 – 0,75 = 4,67 or $4\frac{2}{3}$	✓ S ✓ ΔMNR ||| ΔQPR ✓ S ✓ subst correctly/ *korrek* ✓ RQ = $5\frac{5}{12}$ ✓ NQ = answ/*antw* (6) **[10]**

TOTAL/*TOTAAL*: 149

PolyMathic

Vraestel 8

Mei/Junie Eksamen

PolyMathic

Graad 12 Junie Eksamen
Totaal: 150
Tyd: 3ure

VRAAG 1

'n Toeroperateur het op 'n sekere dag 11 toerbusse na 11 verskillende bestemmings toe gestuur. Die tabel hieronder toon die getal passasiers op elke bus.

8	8	10	12	16	19	20	21	24	25	26

1.1 Bereken die gemiddelde getal passasiers wat in 'n toerbus ry. (2)

1.2 Skryf die vyf-getal-opsomming van die data neer. (3)

1.3 Skets 'n mond-en-snordiagram vir die data. Gebruik die getallelyn wat in die ANTWOORDEBOEK verskaf word. (2)

1.4 Verwys na die mond-en-snordiagram en lewer kommentaar op die skeefheid van die datastel. (1)

1.5 Bereken die standaardafwyking vir hierdie datastel. (2)

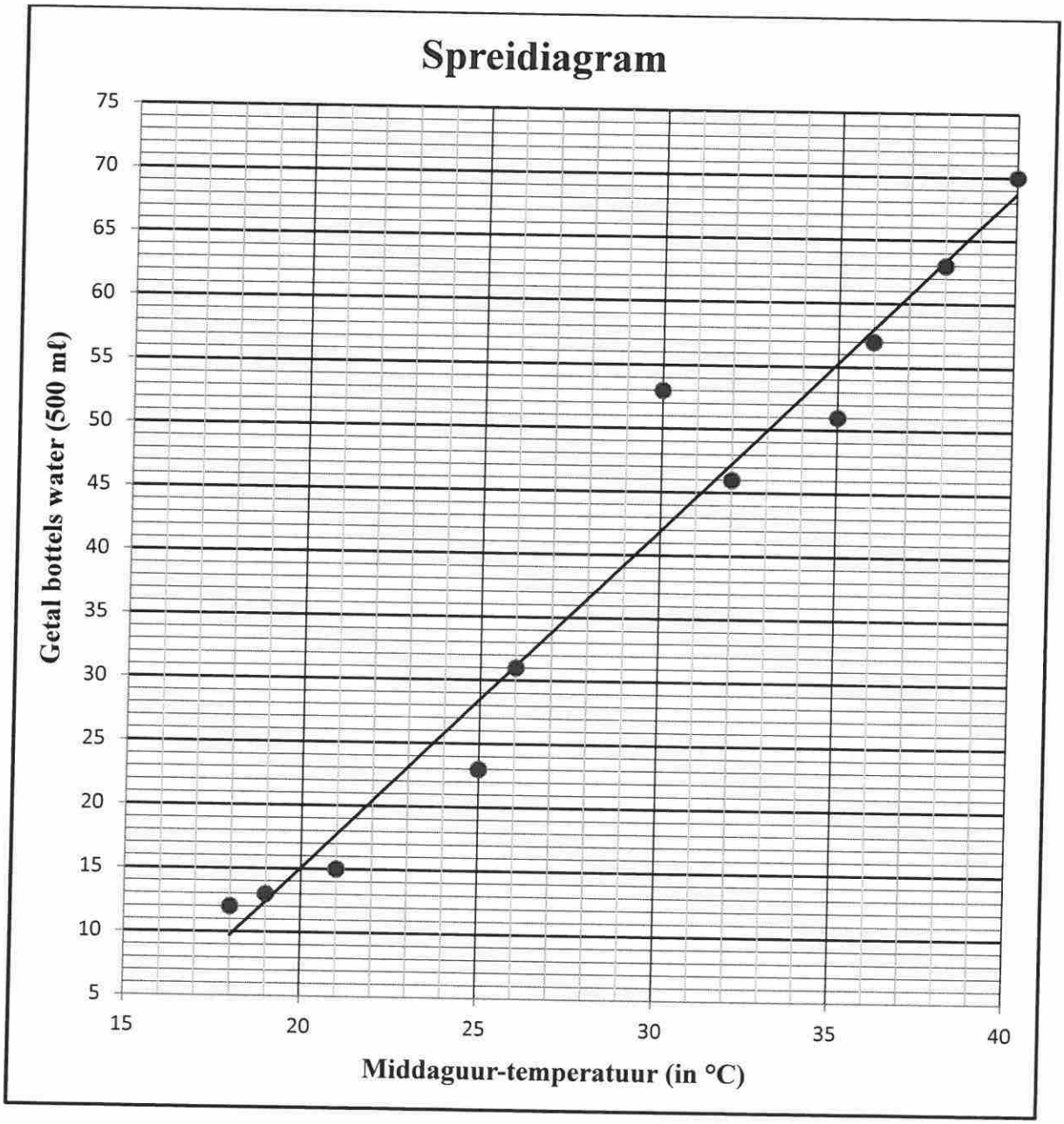
1.6 'n Toer word as gewild beskou indien die getal passasiers op 'n toerbus een standaardafwyking bokant die gemiddelde is. Hoeveel bestemmings was op hierdie spesifieke dag gewild? (2)

[12]

VRAAG 2

Op die eerste skooldag van elke maand word inligting oor die middaguur-temperatuur (in °C) en die getal 500 mℓ-bottels water wat in pouse by die snoepie van 'n sekere skool verkoop word, aangeteken. Die data word in die tabel hieronder getoon en op die spreidiagram voorgestel. Die kleinstekwadrate-regressielyn vir hierdie data is op die spreidiagram geskets.

Middaguur-temperatuur (in °C)	18	21	19	26	32	35	36	40	38	30	25
Getal bottels water (500 mℓ)	12	15	13	31	46	51	57	70	63	53	23



2.1 Identifiseer 'n uitskieter in die data. (1)

2.2 Bepaal die vergelyking van die kleinstekwadrate-regressielyn. (3)

2.3 Skat die getal 500 mℓ-bottels water wat verkoop sal word indien die middaguur-temperatuur 28 °C is. (2)

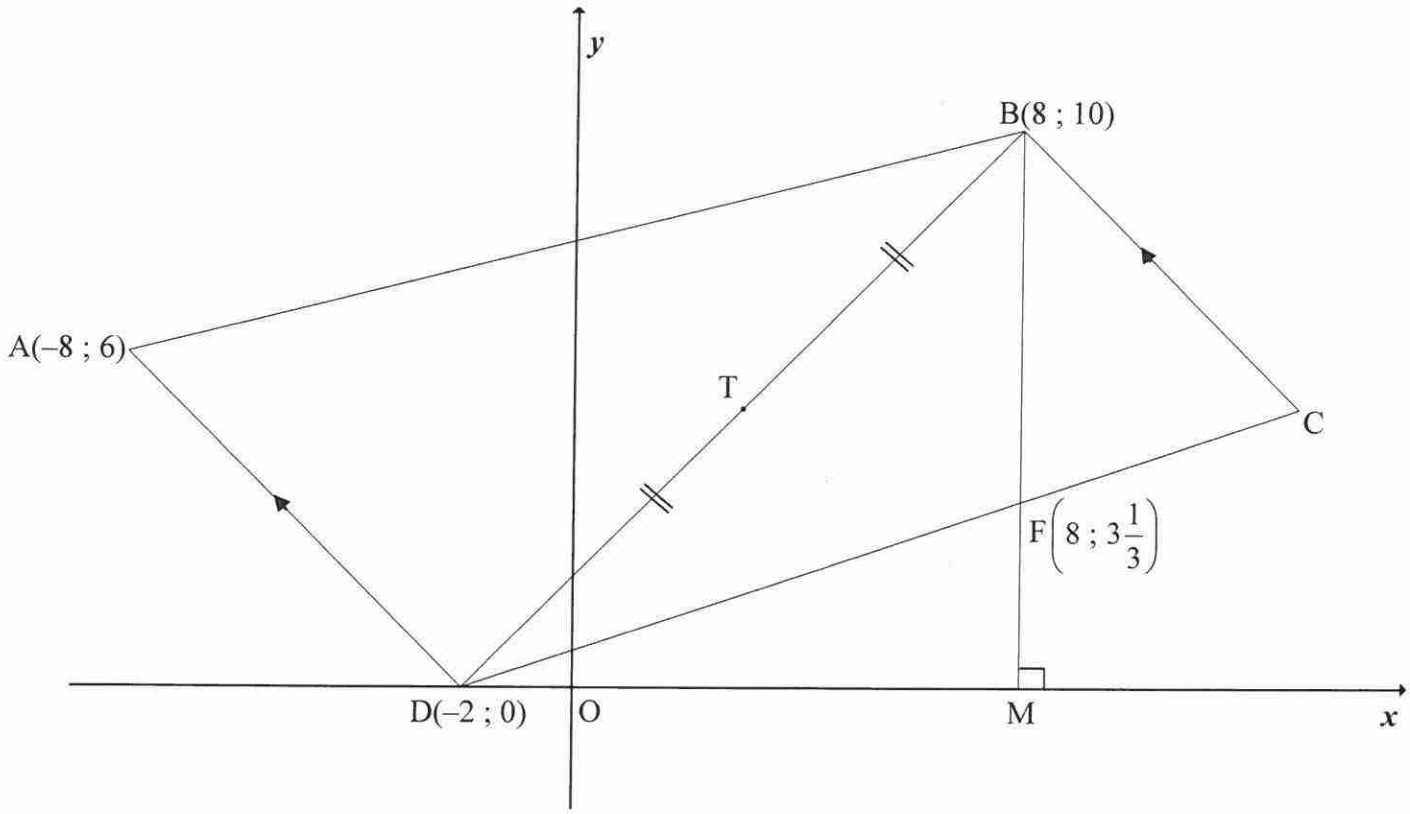
2.4 Verwys na die spreidiagram. Sou jy sê dat die verband tussen die middaguur-temperatuur en die getal 500 mℓ-bottels water wat verkoop word, swak of sterk is? Motiveer jou antwoord. (2)

2.5 Gee 'n rede waarom die waargenome neiging vir hierdie data nie onbepaald kan aanhou nie. (1)

[9]

VRAAG 3

In die diagram hieronder (nie volgens skaal geteken nie) is A(–8 ; 6), B(8 ; 10), C en D(–2 ; 0) die hoekpunte van 'n trapesium met BC | | AD. T is die middelpunt van DB. Die reguitlyn vanaf B, wat ewewydig aan die y-as getrek is, sny DC in $F\left(8 ; 3\frac{1}{3}\right)$ en die x-as in M.



3.1 Bereken die gradiënt van AD. (2)

3.2 Bepaal die vergelyking van BC in die vorm $y = mx + c$. (3)

3.3 Bewys dat BD ⊥ AD. (3)

3.4 Bereken die grootte van $B\hat{D}M$. (2)

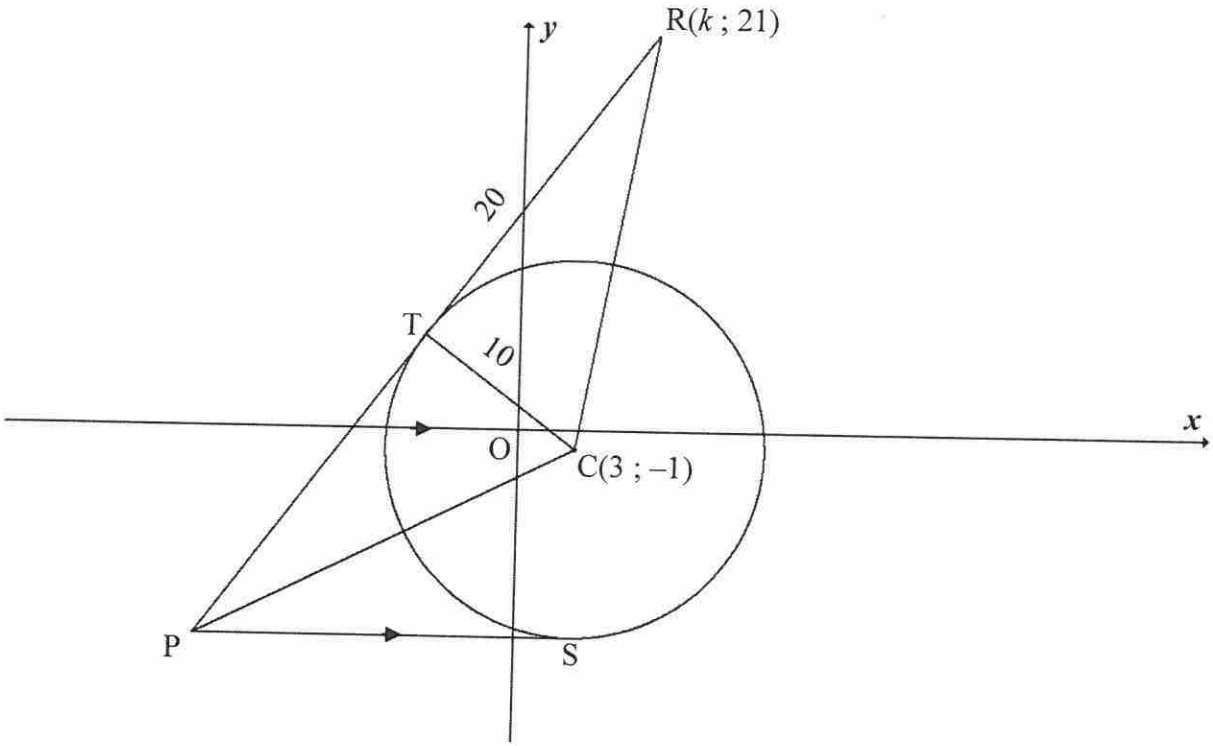
3.5 As gegee word dat TC | | DM en punte T en C simmetries om lyn BM is, bereken die koördinate van C. (3)

3.6 Bereken die oppervlakte van ΔBDF. (5)

[18]

VRAAG 4

'n Sirkel met C(3 ; –1) as middelpunt en 'n radius van 10 eenhede is geskets. PTR is 'n raaklyn aan hierdie sirkel by T. R(k ; 21), C en P is die hoekpunte van 'n driehoek. TR = 20 eenhede.



4.1 Gee 'n rede waarom TC ⊥ TR. (1)

4.2 Bereken die lengte van RC. Laat jou antwoord in wortelvorm. (2)

4.3 Bereken die waarde van k as R in die eerste kwadrant lê. (4)

4.4 Bepaal die vergelyking van die sirkel wat C as middelpunt het en deur T gaan. Skryf jou antwoord in die vorm $(x-a)^2+(y-b)^2=r^2$ (2)

4.5 PS, 'n raaklyn aan die sirkel by S, is ewewydig aan die x-as. Bepaal die vergelyking van PS. (2)

4.6 Die vergelyking van PTR is $3y-4x=35$

4.6.1 Bereken die koördinate van P. (2)

4.6.2 Bereken, deur 'n rede te gee, die lengte van PT. (3)

4.7 Beskou 'n ander sirkel met vergelyking $(x-3)^2+(y+16)^2=16$ en middelpunt M.

4.7.1 Skryf die koördinate van middelpunt M neer. (1)

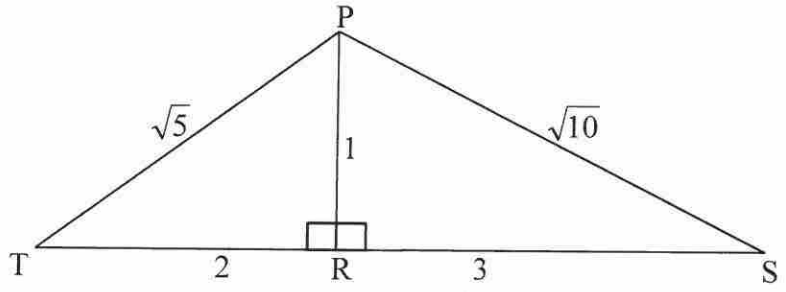
4.7.2 Skryf die lengte van die radius van hierdie sirkel neer. (1)

4.7.3 Bewys dat die sirkel met middelpunt C en die sirkel met middelpunt M mekaar nie sny of raak nie. (3)

[21]

VRAAG 5

5.1 In die diagram is $PR \perp TS$ in stomphoekige driehoek PTS.
$PT = \sqrt{5}$; $TR = 2$; $PR = 1$; $PS = \sqrt{10}$ en $RS = 3$



5.1.1 Skryf die waarde neer van:

(a) $\sin \hat{T}$ (1)

(b) $\cos \hat{S}$ (1)

5.1.2 Bereken, SONDER die gebruik van 'n sakrekenaar, die waarde van $\cos(\hat{T} + \hat{S})$ (5)

5.2 Bepaal die waarde van:

$$\frac{1}{\cos(360° - \theta).\sin(90° - \theta)} - \tan^2(180° + \theta)$$ (6)

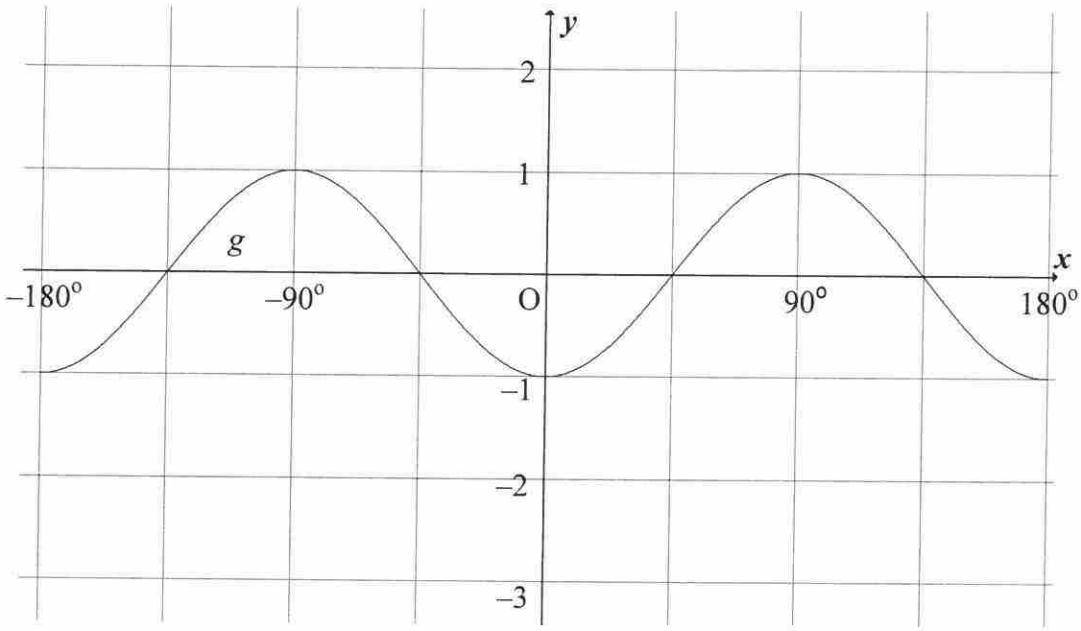
5.3 Indien $\sin x - \cos x = \frac{3}{4}$, bereken die waarde van $\sin 2x$ SONDER die gebruik van 'n sakrekenaar. (5)

[18]

VRAAG 6

6.1 Bepaal die algemene oplossing van $4\sin x + 2\cos 2x = 2$ (6)

6.2 Die grafiek van $g(x) = -\cos 2x$ vir $x \in [-180° ; 180°]$ is hieronder geskets.



6.2.1 Skets die grafiek van $f(x) = 2\sin x - 1$ vir $x \in [-180° ; 180°]$ op die assestelsel wat in die ANTWOORDEBOEK verskaf is. (3)

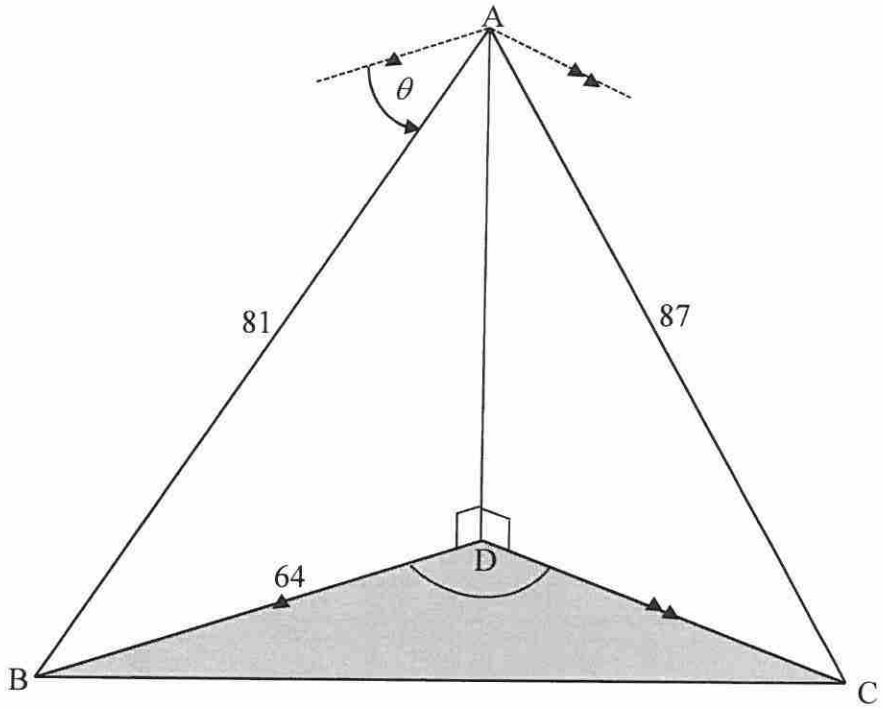
6.2.2 Skryf die waardes van x neer waarvoor g streng afnemend in die interval $x \in [-180° ; 0°]$ is. (2)

6.2.3 Skryf die waarde(s) van x neer waarvoor $f(x + 30°) - g(x + 30°) = 0$ vir $x \in [-180° ; 180°]$ (2)

[13]

VRAAG 7

Vanaf punt A sien 'n waarnemer twee bote, B en C, wat voor anker lê, raak. Die dieptehoek van boot B vanaf A is θ. D is 'n punt direk onder A en is op dieselfde horisontale vlak as B en C. BD = 64 m, AB = 81 m en AC = 87 m.



7.1 Bereken die grootte van θ tot die naaste graad. (3)

7.2 As gegee word dat $B\hat{A}C = 82{,}6°$, bereken BC, die afstand tussen die bote. (3)

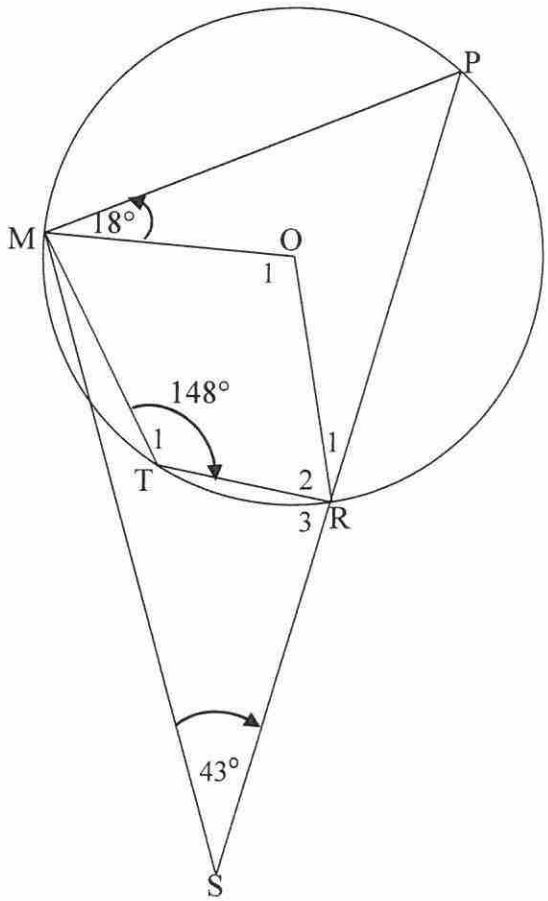
7.3 As $B\hat{D}C = 110°$, bereken die grootte van $D\hat{C}B$. (3)

[9]

Gee redes vir ALLE bewerings in VRAAG 8, 9, 10 en 11.

VRAAG 8

8.1 P, M, T en R is punte op 'n sirkel met O as middelpunt in die diagram hieronder. PR verleng, ontmoet MS by S. Radiusse OM en OR en die koorde MT en TR is getrek. $\hat{T}_1 = 148°$, $P\hat{M}O = 18°$ en $\hat{S} = 43°$



Bereken, met redes, die grootte van:

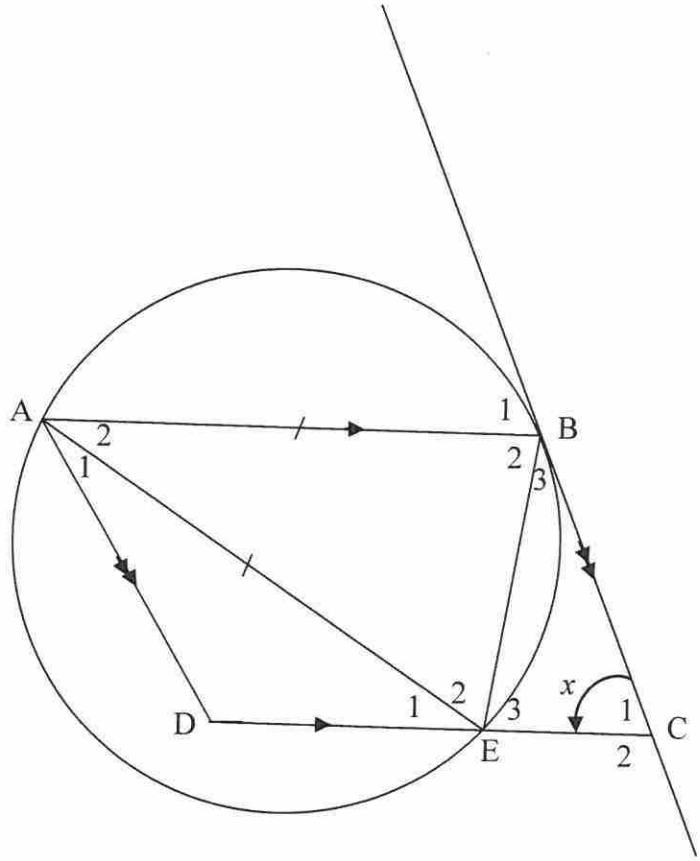
8.1.1 $\hat{P}$ (2)

8.1.2 $\hat{O}_1$ (2)

8.1.3 $O\hat{M}S$ (2)

8.1.4 $\hat{R}_3$ as gegee word dat $T\hat{M}S = 6°$ (2)

8.2 In die diagram hieronder gaan die sirkel deur A, B en E. ABCD is 'n parallelogram. BC is 'n raaklyn aan die sirkel by B. AE = AB. Laat $\hat{C}_1 = x$



8.2.1 Gee 'n rede waarom $\hat{B}_1 = x$ (1)

8.2.2 Noem, met redes, DRIE ander hoeke van dieselfde grootte as x. (6)

8.2.3 Bewys dat ABED 'n koordevierhoek is. (3)

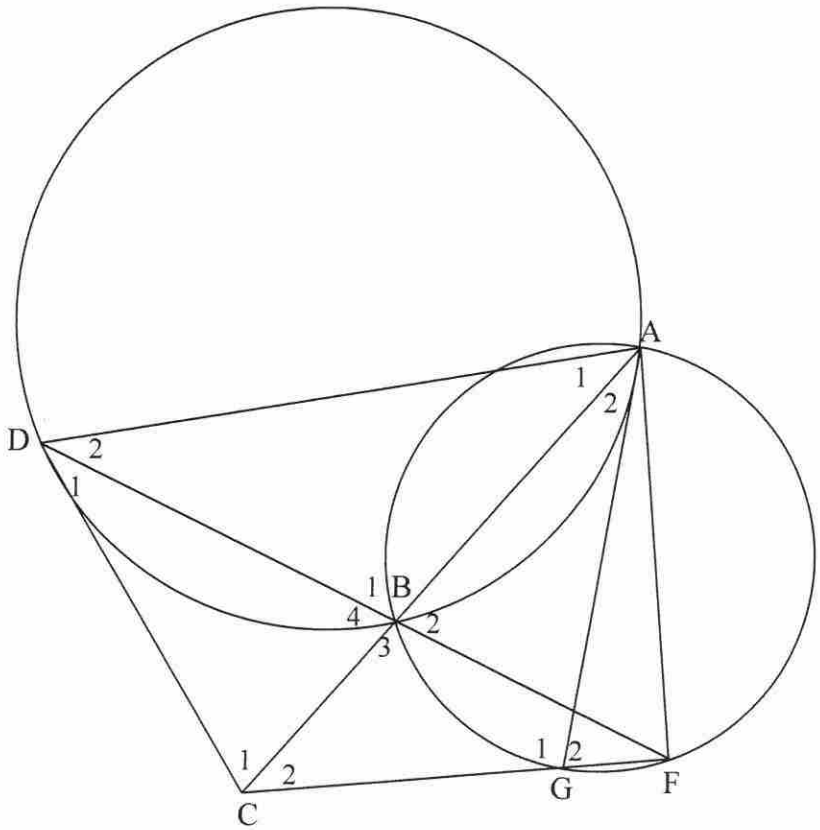
[18]

VRAAG 9

9.1 Voltooi die stelling sodat dit WAAR sal wees:

Die hoek tussen die raaklyn aan 'n sirkel en die koord getrek vanaf die raakpunt, is gelyk aan die hoek ... (1)

9.2 Twee ongelyke sirkels sny in A en B in die diagram hieronder. AB is verleng na C sodat CD 'n raaklyn aan die sirkel ABD by D is. F en G is punte op die kleiner sirkel sodat CGF en DBF reguitlyne is. AD en AG is getrek.



Bewys dat:

9.2.1 $\hat{B}_4 = \hat{D}_1 + \hat{D}_2$ (4)

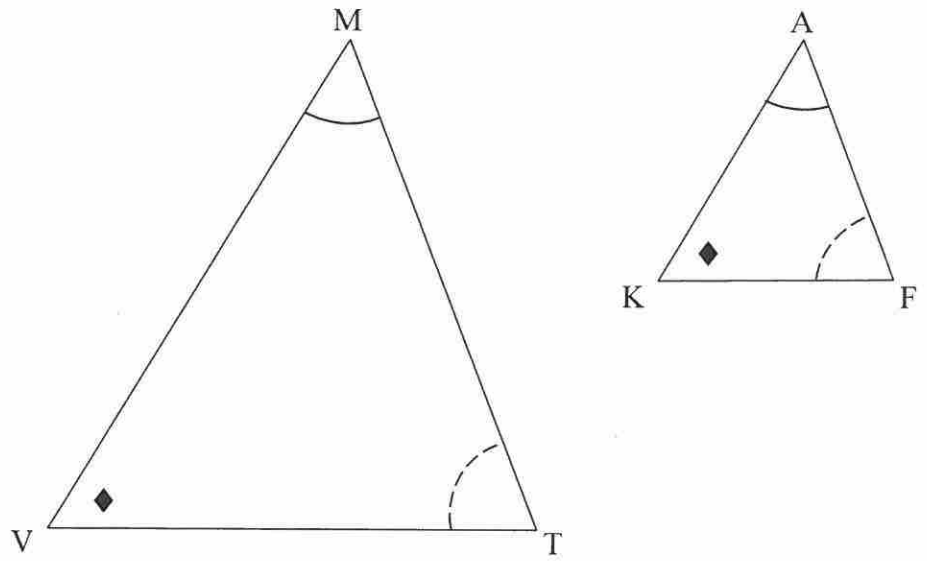
9.2.2 AGCD 'n koordevierhoek is (4)

9.2.3 DC = CF (4)

[13]

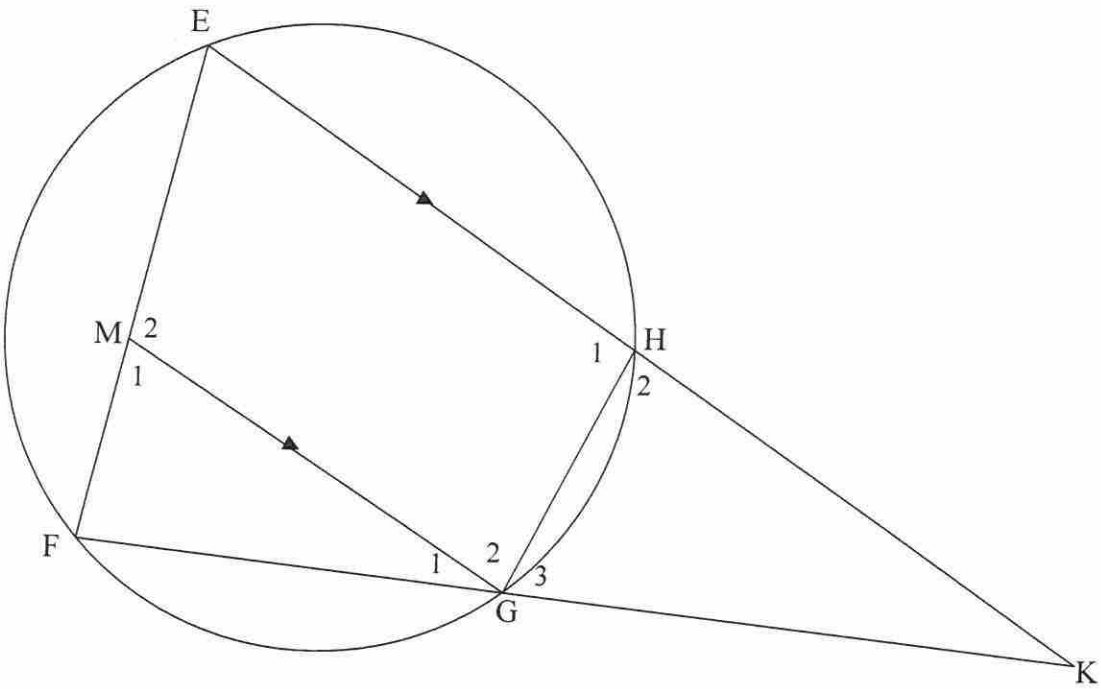
VRAAG 10

10.1 In die diagram hieronder is ΔMVT en ΔAKF geskets sodat $\hat{M} = \hat{A}$, $\hat{V} = \hat{K}$ en $\hat{T} = \hat{F}$



Gebruik die diagram in die ANTWOORDEBOEK om die stelling te bewys wat beweer dat as twee driehoeke gelykhoekig is, dan is die ooreenstemmende sye eweredig, dit wil sê $\frac{MV}{AK} = \frac{MT}{AF}$ (7)

10.2 Koordevierhoek EFGH is in die diagram hieronder geskets. Koord EH verleng en koord FG verleng ontmoet by K. M is 'n punt op EF sodat MG | | EK. Verder is KG = EF



10.2.1 Bewys dat:

(a) $\Delta KGH \,|||\, \Delta KEF$ (4)

(b) $EF^2 = KE \,.\, GH$ (2)

(c) $KG^2 = EM \,.\, KF$ (3)

10.2.2 As gegee word dat KE = 20 eenhede, KF = 16 eenhede en GH = 4 eenhede, bereken die lengte van EM. (3)

[19]

TOTAAL: 150

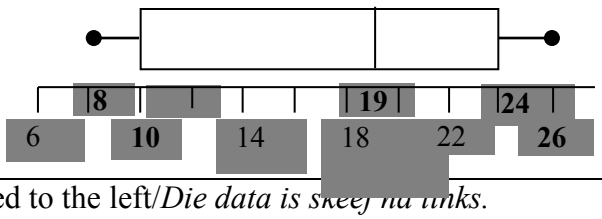
Memo

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n poging om 'n vraag te beantwoord, doodgetrek en nie oorgedoen het nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing. Staak nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.*

QUESTION/*VRAAG* 1

8	8	10	12	16	19	20	21	24	25	26

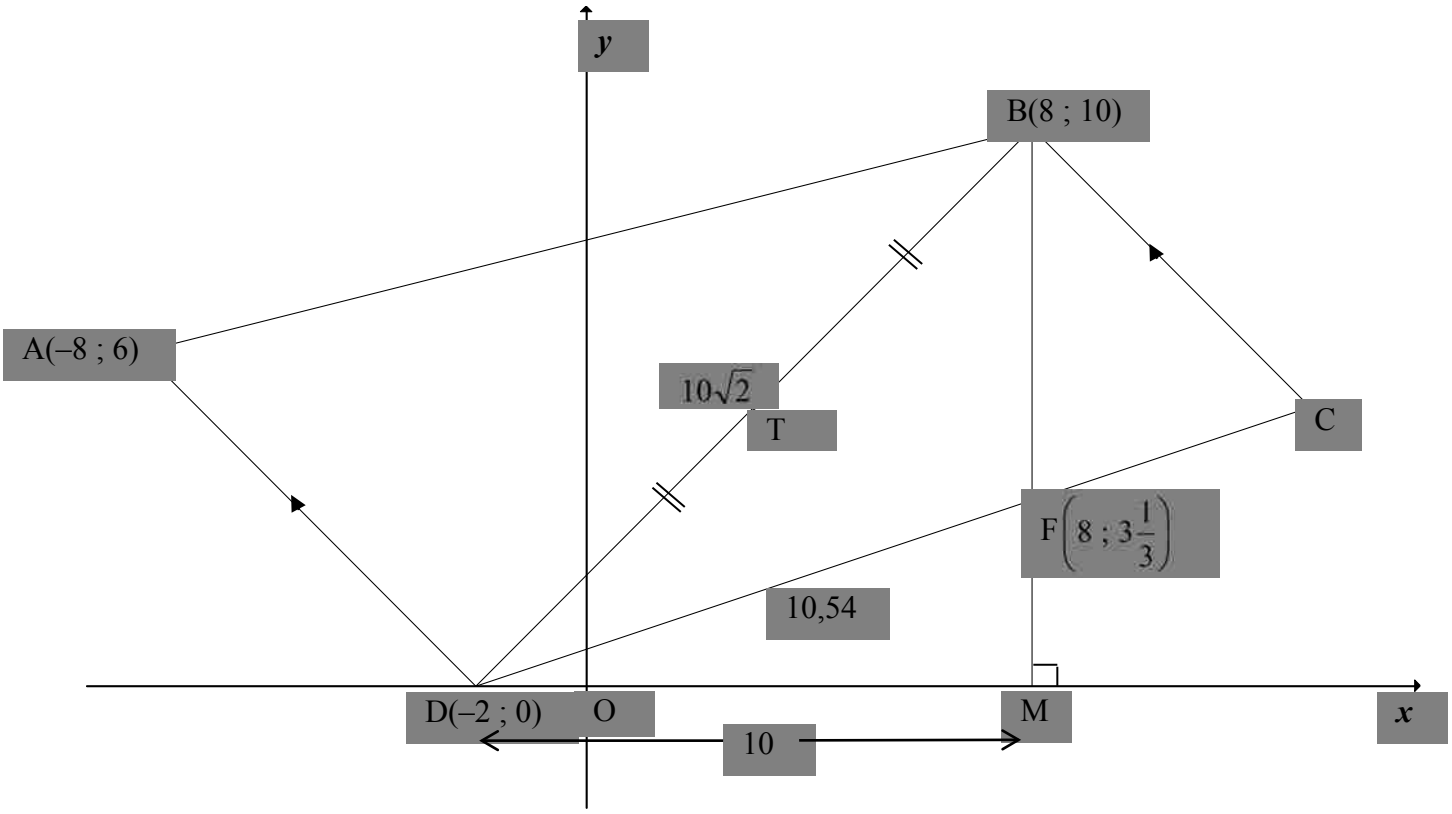
1.1	Mean/*Gemiddelde* $= \frac{189}{11}$ $= 17{,}18$ Answer only: Full marks *Slegs antwoord: Volpunte*	✓189 ✓ answer (2)
1.2	Min = 8, max = 26 Median/*Mediaan* = 19 $Q_1 = 10$, $Q_3 = 24$ ∴ (8 ; 10 ; 19 ; 24 ; 26)	✓ min, max ✓ median ✓ Q_1 & Q_3 (3)
1.3		✓ box/*boks/mond* ✓ whiskers/*snor* (2)
1.4	The data is skewed to the left/*Die data is skeef na links.* **OR/*OF*** Negatively skewed/*Negatief skeef*	✓ answer (1) ✓ answer (1)
1.5	SD/*SA* = 6,46	✓✓ answer (2)
1.6	17,18 + 6,46 = 23,64 ∴ 3 destinations/*bestemmings*	✓ interval ✓ answer (2) **[12]**

QUESTION/*VRAAG* 2

Temperature at midday (in °C) ***Middaguur-temperatuur (in °C)***	18	21	19	26	32	35	36	40	38	30	25
Number of bottles of water (500 mℓ) ***Getal bottels water (500 mℓ)***	12	15	13	31	46	51	57	70	63	53	23

2.1	(30 ; 53)	✓answer (1)
2.2	$a = -38{,}51$ $b = 2{,}68$ $\therefore \hat{y} = 2{,}68x - 38{,}51$	✓ value a ✓ value b ✓ equation (3)
2.3	$\therefore \hat{y} \approx 36{,}53$ bottles **OR/*OF*** $\hat{y} \approx 2{,}68(28) - 38{,}51$ $\approx 36{,}53$ bottles	✓✓ answer (2) ✓ substitution ✓ answer (2)
2.4	Strong/*Sterk* The majority of the points lie **close to** the regression line./*Die meerderheid punte lê naby die regressielyn.* **OR/*OF*** Strong/*Sterk* $r = 0{,}98$	✓ strong/*sterk* ✓ reason/*rede* (2) ✓ strong/*sterk* ✓ reason/*rede* (2)
2.5	Temperature cannot rise beyond a certain point as this would be life threatening **OR** there is only so much water one can consume before it becomes a risk to your health (hyponatremia)./*Temperatuur kan nie hoër as 'n sekere punt styg nie, anders raak dit lewensgevaarlik.* ***OF*** *'n persoon kan net 'n sekere hoeveelheid water inneem, anders raak dit 'n gesondheidsrisiko*	✓ reason/*rede* (1) **[9]**

QUESTION/*VRAAG* 3



3.1	$m_{AD} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{0-6}{-2+8}$ $= \frac{-6}{6} = -1$	✓substitution ✓–1 (2)
3.2	$m_{BC} = -1$ [BC || AD] $y = -x + c$ $10 = -8 + c$ $c = 18$ $y = -x + 18$ **OR/*OF*** $m_{BC} = -1$ [BC || AD] $y - y_1 = m(x - x_1)$ $y - 10 = -(x - 8)$ $y = -x + 18$	✓ gradient ✓substitute *m* and (8 ; 10) ✓ equation (3) ✓ gradient ✓substitute *m* and (8 ; 10) ✓ equation (3)

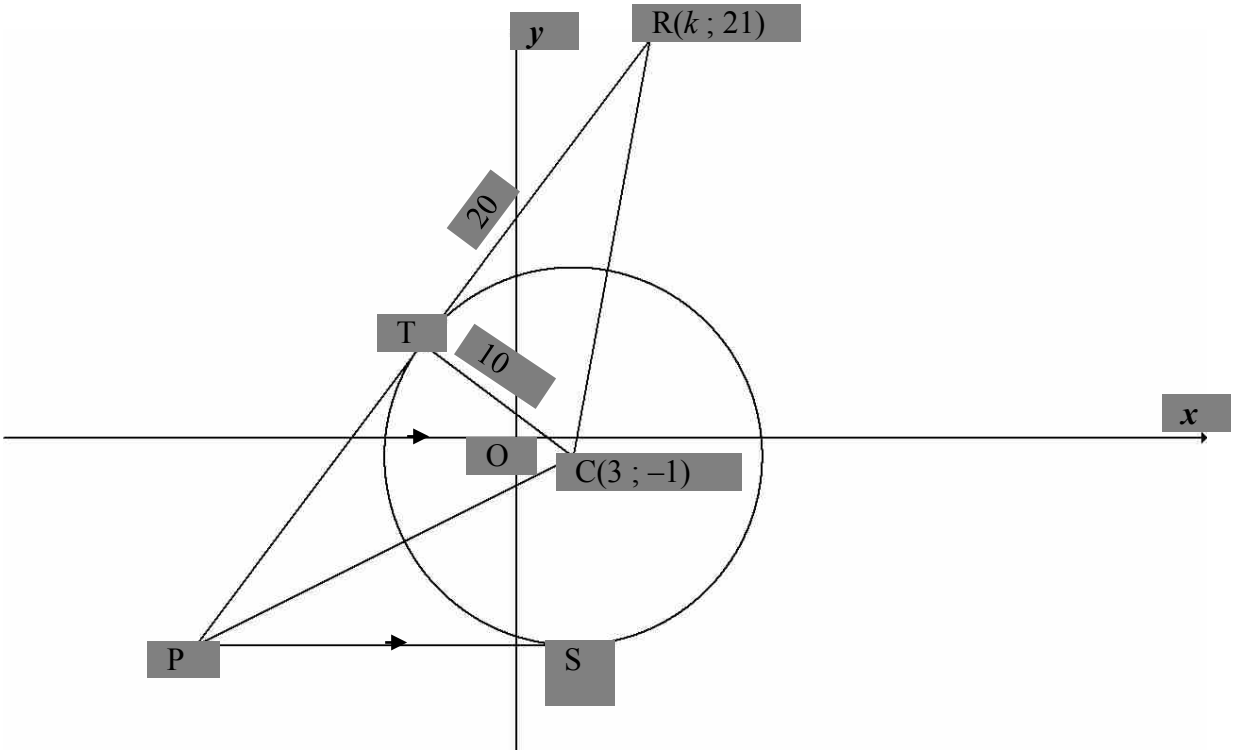
3.3	$m_{BD} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{10-0}{8+2} = 1$ $m_{BD} \times m_{AD} = 1 \times -1 = -1$ $\therefore DB \perp AD$ **OR** $AD^2 = 72$ *or* $AD = \sqrt{72}$ *or* $6\sqrt{2}$ $AB^2 = 272$ *or* $AB = \sqrt{272}$ *or* $4\sqrt{17}$ $BD^2 = 200$ *or* $BD = \sqrt{200}$ *or* $10\sqrt{2}$ $\therefore\ AB^2 = AD^2 + BD^2$ $\therefore\ A\hat{D}B = 90°$ [converse Pyth th/ *omgekeerde Pyth st*]	✓substitution ✓ answer ✓ $m_{BD} \times m_{AD} = -1$ (3) ✓✓ calculating all 3 sides ✓ $AB^2 = AD^2 + BD^2$ (3)
3.4	$\tan B\hat{D}M = m_{BD} = 1$ $\therefore B\hat{D}M = 45°$ **OR** $\sin B\hat{D}M = \frac{BM}{BD} = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}}$ $\therefore B\hat{D}M = 45°$	✓ $\tan B\hat{D}M = m_{BD}$ ✓ answer (2) ✓ $\sin B\hat{D}M = \frac{1}{\sqrt{2}}$ ✓ answer (2)
3.5	$T(x\,;y) = \left(\frac{x_1+x_2}{2}\,;\frac{y_1+y_2}{2}\right)$ $= \left(\frac{-2+8}{2}\,;\frac{0+10}{2}\right)$ $= (3\ ;\ 5)$ T symmetrical about BM/*T is simmetries om BM* $\therefore$ distance of T to BM = 5 units = distance from BM to C $\therefore$ C(13 ; 5) **OR/*OF***	✓T(3 ; 5) ✓value of x ✓ value of y (3)

	$m_{DF} = \frac{3\frac{1}{3} - 0}{8 - (-2)} = \frac{1}{3}$ <u>Equation of DF:</u> $y - y_1 = m(x - x_1)$ $y - 0 = \frac{1}{3}(x + 2)$ $y = \frac{1}{3}x + \frac{2}{3}$ <u>Equation of BC:</u> $y = -x + 18$ $\frac{1}{3}x + \frac{2}{3} = -x + 18$ $4x = 52$ $x = 13$ $\therefore y = -13 + 18 = 5$ $\therefore$ C(13 ; 5)	✓eq of DF ✓value of x ✓ value of y (3)
3.6	area/*opp* ΔBDF = area/*opp* ΔBDM – area/*opp* ΔDFM $= \frac{1}{2}(10)(10) - \frac{1}{2}(10)\left(\frac{10}{3}\right)$ $= \frac{100}{3}$ or $33\frac{1}{3}$ or 33,3 square units/*vk eenh* **OR/*OF*** area/*opp* ΔBDF $= \frac{1}{2}$.BF.DM $= \frac{1}{2}\left(\frac{20}{3}\right)(10)$ $= \frac{100}{3}$ or $33\frac{1}{3}$ or 33,3 square units/*vk eenh* **OR/*OF***	✓ formula/method ✓ 10 (DM) ✓ 10 (BM) ✓ $\frac{10}{3}$ or $3\frac{1}{3}$ ($\perp$h) ✓ answer (5) ✓ formula/method ✓ BF ✓✓ DM ✓ answer (5)

	$\tan F\hat{D}M = m_{DC} = \frac{5-0}{13+2} = \frac{1}{3}$ or $\tan F\hat{D}M = \frac{FM}{DM} = \frac{\frac{10}{3}}{10} = \frac{1}{3}$	✓ gradient/ratio
	$F\hat{D}M = 18{,}43°$ $\therefore B\hat{F}D = 108{,}43°$ [ext ∠ Δ]	✓ $B\hat{F}D$
	$BF = \frac{20}{3}$ or $6\frac{2}{3}$ $DF^2 = (10)^2 + \left(3\frac{1}{3}\right)^2$ [Pyth ΔDFM] $DF = 10{,}54$ or $\frac{\sqrt{1000}}{3}$ or $\frac{10\sqrt{10}}{3}$ $BD = \sqrt{(10-0)^2 + (8+2)^2}$ $= \sqrt{200}$ *or* $10\sqrt{2}$	✓ DF
	$\therefore$ area/*opp* $\Delta BDF = \frac{1}{2}.BF.FD.\sin B\hat{F}D$ $= \frac{1}{2}\left(\frac{20}{3}\right)\left(\frac{10\sqrt{10}}{3}\right)(\sin 108{,}43)$ $= \frac{100}{3}$ or $33\frac{1}{3}$ or 33,33 square units/*vk eenh*	✓correct substitution into area rule ✓ answer (5)
	OR/*OF*	
	$BF = \frac{20}{3}$ or $6\frac{2}{3}$ $BD = \sqrt{(10-0)^2 + (8+2)^2}$ $= \sqrt{200}$ *or* $10\sqrt{2}$ area/*opp* $\Delta BDF = \frac{1}{2}.BF.BD.\sin D\hat{B}F$ $= \frac{1}{2}\left(\frac{20}{3}\right)\left\langle\sqrt{200}\right\rangle(\sin 45°)$ $= \frac{100}{3}$ or $33\frac{1}{3}$ or 33,33 square units/*vk eenh*	✓ BF ✓ BD ✓ formula/method ✓correct substitution into area rule ✓ answer (5)
	OR/*OF* area/*opp* ΔBDF = area/*opp* ΔBCD – area/*opp* ΔBCF $= \frac{1}{2}\left(10\sqrt{2}\right)\left(5\sqrt{2}\right) - \frac{1}{2}\left(\frac{20}{3}\right)(5)$ $= \frac{100}{3}$ or $33\frac{1}{3}$ or 33,33 square units/*vk eenh*	✓ formula/method ✓ BD = $10\sqrt{2}$ ✓ BC = $5\sqrt{2}$ ✓ BF = $\frac{20}{3}$ ✓ answer (5)
	OR/*OF*	

	$\tan F\hat{D}M = m_{DC} = \frac{5-0}{13+2} = \frac{1}{3}$ or $\tan F\hat{D}M = \frac{FM}{DM} = \frac{\frac{10}{3}}{10} = \frac{1}{3}$ $F\hat{D}M = 18{,}43°$ $B\hat{D}F = 26{,}56°$ area / *opp* ΔBDF $= \frac{1}{2}.BD.DF.\sin B\hat{D}F$ $= \frac{1}{2}.(10\sqrt{2}).\left(\frac{10\sqrt{10}}{3}\right).\sin 26{,}56°$ $= \frac{100}{3}$ or $33\frac{1}{3}$ or 33,33 square units/*vk eenh*	✓ gradient/ratio ✓ $B\hat{D}F$ ✓ DF **OR/*OF*** BD ✓correct substitution into area rule ✓ answer (5) **[18]**

QUESTION/*VRAAG* 4



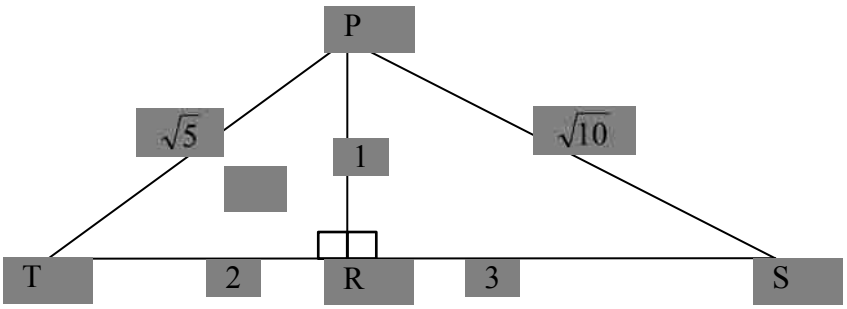
4.1	radius ⊥ tangent /*raaklyn*	✓ R (1)
4.2	$CR^2 = TR^2 + CT^2$ (Pyth) $CR^2 = 20^2 + 10^2 = 500$ $CR = \sqrt{500}$ or $10\sqrt{5}$	✓ substitution ✓ answer (2)
4.3	$CR^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ $500 = (k-3)^2 + (21+1)^2$ $k^2 - 6k + 9 + 484 = 500$ $k^2 - 6k - 7 = 0$ $(k-7)(k+1) = 0$ $k = 7$ or $k \neq -1$ **OR/*OF*** $CR^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ $500 = (k-3)^2 + (21+1)^2$ $(k-3)^2 = 16$ $k - 3 = 4$ or $k - 3 = -4$ $k = 7$ or $k \neq -1$	✓ substitution ✓ standard form ✓ factors ✓ $k = 7$ (4) ✓ substitution ✓ square form ✓ square root ✓ $k = 7$ (4)

4.4	$(x-3)^2+(y+1)^2=100$	✓✓ answer (2)
4.5	CS = 10 and CS $\perp$ PS $\therefore S(3;-11)$ $\therefore y=-11$	✓ $S(3;-11)$ ✓ answer (2)
4.6.1	S(3;–11) $\therefore 3(-11)-4x=35$ $x=-17$ $\therefore P(-17;-11)$ **OR/*OF*** $\frac{4}{3}x+\frac{35}{3}=-11$ $\frac{4}{3}x=\frac{-68}{3}$ $x=-17$ $P(-17;-11)$	✓ substituting ✓ answer (2) ✓ equating ✓ answer (2)
4.6.2	PT = PS [tangents from common point/*rklyne vanaf dies pt*] = 17 + 3 = 20 units **OR** $PC=\sqrt{(-17-3)^2+(-11+1)^2}$ $=\sqrt{500}$ *or* $10\sqrt{5}$ $PT^2=PC^2-TC^2$ [Pyth th] = 500 – 100 = 400 $\therefore$ PT = 20 **OR** $PC=\sqrt{(-17-3)^2+(-11+1)^2}$ $=\sqrt{500}$ *or* $10\sqrt{5}$ $\Delta PTC \equiv \Delta RTC$ [90°HS] $\therefore$ PT = TR $\therefore$ PT = 20	✓ S ✓R ✓ answer (3) ✓ value of PC ✓ using Pyth ✓ answer (3) ✓ value of PC ✓ S/R or proved ✓ answer (3)
4.7.1	M(3;–16)	✓answer (1)

4.7.2	Radius = 4	✓ answer (1)
4.7.3	$r_1 + r_2 = 10 + 4 = 14$ distance CM $= \sqrt{(3-3)^2 + (-1+16)^2}$ $= \sqrt{225}$ $= 15$ CM > $r_1 + r_2$ Therefore the two circles do not intersect or touch./*Daarom sny of raak die twee sirkels nie.*	✓ $r_1 + r_2$ ✓ 15 ✓explanation (3) **[21]**

QUESTION/*VRAAG* 5

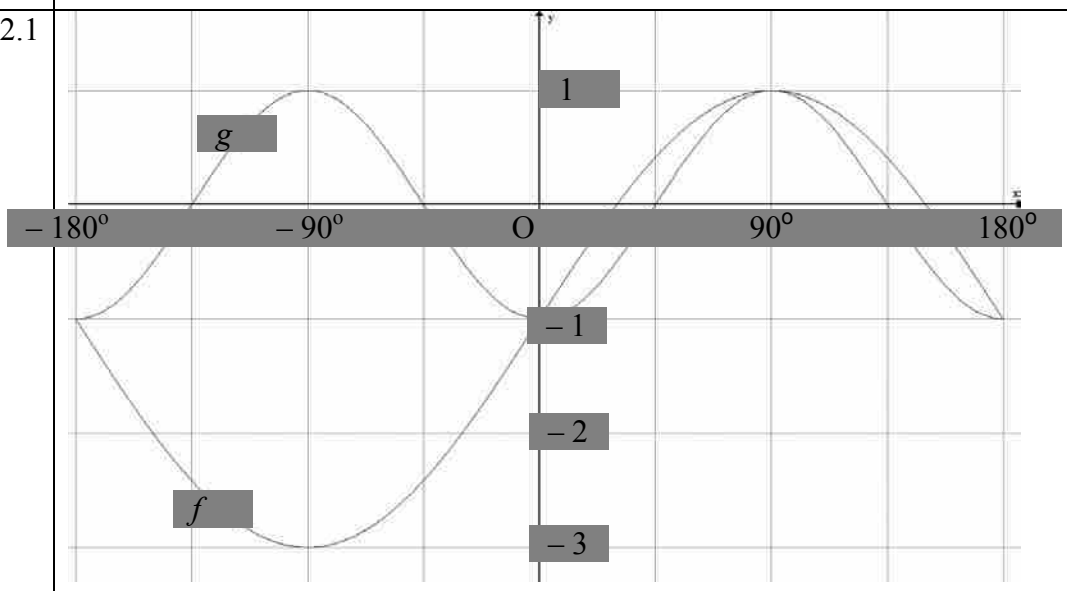
5.1



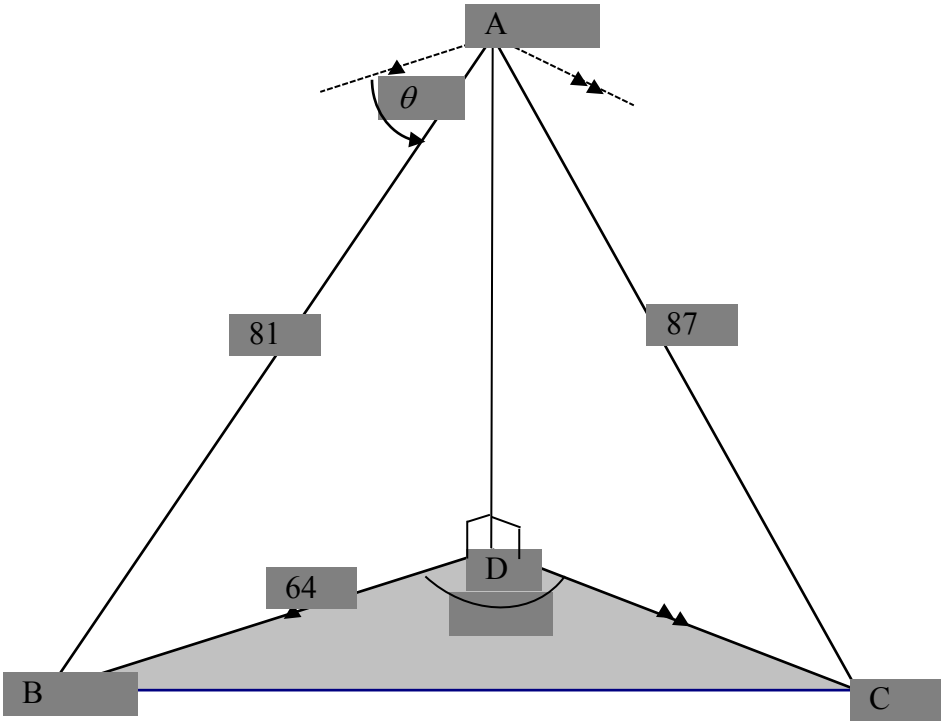
5.1.1(a)	$\sin T = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} = 0{,}45$	✓value (1)
5.1.1(b)	$\cos S = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10} = 0{,}95$	✓value (1)
5.1.2	$\cos(T+S) = \cos T\cos S - \sin T\sin S$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{3}{\sqrt{10}}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{10}}\right)$ $= \frac{6}{\sqrt{50}} - \frac{1}{\sqrt{50}}$ $= \frac{5}{\sqrt{50}}$ *or* $\frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$	✓expansion ✓ $\frac{2}{\sqrt{5}}$ ✓ $\frac{1}{\sqrt{10}}$ ✓ simplification ✓ answer (5)
5.2	$\frac{1}{\cos(360° - \theta)\sin(90° - \theta)} - \tan^2(180° + \theta)$ $= \frac{1}{(\cos\theta)(\cos\theta)} - \tan^2\theta$ $= \frac{1}{\cos^2\theta} - \left(\frac{\sin^2\theta}{\cos^2\theta}\right)$ $= \frac{1-\sin^2\theta}{\cos^2\theta}$ $= \frac{\cos^2\theta}{\cos^2\theta}$ *OR* $\frac{1-\sin^2\theta}{1-\sin^2\theta}$ $= 1$	✓ $\cos\theta$ ✓ $\cos\theta$ ✓ $\tan^2\theta$ ✓ $\frac{\sin^2\theta}{\cos^2\theta}$ ✓identity ✓ answer (6)

5.3	$(\sin x - \cos x)^2 = \left(\frac{3}{4}\right)^2$ $\sin^2 x - 2\sin x\cos x + \cos^2 x = \frac{9}{16}$ $1 - 2\sin x\cos x = \frac{9}{16}$ $2\sin x\cos x = \frac{7}{16}$ $\therefore \sin 2x = \frac{7}{16}$	✓ squaring both sides ✓expanding LHS ✓ using identity ✓ simplifying ✓answer (5) **[18]**

QUESTION/*VRAAG* 6

6.1	$4\sin x + 2\cos 2x = 2$ $2\sin x + \cos 2x - 1 = 0$ $2\sin x + (1 - 2\sin^2 x) - 1 = 0$ $2\sin^2 x - 2\sin x = 0$ $2\sin x(\sin x - 1) = 0$ $2\sin x = 0$ or $\sin x - 1 = 0$ $\sin x = 0$ $\sin x = 1$ $x = k.180°$ or $x = 90° + k.360, k \in Z$	✓using identity ✓ standard form ✓ factors ✓ $\sin x = 0$ or $\sin x = 1$ ✓ $k.180°$ ✓ $90° + k.360, k \in Z$ (6)
6.2.1		✓ turning point (–90° ; –3) ✓ turning point (90° ; 1) ✓ (–180° ; –1) & (0° ; –1) (3)
6.2.2	$(-90°; 0°)$ **OR/*OF*** $-90° < x < 0°$	✓ ✓ answer (2) ✓ ✓ answer (2)
6.2.3	$f(x) = g(x)$ $\therefore -180°; 0°; 90°; 180°$ $f(x + 30°) = g(x + 30°)$ $\therefore x = -30°; 60°; 150°$	✓ any ONE correct ✓ other 2 correct (2) **[13]**

QUESTION/*VRAAG* 7

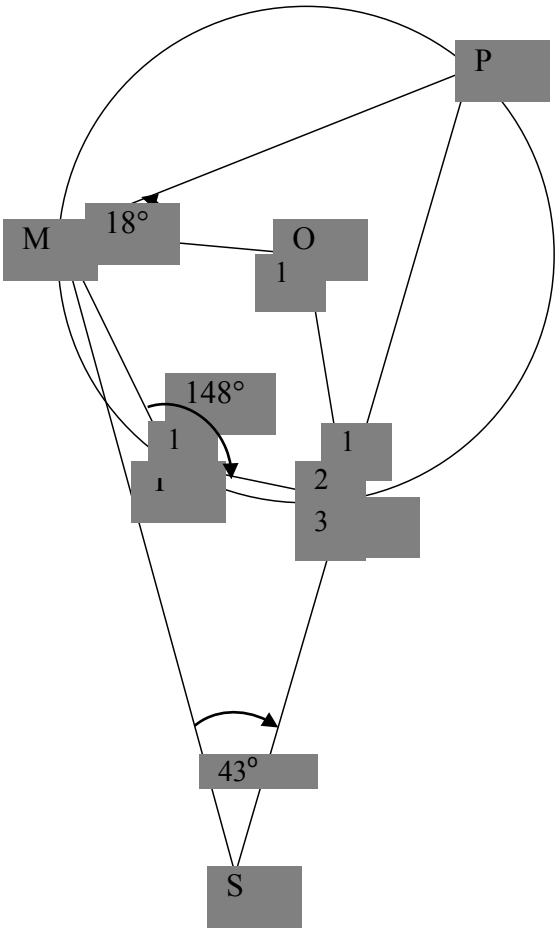


7.1	$A\hat{B}D = \theta$ [alternate $\angle$ s; $\parallel$ lines] $\cos\theta = \frac{BD}{AB} = \frac{64}{81}$ $\theta = 38°$ OR/***OF*** $\sin B\hat{A}D = \frac{64}{81}$ $B\hat{A}D = 52{,}18°$ $\theta = 38°$	✓correct trig ratio ✓substitution into correct ratio ✓ answer (to the nearest degree) (3) ✓correct trig ratio ✓substitution into correct ratio ✓ answer (to the nearest degree) (3)
7.2	$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos B\hat{A}C$ $= 81^2 + 87^2 - 2(81)(87)\cos 82{,}6°$ $= 12314{,}754...$ $BC = 110{,}97\text{ m}$	✓use cosine rule ✓correct substitution into cosine rule ✓answer (3)

7.3	$\frac{\sin D\hat{C}B}{BD} = \frac{\sin B\hat{D}C}{BC}$ $\sin D\hat{C}B = \frac{BD.\sin B\hat{D}C}{BC}$ $\sin D\hat{C}B = \frac{64.\sin 110°}{110,97}$ $\therefore D\hat{C}B = 32,82°$	✓ use sine rule ✓ substitution ✓ answer (3) **[9]**

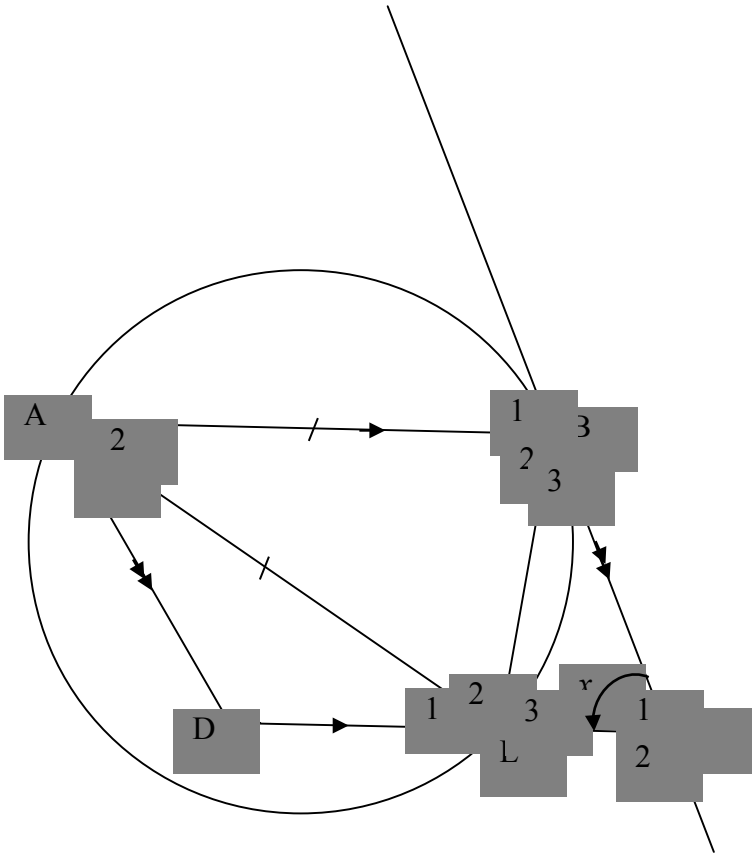
QUESTION/*VRAAG* 8

8.1



8.1.1	$\hat{P} = 32°$ [opp ∠s of cyclic quad/*teenoorst∠e v koordevh*]	✓ S ✓ R (2)
8.1.2	$\hat{O}_1 = 2(32°) = 64°$ [∠ centre = 2 ∠ at circum/*midpts∠ = 2 omtreks∠*] **OR/*OF*** reflex $\hat{O} = 296°$ [∠ centre = 2 ∠ at circum/*midpts∠ = 2 omtreks∠*] $\hat{O}_1 = 64°$ [∠s around a point/*∠e om 'n punt*]	✓ S ✓R (2) ✓ S and R ✓ S (2)
8.1.3	$O\hat{M}S = 180° - (32° + 18° + 43°)$ [sum ∠s Δ / *som ∠e Δ*] $= 87°$	✓ S ✓ S (2)
8.1.4	$\hat{R}_3 = T\hat{M}P$ [ext ∠ cyclic quad/*buite ∠ koordevh*] $= 87° + 18° - 6°$ $= 99°$	✓ R ✓ S (2)

8.2

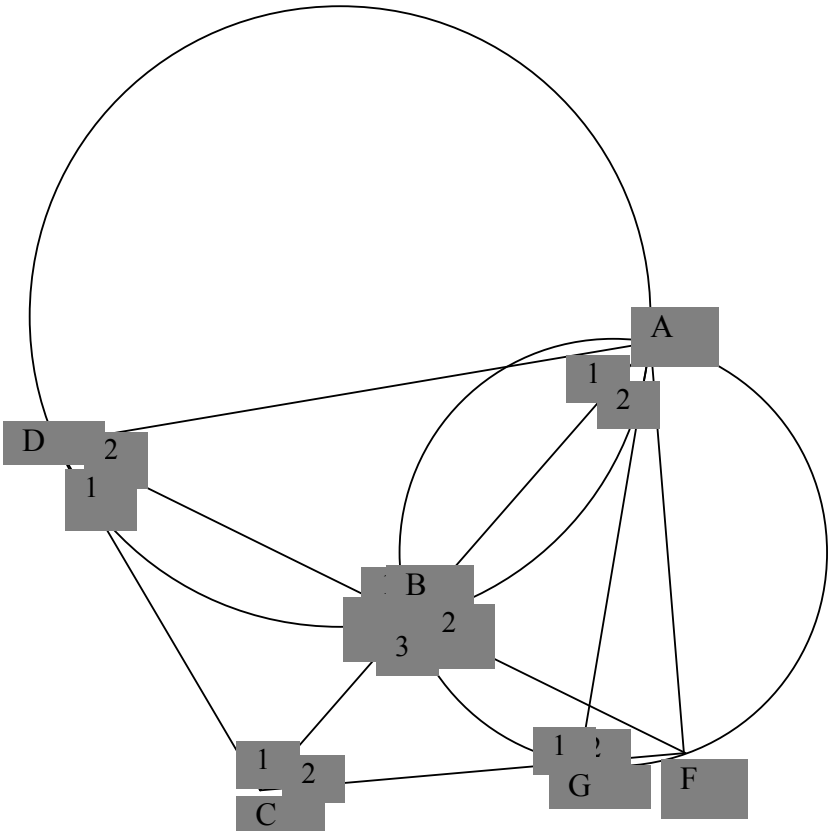


8.2.1	corres ∠s/*ooreenk ∠e*; AB||DC	✓ R (1)
8.2.2	$\hat{E}_2 = x$ [tan - chord theorem/*raakl – koordst*] $\hat{B}_2 = x$ [∠s opp = sides/*∠e teenoor = sye*] $\hat{E}_3 = x$ [alt ∠s/*verwiss ∠e*; AB||DC] $D\hat{A}B = x$ [opp ∠s $\|^m$ /*teenoor ∠e* $\|^m$ **OR/*OF*** alternate/*verwiss* ∠s/*e*; BC ‖AD] Any 3 ∠s correct	✓S ✓R ✓ S ✓ R ✓ S ✓ R (6)
8.2.3	$\hat{D} = 180° - x$ [co - int ∠s suppl/*ko – binne∠e* suppl; AD||BC] $\therefore \hat{B}_2 + \hat{D} = 180°$ ∴ ABED a cyc quad/*kdvh* [converse opp ∠s of cyclic quad/ *omgek teenoorst ∠e koordevh*] **OR/*OF*** $D\hat{A}B = x$ [opp ∠s/*teenoor ∠e* |^m] OR/*OF* [alt∠s/*verwiss ∠e*; BC||AD] $\hat{E}_3 = D\hat{A}B = x$ ∴ ABED a cyc quad/*kdvh* [converse ext ∠ of cyc quad/*omgek buite∠ v koordevh*]	✓S ✓ R ✓ R (3) ✓S ✓ R ✓ R (3) **[18]**

QUESTION/*VRAAG* 9

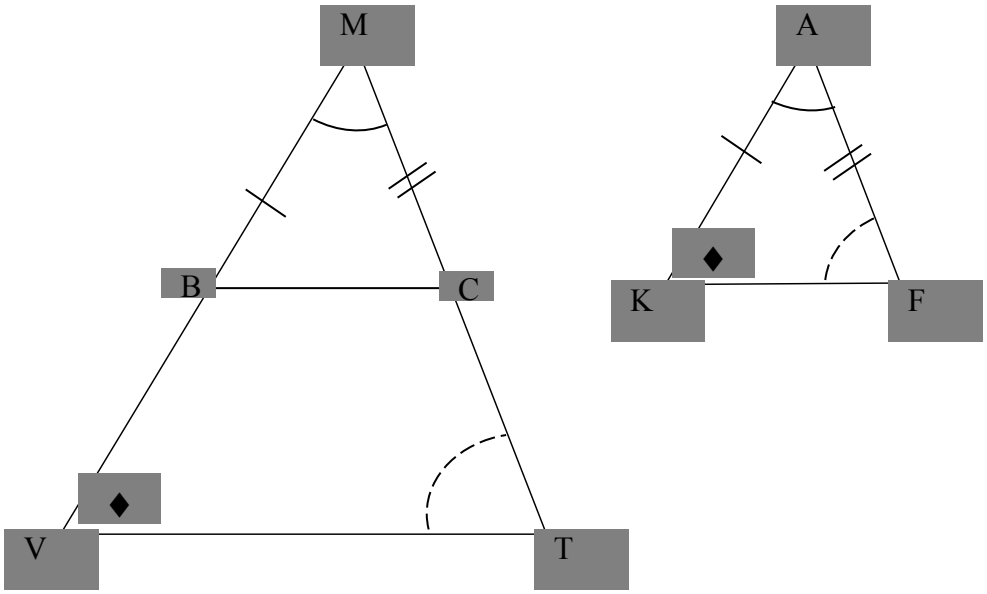
9.1	… in the alternate segment/…*in die(teen)oorstaande segment*	✓ answer (1)

9.2



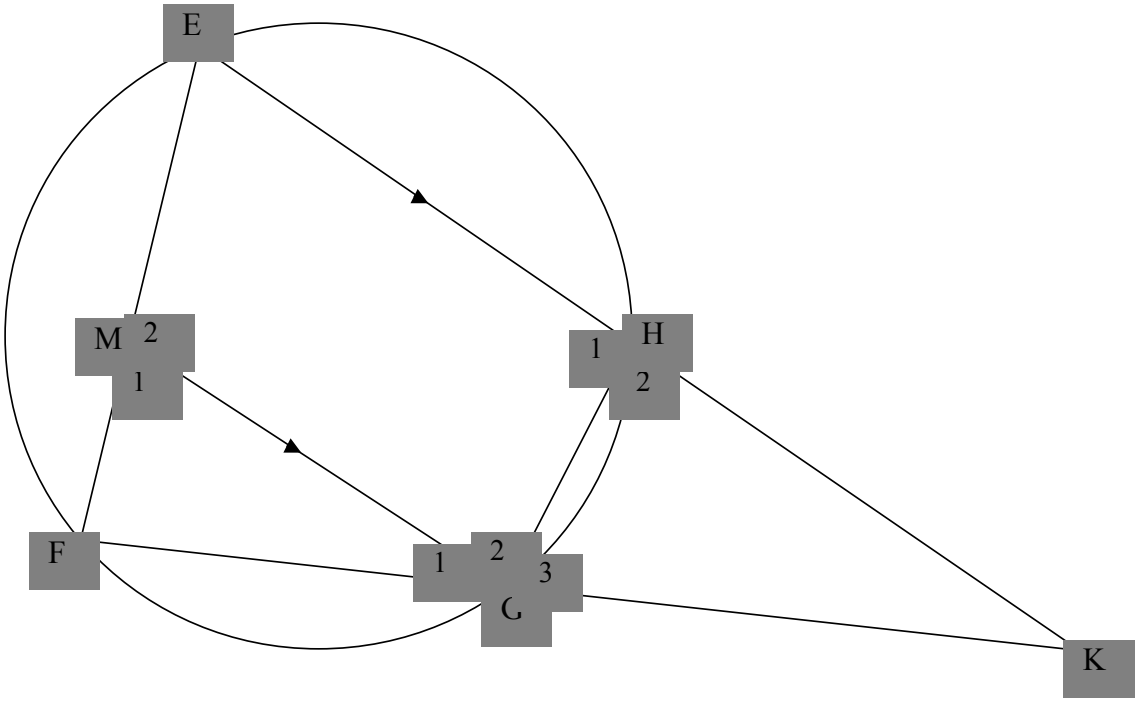
9.2.1	$\hat{A}_1 = \hat{D}_1$ [tan chord theorem/*raakl – koordst*] $\hat{B}_4 = \hat{A}_1 + \hat{D}_2$ [ext ∠ Δ/*buite*∠ Δ] $= \hat{D}_1 + \hat{D}_2$	✓S ✓ R ✓ S ✓ R (4)
9.2.2	$\hat{B}_4 = \hat{B}_2$ [vert opp ∠s/*regoorst ∠e*] $\hat{D}_1 + \hat{D}_2 = \hat{B}_2$ [proven/*bewys*] $= \hat{G}_2$ [∠s in same segment/∠e *in dies segment*] ∴ AGCD is cyc quad/*kvh* [converse ext∠ cyc quad/*omgek buite∠ kvh*]	✓ S ✓ S ✓ R ✓ R (4)
9.2.3	$\hat{D}_1 = \hat{A}_2$ [∠s in same segment/∠*e in dies segment*] $\hat{A}_2 = \hat{F}$ [∠s in same segment/∠*e in dies segment*] $\therefore \hat{D}_1 = \hat{F}$ ∴ DC = CF [sides opp = ∠s/*sye teenoor* = ∠*e*]	✓ S ✓ R ✓ S ✓ R (4) **[13]**

QUESTION/*VRAAG* 10



10.1	Constr/*Konstr* : Draw line BC such that MB = AK and MC = AF *Trek lyn BC sodat MB = AK en MC = AF* Proof/*Bewys* : In ΔBMC and/*en* ΔKAF MB = AK [constr/*konstr*] $\hat{M} = \hat{A}$ [given/*gegee*] MC = AF [constr/*konstr*] ΔBMC ≡ ΔKAF [s∠s] ∴ $M\hat{B}C = A\hat{K}F$ or $M\hat{C}B = A\hat{F}K$ [≡Δ] but /*maar* $\hat{V} = \hat{K}$ or $\hat{T} = \hat{F}$ [given/*gegee*] ∴ $M\hat{B}C = \hat{V}$ or $M\hat{C}B = \hat{T}$ But these are corresponding ∠s/*maar hulle is ooreenk ∠e* ∴ BC || VT [corr ∠s = /*ooreenk∠e* =] ∴ $\frac{MV}{MB} = \frac{MT}{MC}$ [prop theorem/*eweredighst*; BC || VT] but /*maar* MB = AK and MC = AF [constr/*konstr*] ∴ $\frac{MV}{AK} = \frac{MT}{AF}$	✓ constr/*konstr* ✓ S / R ✓ S ✓ S ✓ S / R ✓S ✓R (7)

10.2



10.2.1(a)	In ΔKGH and ΔKEF $\hat{K}$ is common/*gemeen* $\hat{H}_2 = \hat{F}$ [ext $\angle$ cyclic quad/*buite* $\angle$ *koordevh*] $\hat{G}_3 = \hat{E}$ [sum$\angle$s Δ OR ext $\angle$ cyclic quad/*som*$\angle$*e*Δ *OR buite*$\angle$*koordevh*] $\therefore \Delta KGH \parallel\parallel\parallel \Delta KEF$ [$\angle\angle\angle$]	✓ S ✓S ✓R ✓ naming third angle OR $\angle\angle\angle$ (4)
10.2.1(b)	$\frac{EF}{GH} = \frac{KE}{KG}$ [$\parallel\parallel\parallel \Delta$s] $\therefore \frac{EF}{GH} = \frac{KE}{EF}$ [$KG = EF$] $\therefore EF^2 = KE.GH$	✓ S ✓ S (2)
10.2.1(c)	$\frac{KG}{KF} = \frac{EM}{EF}$ [prop theorem/*eweredighst*; $MG \parallel EK$] but $EF = KG$ [given/*gegee*] $\frac{KG}{KF} = \frac{EM}{KG}$ $KG^2 = EM.KF$	✓ S ✓ R ✓ S (3)
10.2.2	$KE.GH = EM.KF$ $EM = \frac{20 \times 4}{16}$ $= 5$ units	✓ $KE.GH = EM.KF$ ✓ substitution ✓ answer (3) **[19]**

TOTAL/*TOTAAL*: 150

PolyMathic

Vraestel 9

Okt/Nov

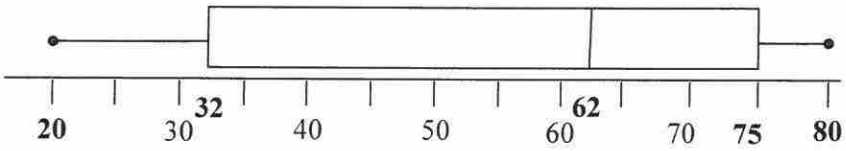
Eksamen

PolyMathic

Graad 12 EindEksamen
Totaal: 150
Tyd: 3ure

VRAAG 1

Die mond-en-snordiagram hieronder toon die punte (uit 80) wat 'n klas van nege leerders in 'n Geskiedenistoets behaal het.



1.1 Lewer kommentaar op die skeefheid van die data. (1)

1.2 Skryf die omvang (variasiewydte) neer van die punte wat behaal is. (2)

1.3 Indien die leerders 32 punte moes behaal het om die toets te slaag, beraam watter persentasie van die klas die toets gedruip het. (2)

1.4 In stygende volgorde is die tweede punt 28, die derde punt 36 en die sesde punt 69. Die sewende en agste punte is dieselfde. Die gemiddelde punt vir hierdie toets is 54.

	28	36			69			

Vul die punte van die oorblywende leerders in stygende volgorde in. (6)
[11]

VRAAG 2

'n Maatskappy het die getal boodskappe aangeteken wat oor 'n tydperk van 60 werksdae per e-pos gestuur is. Die data word in die tabel hieronder getoon.

GETAL BOODSKAPPE	GETAL DAE
$10 < x \leq 20$	2
$20 < x \leq 30$	8
$30 < x \leq 40$	5
$40 < x \leq 50$	10
$50 < x \leq 60$	12
$60 < x \leq 70$	18
$70 < x \leq 80$	3
$80 < x \leq 90$	2

2.1 Benader die gemiddelde getal boodskappe wat per dag gestuur is, afgerond tot TWEE desimale plekke. (3)

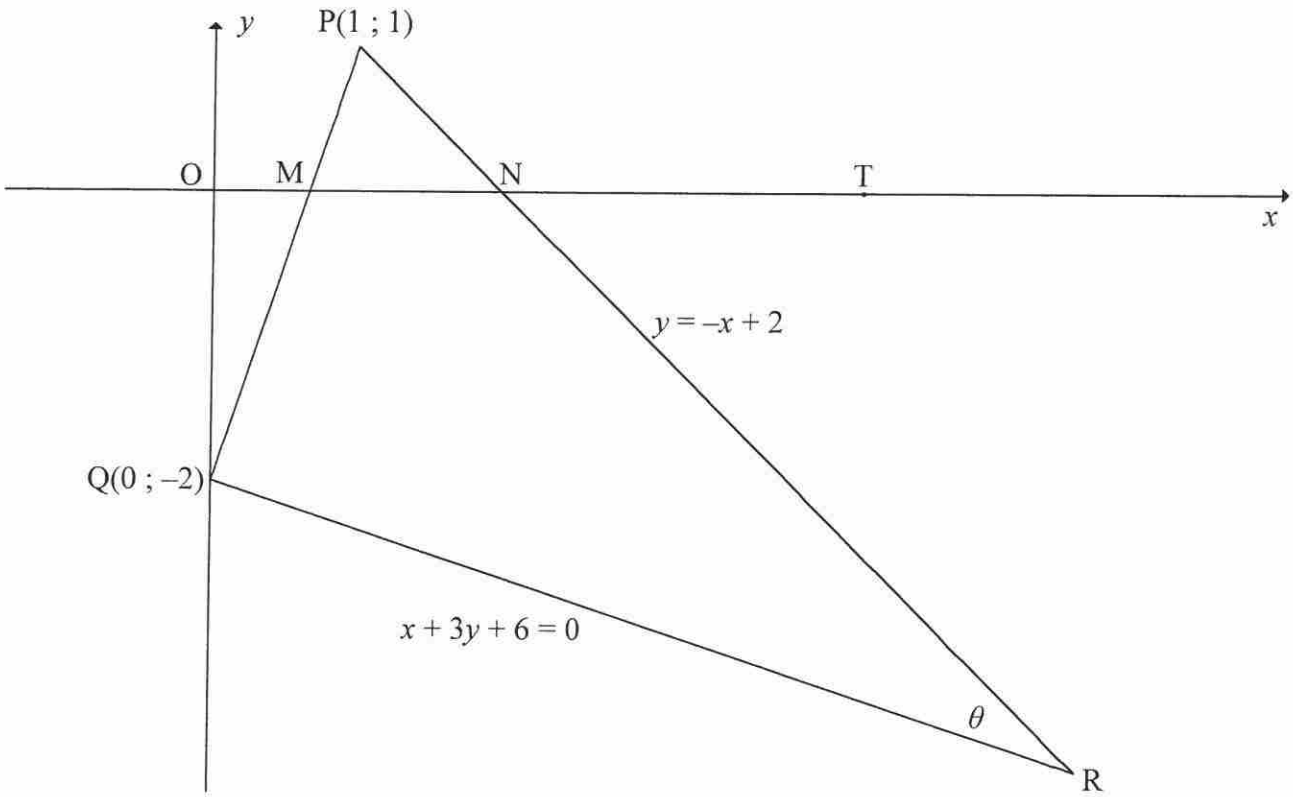
2.2 Teken 'n kumulatiewefrekwensie-grafiek (ogief) van die data op die rooster wat in die ANTWOORDEBOEK verskaf word. (4)

2.3 Benader vervolgens die getal dae waarop 65 of meer boodskappe gestuur is. (2)

[9]

VRAAG 3

In die diagram hieronder is P(1 ; 1), Q(0 ; –2) en R die hoekpunte van 'n driehoek en $P\hat{R}Q = \theta$. Die x-afsnitte van PQ en PR is M en N onderskeidelik. Die vergelykings van die sye PR en QR is $y = -x + 2$ en $x + 3y + 6 = 0$ onderskeidelik. T is 'n punt op die x-as, soos getoon.



3.1 Bepaal die gradiënt van QP. (2)

3.2 Bewys dat $P\hat{Q}R = 90°$. (2)

3.3 Bepaal die koördinate van R. (3)

3.4 Bereken die lengte van PR. Laat jou antwoord in wortelvorm. (2)

3.5 Bepaal die vergelyking van 'n sirkel wat deur P, Q en R gaan in die vorm $(x-a)^2 + (y-b)^2 = r^2$. (6)

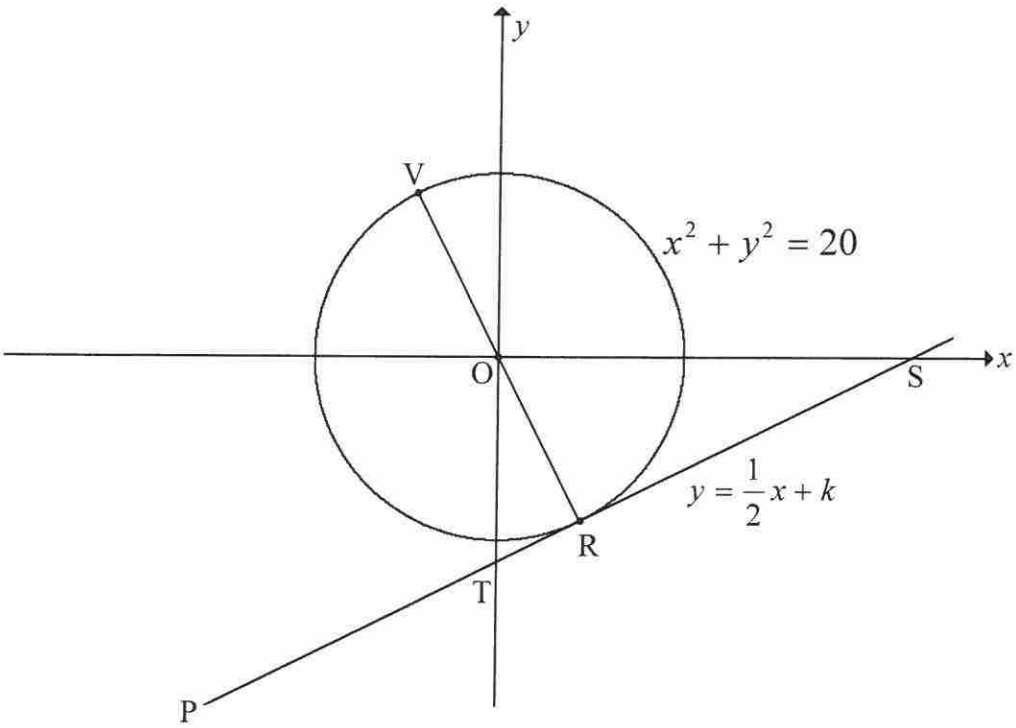
3.6 Bepaal die vergelyking van 'n raaklyn aan die sirkel wat deur P, Q en R by punt P gaan, in die vorm $y = mx + c$. (3)

3.7 Bereken die grootte van θ. (5)

[23]

VRAAG 4

In die diagram hieronder is $x^2 + y^2 = 20$ die vergelyking van die sirkel met middelpunt O. Die raaklyn PRS aan die sirkel by R het die vergelyking $y = \frac{1}{2}x + k$. PRS sny die y-as by T en die x-as by S.



4.1 Bepaal, met redes, die vergelyking van OR in die vorm $y = mx + c$. (3)

4.2 Bepaal die koördinate van R. (4)

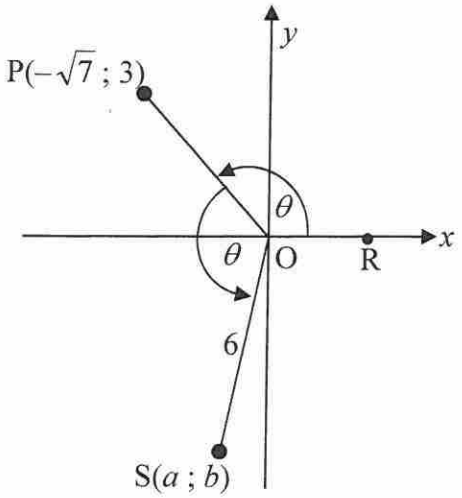
4.3 Bepaal die oppervlakte van ΔOTS, gegee dat R(2 ; –4). (6)

4.4 Bereken die lengte van VT. (4)

[17]

VRAAG 5

5.1 $P(-\sqrt{7}\,;3)$ en $S(a\,;b)$ is punte in die Cartesiese vlak soos in die diagram hieronder getoon. $P\hat{O}R = P\hat{O}S = \theta$ en $OS = 6$.



Bepaal, SONDER die gebruik van 'n sakrekenaar, die waarde van:

5.1.1 $\tan\theta$ (1)

5.1.2 $\sin(-\theta)$ (3)

5.1.3 a (4)

5.2 5.2.1 Vereenvoudig $\dfrac{4\sin x\cos x}{2\sin^2 x-1}$ tot 'n enkele trigonometriese verhouding. (3)

5.2.2 Bereken vervolgens die waarde van $\dfrac{4\sin 15°\cos 15°}{2\sin^2 15°-1}$ SONDER om 'n sakrekenaar te gebruik. (Laat jou antwoord in die eenvoudigste wortelvorm.) (2)

[13]

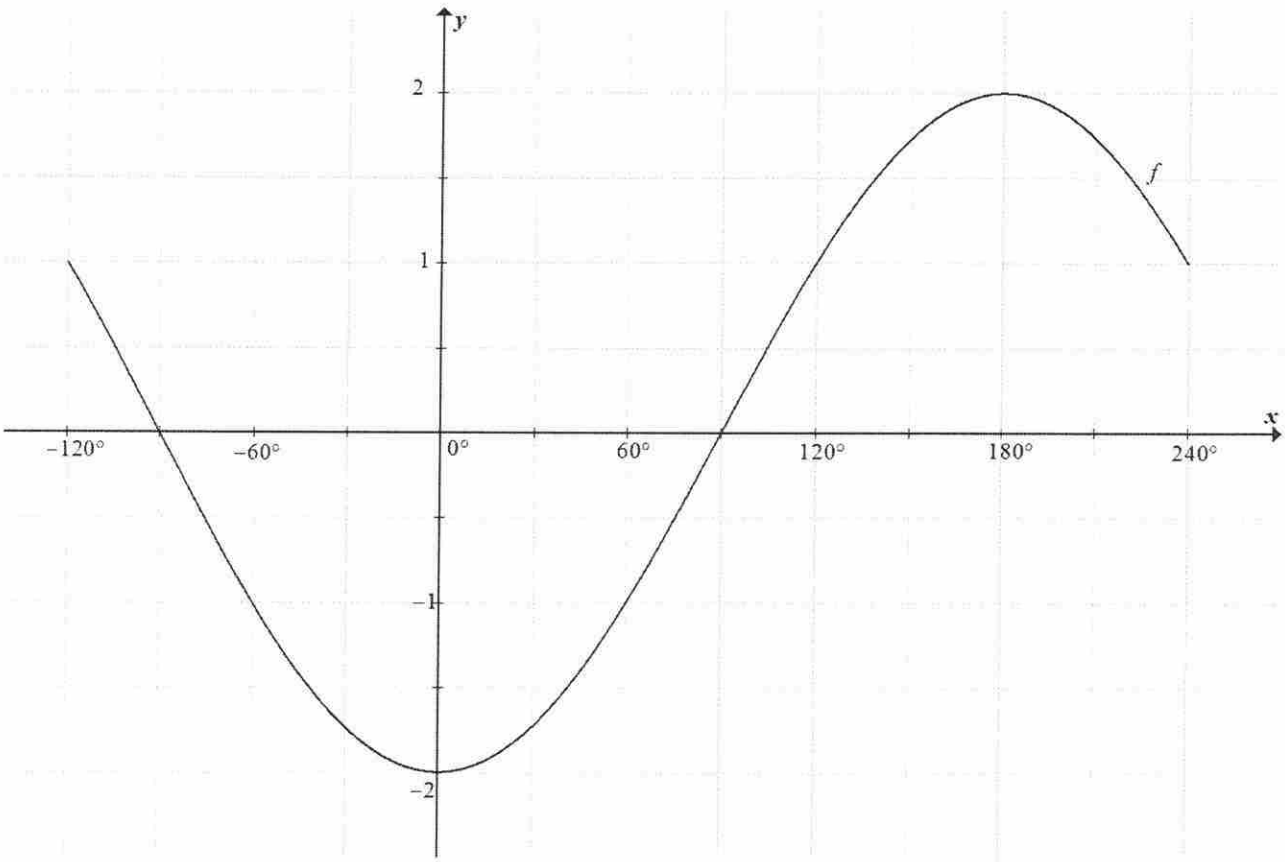
VRAAG 6

Gegee die vergelyking: $\sin(x + 60°) + 2\cos x = 0$

6.1 Toon dat die vergelyking ook as $\tan x = -4 - \sqrt{3}$ geskryf kan word. (4)

6.2 Bepaal die oplossings van die vergelyking $\sin(x + 60°) + 2\cos x = 0$ in die interval $-180° \leq x \leq 180°$. (3)

6.3 In die diagram hieronder is die grafiek van $f(x) = -2\cos x$ vir $-120° \leq x \leq 240°$ geskets.



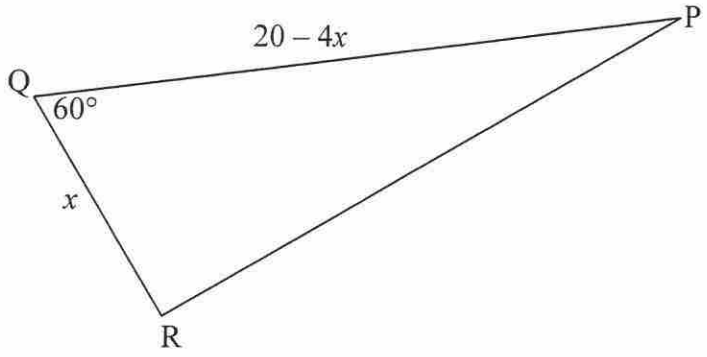
6.3.1 Skets die grafiek van $g(x) = \sin(x + 60°)$ vir $-120° \leq x \leq 240°$ op die rooster wat in die ANTWOORDEBOEK verskaf word. (3)

6.3.2 Bepaal die waardes van x in die interval $-120° \leq x \leq 240°$ waarvoor $\sin(x + 60°) + 2\cos x > 0$. (3)

[13]

VRAAG 7

7.1 In die diagram hieronder is ΔPQR geskets met $PQ = 20 - 4x$, $RQ = x$ en $\hat{Q} = 60°$.

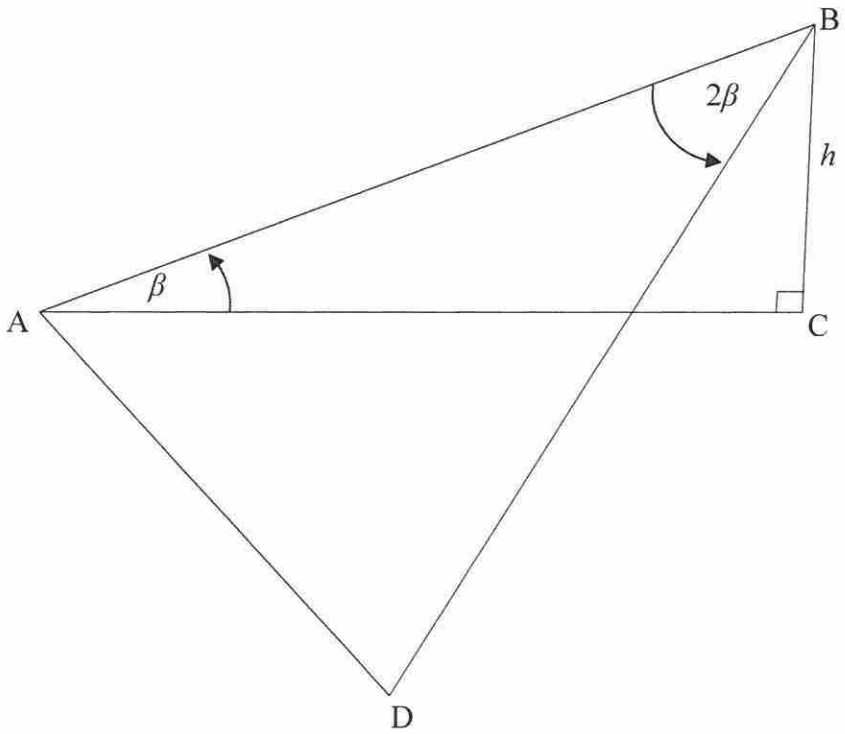


7.1.1 Toon dat die oppervlakte van $\Delta PQR = 5\sqrt{3}x - \sqrt{3}x^2$. (2)

7.1.2 Bepaal die waarde van x waarvoor die oppervlakte van ΔPQR 'n maksimum sal wees. (3)

7.1.3 Bereken die lengte van PR indien die oppervlakte van ΔPQR 'n maksimum is. (3)

7.2 In die diagram hieronder is BC 'n mas wat deur twee kabels by A en D geanker is. A, D en C is in dieselfde horisontale vlak. Die hoogte van die mas is h en die hoogtehoek vanaf A na die bopunt van die mas, B, is β. $A\hat{B}D = 2\beta$ en BA = BD.



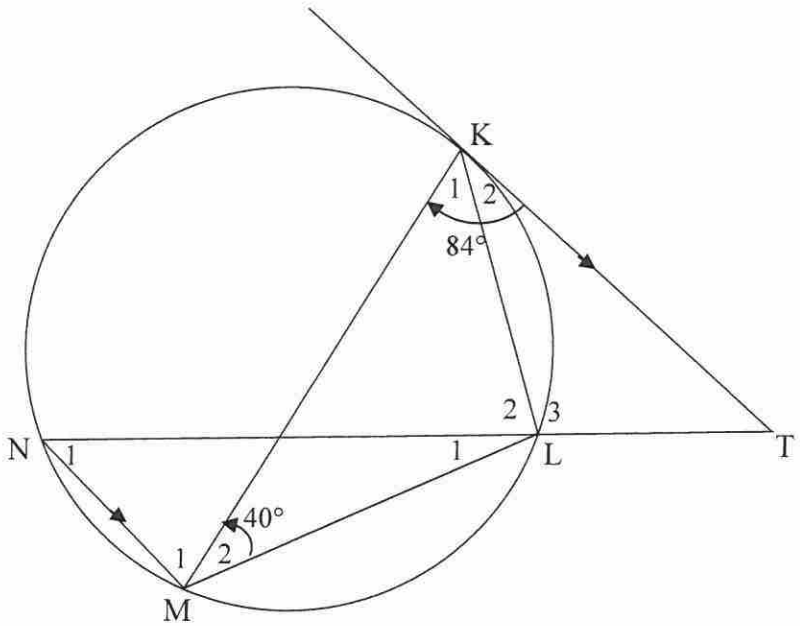
Bepaal die afstand AD tussen die twee ankerpunte in terme van h. (7)

[15]

Gee redes vir ALLE bewerings in VRAAG 8, 9 en 10.

VRAAG 8

8.1 In die diagram hieronder is raaklyn KT aan die sirkel by K ewewydig aan die koord NM. NT sny die sirkel by L. ΔKML is getrek. $\hat{M}_2 = 40°$ en $M\hat{K}T = 84°$.



Bepaal, met redes, die grootte van:

8.1.1 $\hat{K}_2$ (2)

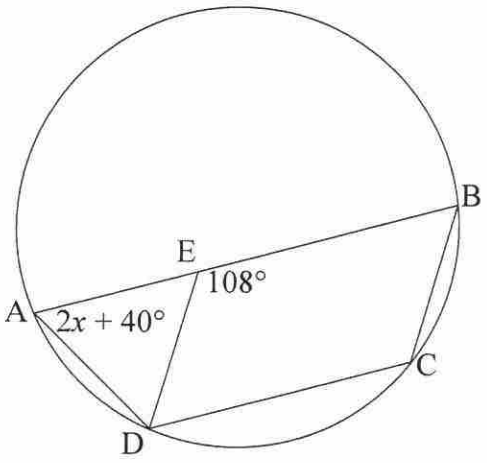
8.1.2 $\hat{N}_1$ (3)

8.1.3 $\hat{T}$ (2)

8.1.4 $\hat{L}_2$ (2)

8.1.5 $\hat{L}_1$ (1)

8.2 In die diagram hieronder is AB en DC koorde van 'n sirkel. E is 'n punt op AB sodat BCDE 'n parallelogram vorm. $D\hat{E}B = 108°$ en $D\hat{A}E = 2x + 40°$.

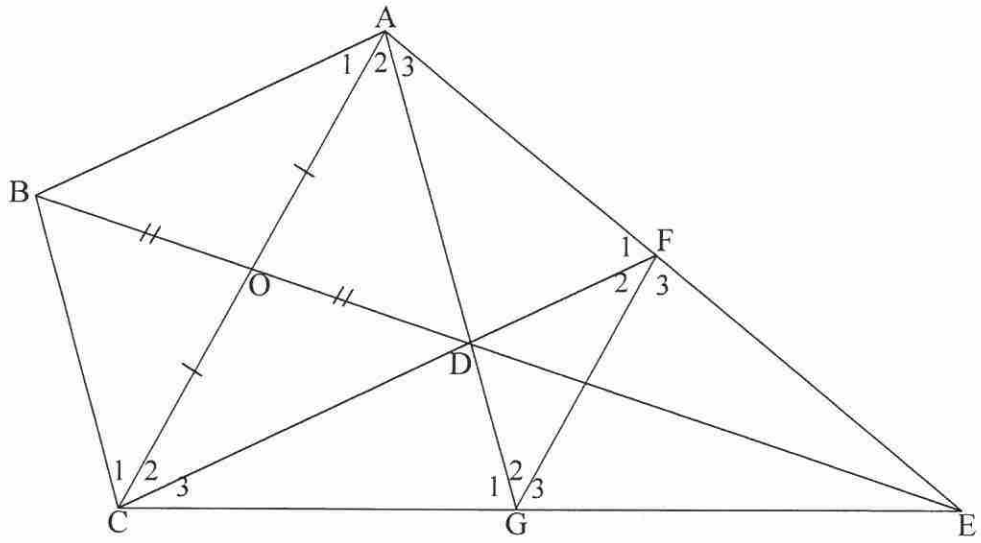


Bereken, met redes, die waarde van x. (5)

[15]

VRAAG 9

In die diagram hieronder halveer EO die sy AC van ΔACE. EDO is verleng na B sodat BO = OD. AD en CD verleng, ontmoet EC en EA by G en F onderskeidelik.



9.1 Gee 'n rede waarom ABCD 'n parallelogram is. (1)

9.2 Skryf neer, met redes, TWEE verhoudings wat elk aan $\frac{ED}{DB}$ gelyk is. (4)

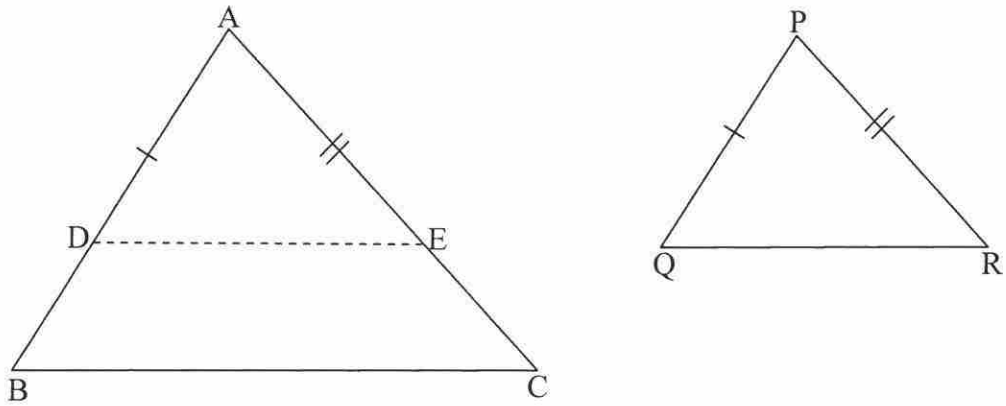
9.3 Bewys dat $\hat{A}_1 = \hat{F}_2$. (5)

9.4 Dit word verder gegee dat ABCD 'n ruit is. Bewys dat ACGF 'n koordevierhoek is. (3)

[13]

VRAAG 10

10.1 In die diagram hieronder word ΔABC en ΔPQR gegee met $\hat{A} = \hat{P}$, $\hat{B} = \hat{Q}$ en $\hat{C} = \hat{R}$.



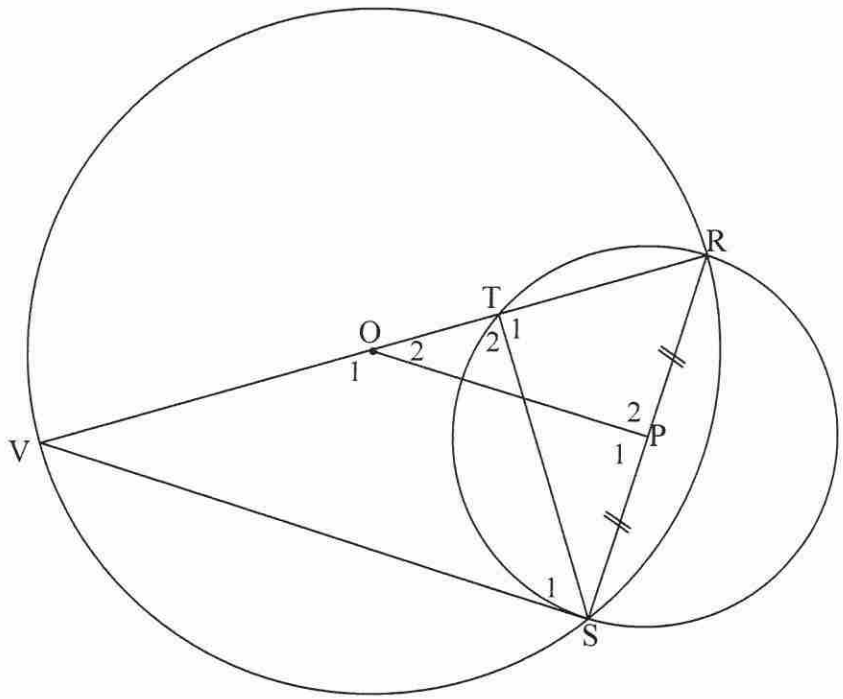
DE word getrek sodat $AD = PQ$ en $AE = PR$.

10.1.1 Bewys dat $\Delta ADE \equiv \Delta PQR$. (2)

10.1.2 Bewys dat $DE \mid\mid BC$. (3)

10.1.3 Bewys vervolgens dat $\frac{AB}{PQ} = \frac{AC}{PR}$. (2)

10.2 In die diagram hieronder is VR 'n middellyn van 'n sirkel met middelpunt O. S is enige punt op die omtrek. P is die middelpunt van RS. Die sirkel met RS as middellyn sny VR by T. ST, OP en SV is geteken.



10.2.1 Waarom is OP ⊥ PS? (1)

10.2.2 Bewys dat ΔROP ||| ΔRVS. (4)

10.2.3 Bewys dat ΔRVS ||| ΔRST. (3)

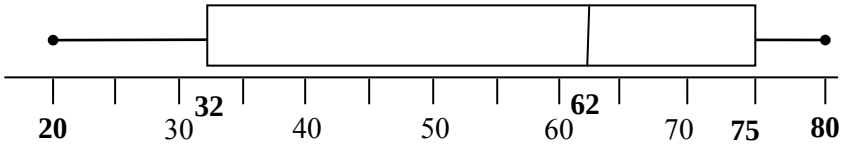
10.2.4 Bewys dat $ST^2 = VT \cdot TR$. (6)

[21]

TOTAAL: 150

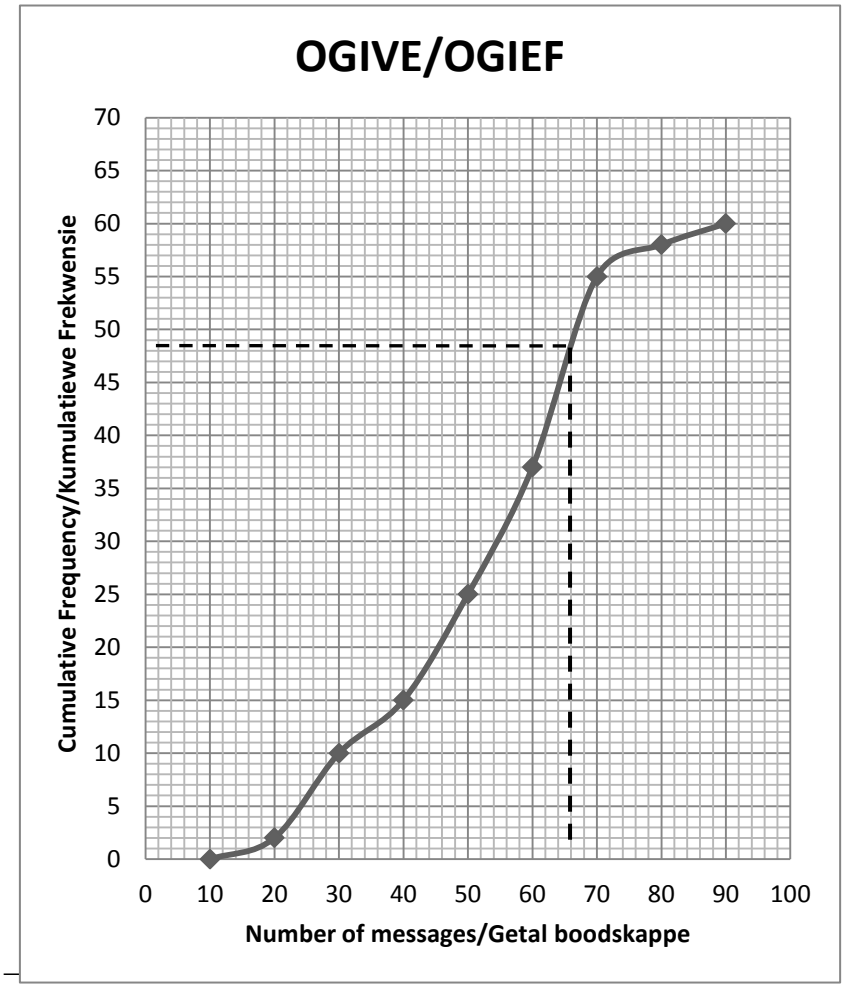
Memo

QUESTION/*VRAAG* 1

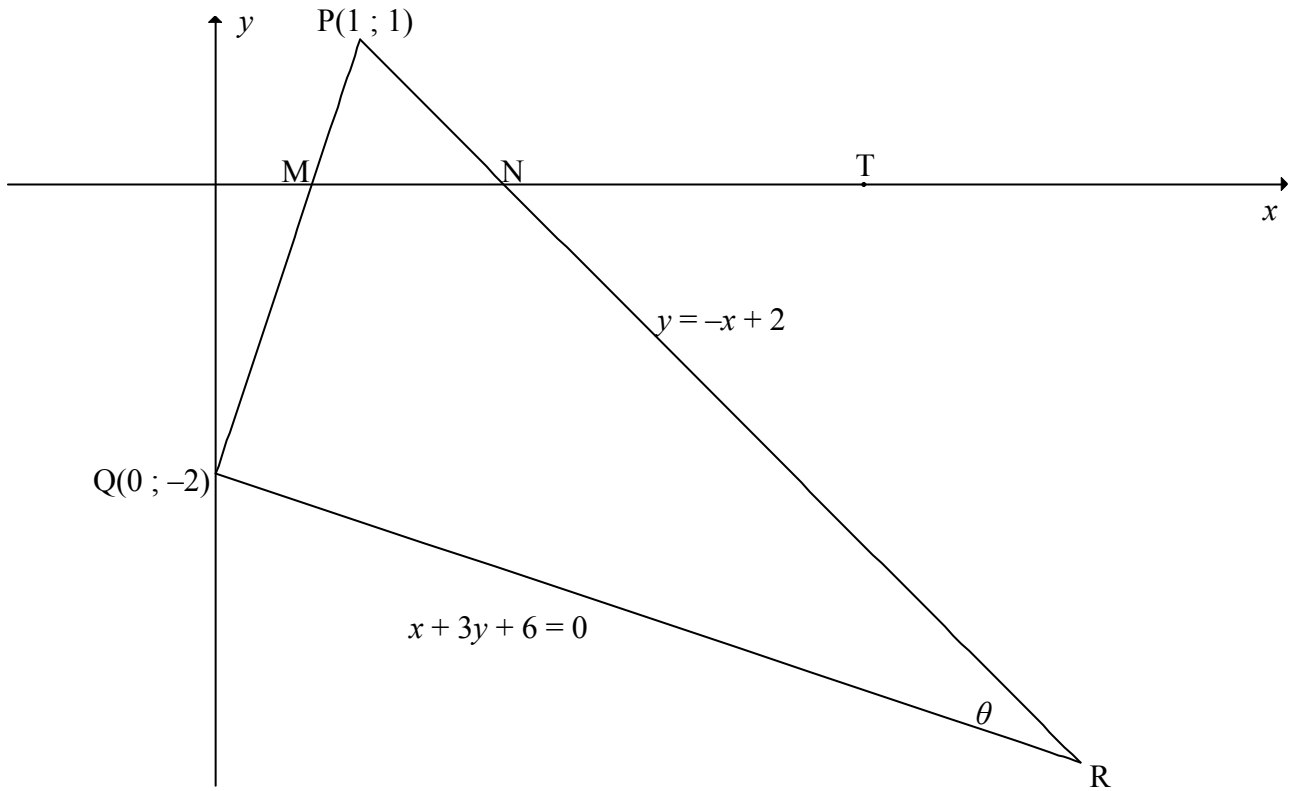


1.1	The data is skewed to the left/*Die data is skeef na links.* **OR/*OF*** The data is negatively skewed/*Die data is negatief skeef.*	✓answ/*antw* ✓answ/*antw* (1)
1.2	Range/*Omvang* = 80 – 20 = 60	✓ max. – min. ✓answ/*antw* (2)
1.3	25% of the learners failed/*van die leerders het gedruip*	✓ ✓answ/*antw* (2)
1.4	$54 = \frac{445 + T_4}{9}$ $T_4 = 41$ **20** 28 36 **41** **62** 69 **75** **75** **80**	✓ 20 ✓✓ 41 ✓ 62 ✓ 75 ✓ 80 (6) **[11]**

QUESTION/*VRAAG* 2

2.1	Mean/*Gemiddelde* = $\frac{2(15)+8(25)+...2(85)}{60} = \frac{3080}{60}$ = 51,33 messages per day/*boodskappe per dag*	✓ 3 080 ✓ $\frac{3080}{60}$ ✓answ/*antw* (3)
2.2		✓grounding at (10 ; 0) ✓plotting at upper limits ✓plotting. cumulative *f* ✓smooth shape of curve ✓*geanker by* (10 ; 0) ✓*stip by boonste limiete* ✓*plot kumulatiewe f* ✓*gladde vorm van kurwe* (4)
2.3	Number of days/*Getal dae* = 60 – 46 (see on graph above/*sien op grafiek hierbo*) = 14 days/*dae* **OR/*OF*** Number of days/*Getal dae* = 2 + 3 + $\frac{1}{2} \times 18$ = 14 days/*dae*	✓46 (accept 45 – 49) ✓answ/*antw* (accept 11 – 15) (2) ✓add correct values/*tel korrekte waardes by* ✓answ/*antw* (2) **[9]**

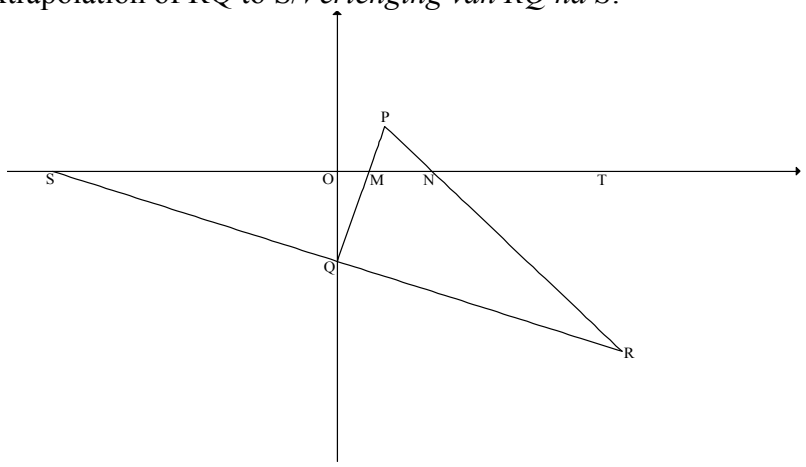
QUESTION/*VRAAG* 3



3.1	$m_{PQ} = \frac{1-(-2)}{1-0}$ $= 3$	✓subst (1 ; 1) & (0 ; –2) ✓answ/*antw* (2)
3.2	QR: $y = -\frac{1}{3}x - 2$ $\therefore m_{QR} = -\frac{1}{3}$ $m_{PQ} \times m_{QR} = 3 \times -\frac{1}{3}$ $= -1$ $\therefore PQ \perp QR \quad \therefore P\hat{Q}R = 90°$	✓ $m_{QR} = -\frac{1}{3}$ ✓ $m_{PQ} \times m_{QR} = -1$ (2)

3.3	$-\frac{1}{3}x-2=-x+2$ $\frac{2}{3}x=4$ $x=6$ $y=-4$ $\therefore R(6\ ;-4)$	✓equating/*gelyk stel* ✓x-value/*waarde* ✓y-value/*waarde* (3)
3.4	$PR=\sqrt{(1-6)^2+(1-(-4))^2}$ $=\sqrt{50}=5\sqrt{2}$ **OR/*OF*** $PR^2=(1-6)^2+(1-(-4))^2$ $=50$ $\therefore PR=\sqrt{50}=5\sqrt{2}$	✓subst into/*in* distance formula/ *afstandsformule* ✓answ/*antw* in surd form/ *wortelvorm* (2) ✓subst into/*in* distance formula/ *afstandsformule* ✓answ/*antw* in surd form/ *wortelvorm* (2)
3.5	PR is a diameter/*'n middellyn* [chord subtends/*kd onderspan* 90°] Centre of circle/*Midpt v sirkel*: $\left(\frac{1+6}{2};\frac{1-4}{2}\right)$ $=\left(3\frac{1}{2};-1\frac{1}{2}\right)$ $r=\frac{\sqrt{50}}{2}$ OR $\frac{5\sqrt{2}}{2}$ OR 3,54 $\therefore\left(x-\frac{7}{2}\right)^2+\left(y+\frac{3}{2}\right)^2=\frac{50}{4}$ OR $\frac{25}{2}$ OR 12,5	✓✓S ✓✓ $\left(3\frac{1}{2};-1\frac{1}{2}\right)$ ✓r-value/*waarde* ✓answ/*antw* (6)
3.6	m of/*van* radius $=-1$ $\therefore m$ of/*van* tangent/*raaklyn* $=1$ Equation of tangent/*Vgl van raaklyn*: $y-y_1=(x-x_1)$ $y-1=x-1$ $\therefore y=x$ **OR/*OF*** $y=x+c$ $1=1+c$ $y=x$	✓m of tang/*rkl* ✓subst m & P(1 ; 1) into/*in* eq of line/*vgl v lyn* ✓answ/*antw* (3)
3.7	$\tan P\hat{N}T=m_{PR}=-1$ $\therefore P\hat{N}T=135°$ $\tan P\hat{M}T=m_{PQ}=3$ $\therefore P\hat{M}T=71{,}57°$ $\hat{P}=63{,}43°$ [ext ∠ of Δ/*buite ∠ v* Δ] $\therefore\theta=26{,}57°$ [sum of ∠s in Δ/*som v ∠e in* Δ] **OR/*OF***	✓ $\tan P\hat{N}T=-1$ ✓ $P\hat{N}T=135°$ ✓ $P\hat{M}T=71{,}57°$ ✓ $\hat{P}=63{,}43°$ ✓answ/*antw* (5)

Extrapolation of RQ to S/*Verlenging van RQ na S*:



$\tan \text{P}\hat{\text{N}}\text{T} = m_{\text{PR}} = -1$

$\therefore \text{S}\hat{\text{N}}\text{R} = 135°$

$\tan \text{N}\hat{\text{S}}\text{R} = m_{\text{RS}} = -\frac{1}{3}$

$\therefore \text{N}\hat{\text{S}}\text{R} = 18{,}43°$

$\theta = 180° - (135° + 18{,}43°)$ [sum of ∠s in Δ/*som v ∠e in* Δ]

$= 26{,}57°$

✓ $\tan \text{P}\hat{\text{N}}\text{T} = -1$
✓ $\text{S}\hat{\text{N}}\text{R} = 135°$
✓ $\tan \text{N}\hat{\text{S}}\text{R} = -\frac{1}{3}$
✓ $\text{N}\hat{\text{S}}\text{R} = 18{,}43°$

✓answ/*antw*
(5)

OR/*OF*

$\text{PQ}^2 = 1^2 + 3^2 = 10$

$\text{PQ} = \sqrt{10}$

$\therefore \sin\theta = \frac{\text{PQ}}{\text{PR}} = \frac{\sqrt{10}}{\sqrt{50}} = \frac{1}{\sqrt{5}}$

$\therefore \theta = 26{,}57°$

✓subst into/*in* distance formula/ *afstandsformule*
✓distance/*afst* PQ
✓correct trig ratio/ *korrekte trig vh*
✓correct trig eq/ *korrekte trig vgl*
✓answ/*antw*
(5)

OR/*OF*

$\text{QR}^2 = 6^2 + 2^2 = 40$

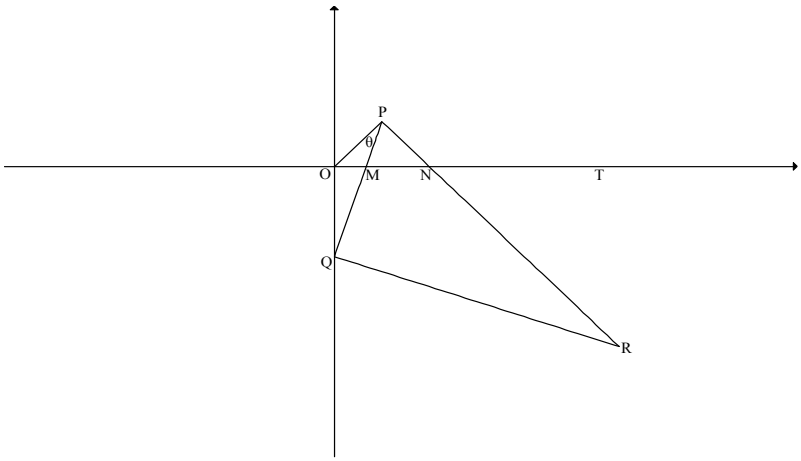
$\text{QR} = 2\sqrt{10}$

$\therefore \cos\theta = \frac{2\sqrt{10}}{\sqrt{50}} = \frac{2}{\sqrt{5}}$

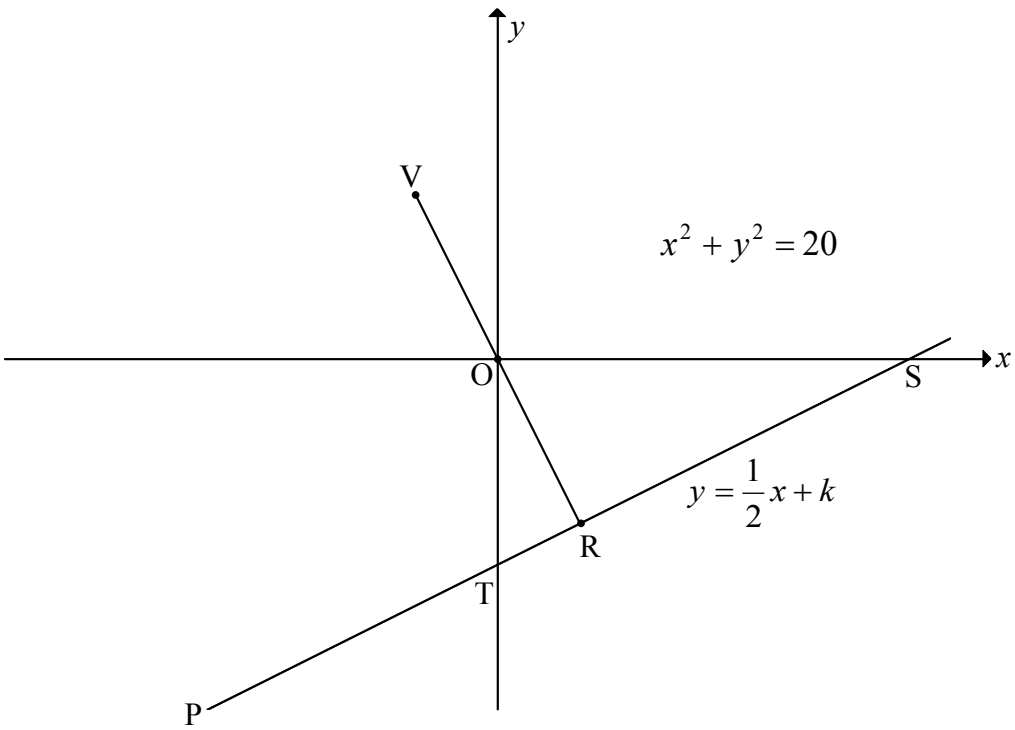
$\therefore \theta = 26{,}57°$

✓subst into/*in* distance formula/ *afstandsformule*
✓distance/*afst* PQ
✓correct trig ratio/ *korrekte trig vh*
✓correct trig eq/ *korrekte trig vgl*
✓answ/*antw*
(5)

OR/*OF*

	$\tan\theta = \dfrac{m_{RQ} - m_{PR}}{1 + m_{RQ}.m_{PR}}$ $= \dfrac{-\frac{1}{3} - (-1)}{1 + (-\frac{1}{3})(-1)}$ $= \dfrac{1}{2}$ $\therefore\ \theta = 26{,}57°$	✓correct formula/ *korrekte formule* ✓ $m_{RQ} = -\frac{1}{3}$ ✓correct subst/ *subst korrek* ✓tan $\theta = \dfrac{1}{2}$ ✓ $\theta = 26{,}57°$ (5)
		
	tangent OP goes through the origin/*raakl OP gaan deur oorsprong* $P\hat{O}M = 45°$ $O\hat{P}M = \theta = \hat{P}$ [tan-chord theorem/*raakl-kdst*] $\tan P\hat{M}T = m_{PQ} = 3$ $\therefore P\hat{M}T = 71{,}57°$ $\therefore\ \theta + 45° = 71{,}57°$ [ext ∠ of Δ/*buite-∠ v* Δ] $\therefore\ \theta = 26{,}57°$	✓ $P\hat{O}M = 45°$ ✓R ✓ $P\hat{M}T = 71{,}57°$ ✓S ✓ $\theta = 26{,}57°$ (5) **[23]**

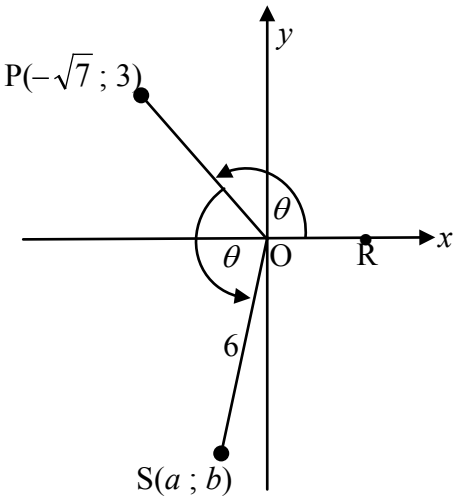
QUESTION/*VRAAG* 4



4.1	OR ⊥ TR [radius ⊥ tangent/*raakl*] $\therefore\ m_{TR} \times m_{OR} = -1$ $\therefore\ m_{OR} = -2$ $\therefore\ y = -2x$	✓S/R ✓*m* of/*van* OR ✓equation/*vgl* (3)
4.2	$x^2 + (-2x)^2 = 20$ $x^2 + 4x^2 = 20$ $5x^2 - 20 = 0$ $x^2 - 4 = 0$ $(x+2)(x-2) = 0$ $\therefore\ x = 2$ $y = -2(2) = -4$ $\therefore$ R(2 ; –4)	✓subst eq of OR into circle eq/ *subst vgl OR in sirkelvgl* ✓st. form/*st. vorm* ✓*x*-value/*waarde* ✓*y*-value/*waarde* (4)

4.3	Subst R(2 ; –4) into the equation of/*in vgl van* PRS: $-4 = \frac{1}{2}(2) + k$ $k = -5$ $\therefore$ OT = 5 $0 = \frac{1}{2}x - 5$ $x = 10$ $\therefore$ OS = 10 Area/*Oppervlakte* = $\frac{1}{2}$OS . OT $= \frac{1}{2}(10)(5)$ = 25 sq units/*vk eenh*	✓correct subst/ *korrekte subst* ✓value of k ✓$y = 0$ ✓x-intercept/*afsnit* ✓correct subst into area form/ *subst korrek in opp-formule* ✓answ/*antw* (6)
4.4	$0 = \frac{x_V + 2}{2}$ and/*en* $0 = \frac{y_V - 4}{2}$ $\therefore$ V(–2 ; 4) T(0 ; –5) .... from/*van* 4.3 $VT = \sqrt{(-2-0)^2 + (4-(-5))^2}$ $= \sqrt{4+81}$ $= \sqrt{85}$	✓x-value/*waarde*V ✓y-value/*waarde*V ✓subst of points V and T into distance formula/ *subst punte V en T in afst-form* ✓answ/*antw* (4) **[17]**

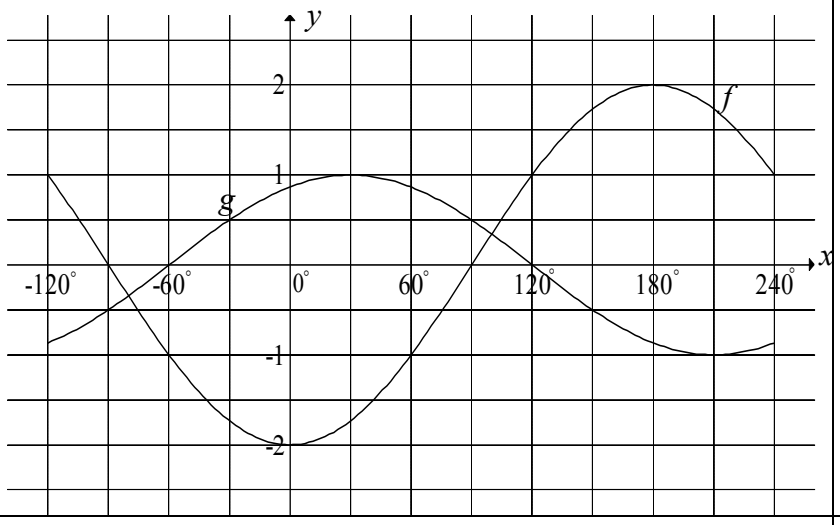
QUESTION/*VRAAG* 5



5.1.1	$\tan\theta = -\frac{3}{\sqrt{7}}$	✓answ/*antw* (1)
5.1.2	$\sin(-\theta) = -\sin\theta$ $OP^2 = \left(-\sqrt{7}\right)^2 + 3^2$ $OP^2 = 16$ $OP = 4$ $\sin(-\theta) = -\frac{3}{4}$	✓reduction/ *reduksie* ✓ $OP = 4$ ✓answ/*antw* (3)
5.1.3	$\frac{a}{6} = \cos 2\theta$ $a = 6(1 - 2\sin^2\theta)$ $= 6 - 12\left(\frac{3}{4}\right)^2$ $= \frac{24}{4} - \frac{27}{4}$ $= -\frac{3}{4}$ **OR/*OF*** $\frac{a}{6} = \cos 2\theta$ $a = 6(2\cos^2\theta - 1)$ $= 12\left(\frac{-\sqrt{7}}{4}\right)^2 - 6$ $= \frac{21}{4} - \frac{24}{4}$ $= -\frac{3}{4}$ **OR/*OF***	✓trig ratio/*verh* ✓expansion/ *uitbreiding* ✓ $\sin\theta = \frac{3}{4}$ ✓answ/*antw* (4) ✓trig ratio/*verh* ✓expansion/ *uitbreiding* ✓ $\cos\theta = \frac{-\sqrt{7}}{4}$ ✓answ/*antw* (4)

	$\frac{a}{6} = \cos 2\theta$ $a = 6(\cos^2\theta - \sin^2\theta)$ $= 6\left[\left(\frac{-\sqrt{7}}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right]$ $= 6\left(-\frac{2}{16}\right)$ $= -\frac{3}{4}$	✓ trig ratio/*verh* ✓expansion/ *uitbreiding* ✓ $\cos\theta = \frac{-\sqrt{7}}{4}$ & $\sin\theta = \frac{3}{4}$ ✓answ/*antw* (4)
5.2.1	$\frac{4\sin x.\cos x}{2\sin^2 x - 1} = \frac{2(2\sin x.\cos x)}{-(1-2\sin^2 x)}$ $= \frac{2\sin 2x}{-\cos 2x}$ $= -2\tan 2x$	✓$2\sin 2x$ ✓$-\cos 2x$ ✓answ/*antw* (3)
5.2.2	$\frac{4\sin 15°\cos 15°}{2\sin^2 15° - 1} = -2\tan 2(15°)$ $= -2\tan 30°$ $= -2\left(\frac{1}{\sqrt{3}}\right)$ $= -\frac{2}{\sqrt{3}}$ **OR/*OF*** $-\frac{2\sqrt{3}}{3}$	✓ $-2\tan 2(15°)$ ✓answ/*antw* (2) **[13]**

QUESTION/*VRAAG* 6

6.1	$\sin(x + 60°) + 2\cos x = 0$ $\sin x \cos 60° + \cos x \sin 60° + 2\cos x = 0$ $\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x + 2\cos x = 0$ $\frac{1}{2}\sin x = -2\cos x - \frac{\sqrt{3}}{2}\cos x$ $\sin x = -4\cos x - \sqrt{3}\cos x$ $\sin x = \cos x(-4 - \sqrt{3})$ $\frac{\sin x}{\cos x} = \frac{\cos x(-4 - \sqrt{3})}{\cos x}$ $\therefore \tan x = -4 - \sqrt{3}$	✓expansion/*uitbreiding* ✓special angle values/ *spesiale ∠-waardes* ✓simpl/*vereenv* ✓ $\sin x = \cos x(-4 - \sqrt{3})$ (4)
6.2	$\tan x = -4 - \sqrt{3}$ $\tan x = -(4 + \sqrt{3})$ *ref* ∠ = 80,10° $x = -80{,}1°$ or/*of* 99,9°	✓80,10° ✓99,90° ✓–80,1° (3)
6.3.1		✓(30° ; 1) ✓(–60° ; 0) ✓shape/*vorm* (3)
6.3.2	$\therefore \sin(x + 60°) > -2\cos x$ $x \in (-80{,}10° ; 99{,}90°)$ **OR/*OF*** $-80{,}10° < x < 99{,}90°$	✓✓critical values/ *kritiese waardes* ✓notation/*notasie* (3) **[13]**

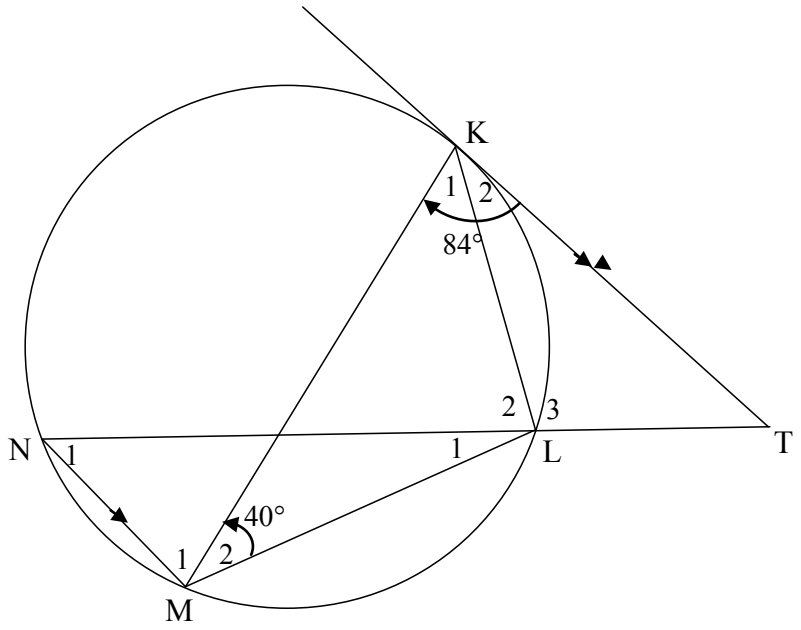
QUESTION/*VRAAG* 7

7.1.1	Area of/*Oppervlakte van* $\Delta PQR = \frac{1}{2}PQ.QR.\sin\hat{Q}$ $= \frac{1}{2}x(20-4x)(\sin 60°)$ $= 10x - 2x^2\left(\frac{\sqrt{3}}{2}\right)$ $= 5\sqrt{3}x - \sqrt{3}x^2$	✓subst into area rule/ *subst in opp-reël* ✓subst & simpl/ *subst en vereenv* (2)
7.1.2	For maximum area/*Vir maksimum opp:* $(\text{Area } \Delta PQR)' = 0$ $5\sqrt{3} - 2\sqrt{3}x = 0$ $2\sqrt{3}x = 5\sqrt{3}$ $\therefore\ x_{max} = \frac{5}{2}$ or $2\frac{1}{2}$ or/*of* 2,5 **OR/*OF*** $x_{max} = -\frac{b}{2a}$ $= -\frac{5\sqrt{3}}{2(-\sqrt{3})} = \frac{5}{2}$ or $2\frac{1}{2}$ or 2,5 **OR/*OF*** $5\sqrt{3}x - \sqrt{3}x^2 = 0$ $\sqrt{3}x(5-x) = 0$ $\therefore x = 0$ or 5 $\therefore\ x_{max} = \frac{0+5}{2} = \frac{5}{2}$ or/*of* 2,5	✓$(\text{Area } \Delta PQR)' = 0$ ✓ $5\sqrt{3} - 2\sqrt{3}x$ ✓answ/*antw* (3) ✓formula/*e* ✓ subst ✓answ/*antw* (3) ✓*x*-intercepts/ *x-afsnitte* ✓subst ✓answ/*antw* (3)
7.1.3	$RP^2 = QP^2 + QR^2 - 2.QP.QR.\cos Q$ $= 10^2 + 2{,}5^2 - 2(10)(2{,}5)\cos 60°$ $= 81{,}25$ $\therefore RP = 9{,}01$	✓subst into cosine rule/*in cos-reël* ✓simpl/*vereenv* ✓answ/*antw* (3)

7.2	In ΔABC: $\sin\beta = \frac{h}{AB}$ $\therefore AB = \frac{h}{\sin\beta}$ In ΔABD: $AB = BD$ and/*en* $A\hat{D}B = 90° - \beta$ [$\angle$s of/*v* $\Delta = 180°$] $\frac{\sin 2\beta}{AD} = \frac{\sin(90° - \beta)}{AB}$ $AD = \frac{AB.\sin 2\beta}{\sin(90° - \beta)}$ $= \frac{h}{\sin\beta} \times \frac{2\sin\beta.\cos\beta}{\cos\beta}$ $= 2h$	✓AB ito *h* and/*en* β ✓ $A\hat{D}B = 90° - \beta$ ✓correct subst into cosine rule/*subst korrek in cos-reël* ✓AD as subject/ *onderwerp* ✓expansion/*uitbrei* ✓sin $(90° - \beta)$ $= \cos\beta$ ✓answer ito *h* (7)
	OR/*OF* In ΔABC: $\sin\beta = \frac{h}{AB}$ $\therefore AB = \frac{h}{\sin\beta}$ In ΔABD: $AB = BD$ $AD^2 = AB^2 + AB^2 - 2AB.AB.\cos 2\beta$ $= \left(\frac{h}{\sin\beta}\right)^2 + \left(\frac{h}{\sin\beta}\right)^2 - 2\left(\frac{h}{\sin\beta}\right)^2.\cos 2\beta$ $= \left(\frac{h}{\sin\beta}\right)^2 + \left(\frac{h}{\sin\beta}\right)^2 - 2\left(\frac{h}{\sin\beta}\right)^2(1 - 2\sin^2\beta)$ $= \left(\frac{h}{\sin\beta}\right)^2 + \left(\frac{h}{\sin\beta}\right)^2 - 2\left(\frac{h}{\sin\beta}\right)^2 + 4h^2$ $= 4h^2$ $\therefore AD = 2h$	✓AB ito *h* and/*en* β ✓correct subst into cosine rule/*subst korrek in cos-reël* ✓expansion/*uitbrei* ✓multiplication/ *vermenigv* ✓simpl/*vereenv* ✓answer ito *h* (7)
	OR/*OF* Split isosceles triangle ABQ into two congruent triangles AEB and DEB. Then $\Delta ABC \equiv \Delta BAE$ ($AB = AC$, $A\hat{B}E = B\hat{A}C = \beta$, h) $\therefore AE = ED = BC = h$ $\therefore AD = 2h$	(7)
		[15]

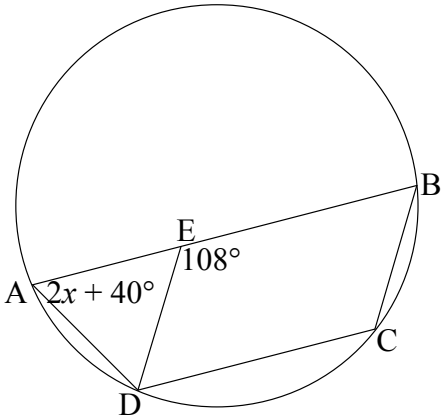
QUESTION/*VRAAG* 8

8.1



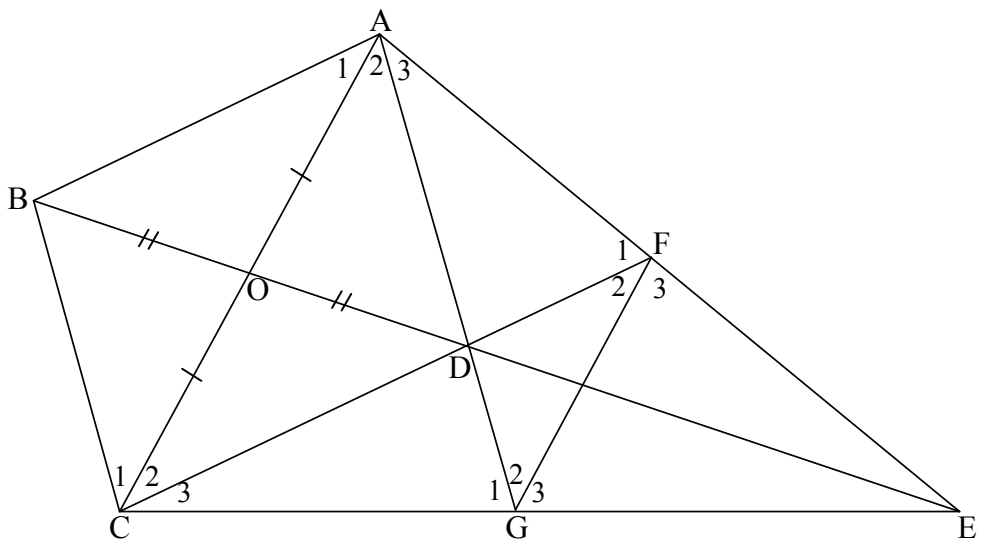
8.1.1	$\hat{K}_2 = \hat{M}_2 = 40°$ [tan chord theorem/*raakl-kdst*]	✓S ✓R (2)
8.1.2	$\hat{N}_1 = \hat{K}_1$ [∠s in the same seg/*∠e in dies segm*] $\hat{K}_1 = 84° - 40° = 44°$ $\therefore\ \hat{N}_1 = 44°$	✓S ✓ R ✓S (3)
8.1.3	$\hat{T} = \hat{N}_1 = 44°$ [alt/*verw* ∠s/*e*; KT | | NM]	✓S ✓ R (2)
8.1.4	$\hat{L}_2 = \hat{K}_2 + \hat{T}$ [ext ∠ of Δ/*buite ∠ v Δ*] $= 40° + 44°$ $= 84°$	✓R ✓S (2)
8.1.5	In ΔKLM: $44° + 84° + 40° + \hat{L}_1 = 180°$ [∠s sum in Δ/*∠e som in* Δ] $\therefore \hat{L}_1 = 12°$	✓S (1)

8.2



8.2	$\hat{C} = 108°$ [opp∠s of ||m/*tos ∠e v ||m*] $2x + 40° + 108° = 180°$ [opp∠s of cyc quad/*tos∠e v kdvh*] $2x = 32°$ $x = 16°$ **OR/*OF*** $\hat{C} = 180° - (2x + 40°)$ [opp∠s of cyc quad/*tos∠e v kdvh*] $180° - (2x + 40°) = 108°$ [opp∠s of ||m/*tos ∠e v ||m*] $2x = 32°$ $x = 16°$	✓S ✓R ✓S ✓R ✓answ/*antw* (5) ✓S ✓R ✓S ✓R ✓answ/*antw* (5) **[15]**

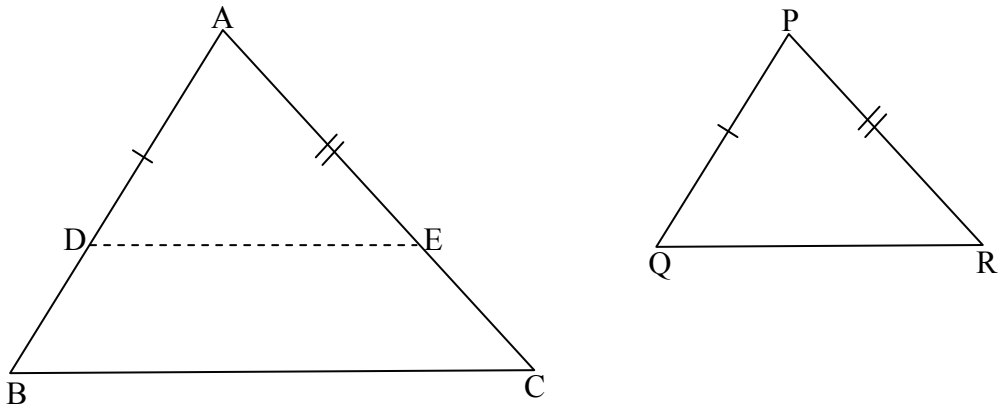
QUESTION/*VRAAG* 9



9.1	ABCD is a ‖m [diags of quad bisect each other/ *hoekl v vh halveer mekaar*]	✓ R (1)
9.2	$\frac{ED}{DB} = \frac{FE}{AF}$ [Prop Th/*Eweredigh st*; DF || BA] $\frac{ED}{DB} = \frac{GE}{CG}$ [Prop Th/*Eweredigh st*; DG || BC]	✓S ✓ R ✓S ✓ R (4)
9.3	$\frac{FE}{AF} = \frac{GE}{CG}$ [proved/*bewys*] $\therefore$ AC || FG [line divides two sides of Δ in prop/ *lyn verdeel 2 sye van Δ eweredig*] $\hat{C}_2 = \hat{F}_2$ [alt/*verw* ∠s/*e*; AC || FG] $\hat{A}_1 = \hat{C}_2$ [alt/*verw* ∠s/*e*; AB || CD] $\therefore \hat{A}_1 = \hat{F}_2$	✓S ✓ S ✓R ✓S ✓ S (5)
9.4	$\hat{A}_1 = \hat{A}_2$ [diags of rhombus/*hoekl v ruit*] $\hat{A}_2 = \hat{F}_2$ [$\hat{A}_1 = \hat{F}_2$] $\therefore$ ACGF = cyc quad/*kdvh* [∠s in the same seg =/ *∠e in dies segm* =] **OR/*OF*** $\hat{C}_2 = \hat{A}_2$ [∠s opp equal sides of rhombus/ *∠e to gelyke sye v ruit*] $\hat{A}_2 = \hat{G}_2$ [alt/*verw*-∠s/*e*; AC || FG] $\therefore \hat{C}_2 = \hat{G}_2$ $\therefore$ ACGF is a cyc quad/*kdvh* [∠s in the same seg =/ *∠e in dies segm* =]	✓S ✓ S ✓R (3) ✓S ✓ S ✓R (3) **[13]**

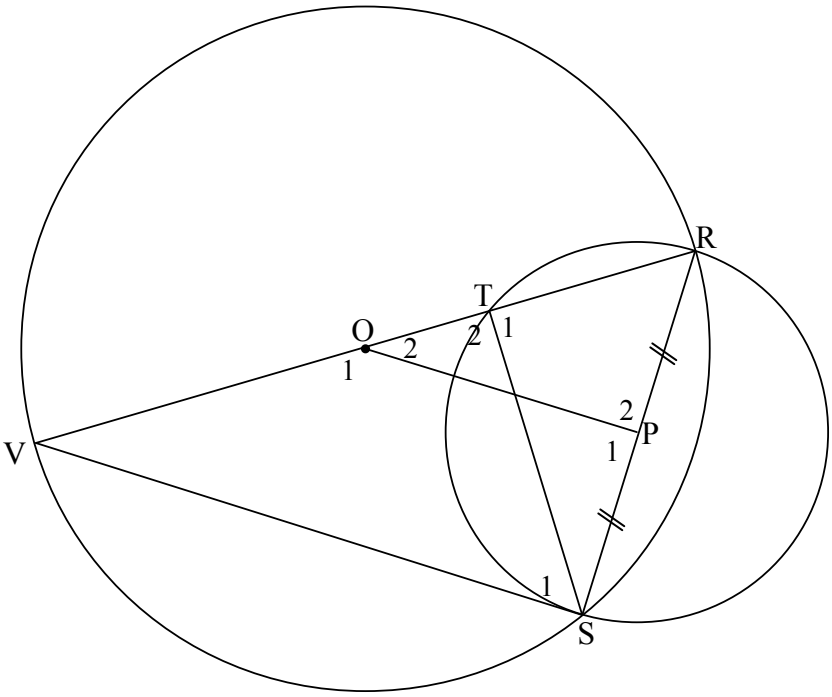
QUESTION/*VRAAG* 10

10.1



10.1.1	In ΔADE and/*en* ΔPQR: AD = PQ [construction/*konstr*] $\hat{A} = \hat{P}$ [given/*gegee*] AE = PR [construction/*konstr*] $\therefore \Delta ADE \equiv \Delta PQR$ [S$\angle$S]	✓all/*al* 3 S's/*e* ✓reason/*rede* (2)
10.1.2	$A\hat{D}E = \hat{Q}$ [Δs $\equiv$ $\therefore$ corres/*ooreenk* $\angle$s/*e* =] But $\hat{B} = \hat{Q}$ [given/*gegee*] $\therefore A\hat{D}E = \hat{B}$ $\therefore$ DE | | BC [corres/*ooreenk* $\angle$s/*e* =]	✓ $A\hat{D}E = \hat{Q}$ ✓ $A\hat{D}E = \hat{B}$ ✓reason/*rede* (3)
10.1.3	$\frac{AB}{AD} = \frac{AC}{AE}$ [Prop Th/*Eweredigh st*; DE | | BC] But/*Maar* AD = PQ and/*en* AE = PR [construction/*konstr*] $\therefore \frac{AB}{PQ} = \frac{AC}{PR}$	✓S/R ✓S (2)

10.2



10.2.1	line from centre to midpt of chord/*lyn van midpt na midpt van koord*	✓answ/*antw* (1)
10.2.2	OP || VS [Midpt Theorem/*Midpt-stelling*] In ΔROP and/*en* ΔRVS: $\hat{R} = \hat{R}$ [common/*gemeen*] $\hat{O}_2 = \hat{V}$ [corresp/*ooreenk* ∠s/*e*; OP || VS] ∴ΔROP ||| ΔRVS [∠,∠,∠] **OR/*OF*** In ΔROP and/*en* ΔRVS: $\hat{P}_2 = V\hat{S}R$ [corresponding ∠s/ *ooreenkomstige* ∠'e] $\hat{R} = \hat{R}$ [common/*gemeen*] ∴ΔROP ||| ΔRVS [∠,∠,∠]	✓S ✓R ✓S ✓S & ∠;∠;∠ **OR/*OF*** 3 angles/*hoeke* (4) ✓S ✓R ✓S ✓S & ∠;∠;∠ **OR/*OF*** 3 angles/*hoeke* (4)

10.2.3	In ΔRVS and/*en* ΔRST: $V\hat{S}R = S\hat{T}R = 90°$ [$\angle$ in semi-circle/$\angle$ *in halfsirkel*] $\hat{R}$ is common/*gemeen* $\hat{V} = T\hat{S}R$ $\therefore \Delta RVS \mid\mid\mid \Delta RST$ [$\angle,\angle,\angle$]	✓S ✓ R ✓S & $\angle;\angle;\angle$ **OR/*OF*** 3 angles/*hoeke* (3)
10.2.4	In ΔRTS and/*en* ΔSTV: $R\hat{T}S = V\hat{T}S = 90°$ [$\angle$ s on straight line/$\angle$*e op rt lyn*] $\hat{R} = 90° - T\hat{S}R$ $= T\hat{S}V$ $T\hat{S}R = \hat{V}$ $\therefore \Delta RTS \mid\mid\mid \Delta STV$ [$\angle,\angle,\angle$] $\therefore \frac{RT}{ST} = \frac{TS}{VT}$ $\therefore ST^2 = VT.TR$	✓ΔRTS & ΔSTV ✓S ✓S ✓S (with justification/*met motivering*) ✓$\Delta RTS \mid\mid\mid \Delta STV$ ✓ratio/*verh* (6)
		[21]

TOTAL/*TOTAAL*: 150

PolyMathic

Vraestel 10
Okt/Nov
Eksamen

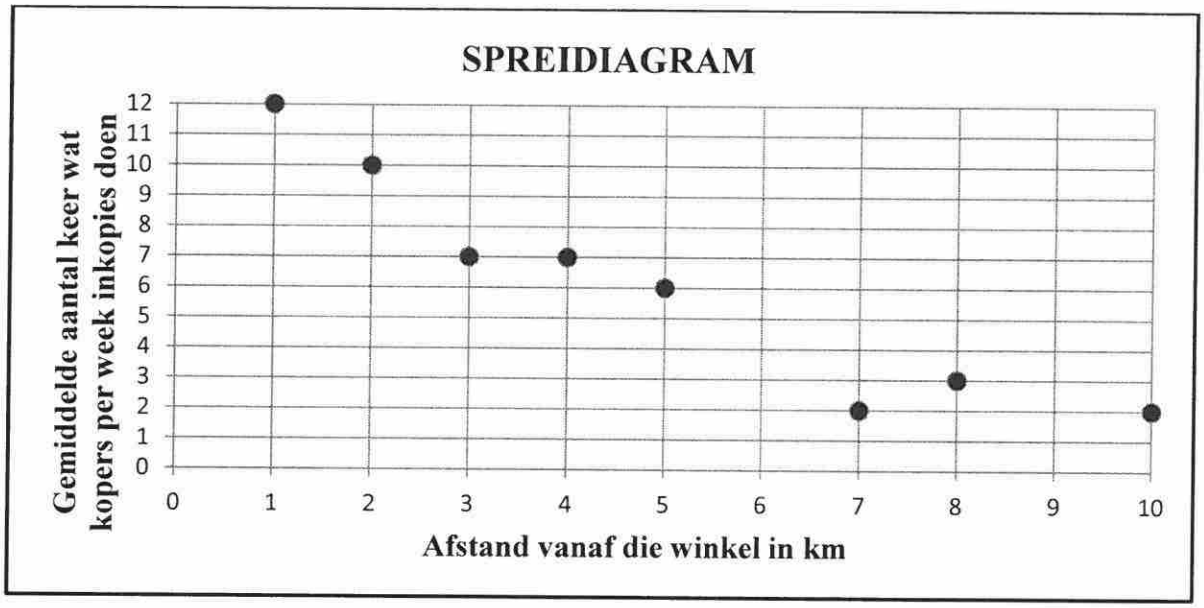
PolyMathic

Graad 12 EindEksamen
Totaal: 150
Tyd: 3ure

VRAAG 1

'n Opname is by 'n plaaslike supermark gemaak oor die afstand wat kopers vanaf die winkel woon en die gemiddelde aantal keer wat hierdie kopers in 'n week inkopies doen. Die resultate word in die tabel hieronder getoon.

Afstand vanaf die winkel in km	1	2	3	4	5	7	8	10
Gemiddelde aantal keer wat kopers per week inkopies doen	12	10	7	7	6	2	3	2



1.1 Gebruik die spreidiagram om kommentaar te lewer op die sterkte van die verband tussen die afstand wat 'n koper vanaf die winkel woon en die gemiddelde aantal keer wat sy/hy in 'n week by die winkel inkopies doen. (1)

1.2 Bereken die korrelasiekoëffisiënt van die data. (1)

1.3 Bereken die vergelyking van die kleinstekwadrate-regressielyn van die data. (3)

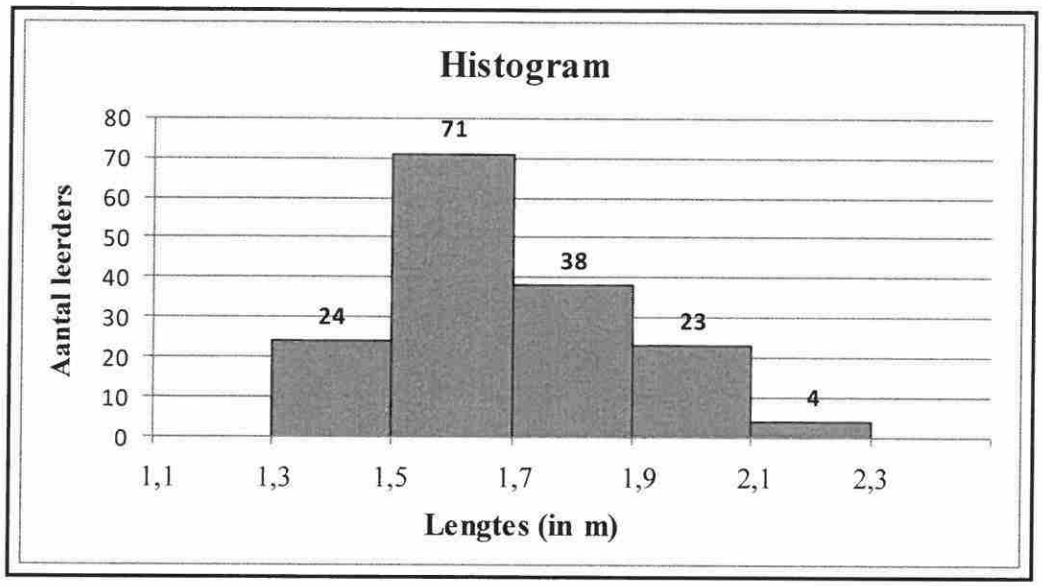
1.4 Gebruik jou antwoord by VRAAG 1.3 om die gemiddelde aantal keer te beraam wat 'n koper, wat 6 km vanaf die supermark woon, die winkel in 'n week sal besoek. (2)

1.5 Skets die kleinstekwadrate-regressielyn op die spreidiagram wat in die ANTWOORDEBOEK verskaf is. (2)

[9]

VRAAG 2

Die lengtes van 160 leerders in 'n skool word gemeet. Die lengte van die kortste leerder is 1,39 m en dié van die langste leerder is 2,21 m. Die lengtes word in die histogram hieronder voorgestel.



2.1 Beskryf die skeefheid van die data. (1)

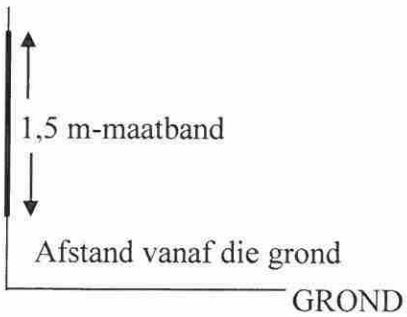
2.2 Bereken die omvang (variasiewydte) van die lengtes. (2)

2.3 Voltooi die kumulatiewefrekwensie-kolom in die tabel wat in die ANTWOORDEBOEK gegee word. (2)

2.4 Skets 'n ogief (kumulatiewefrekwensie-kromme), wat die data voorstel, op die rooster wat in die ANTWOORDEBOEK gegee word. (4)

2.5 Tagtig leerders se lengtes is minder as x meter. Skat x. (2)

2.6 Die persoon wat die metings geneem het, het slegs 'n 1,5 m-maatband beskikbaar gehad. Om te kompenseer vir die kort maatband het hy besluit om die band teen 'n muur, 1 m vanaf die grond, te monteer. Nadat die metings aangeteken is, het hy ontdek dat die maatband 1,1 m vanaf die grond in plaas van 1 m, gemonteer was.



Watter invloed het hierdie fout op die volgende:

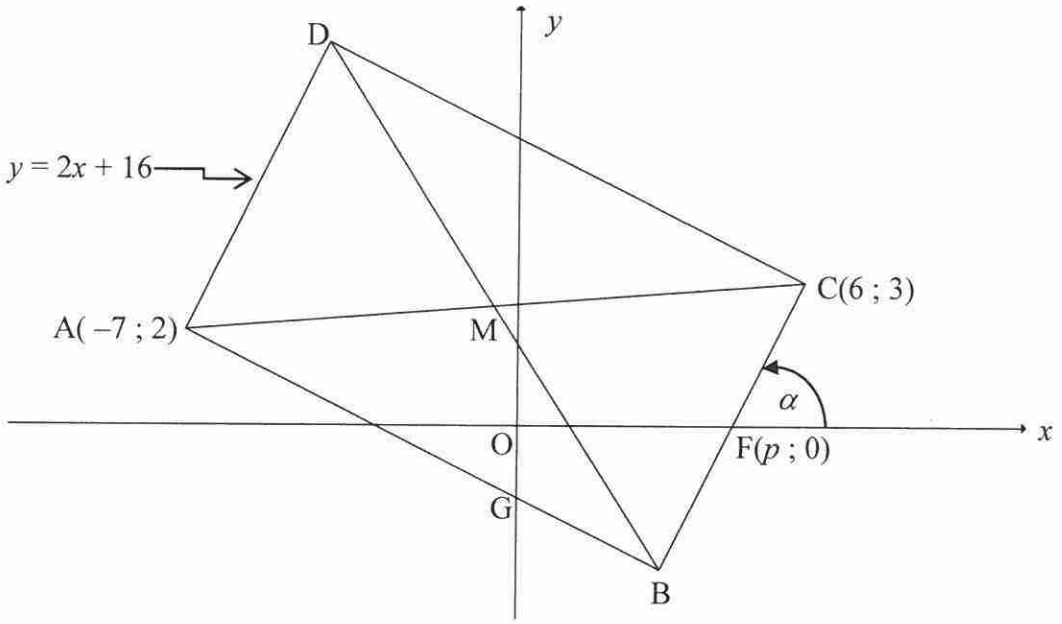
2.6.1 Gemiddelde van die datastel (1)

2.6.2 Standaardafwyking van die datastel (1)

[13]

VRAAG 3

In die diagram is A(–7 ; 2), B, C(6 ; 3) en D die hoekpunte van reghoek ABCD. Die vergelyking van AD is $y = 2x + 16$. Lyn AB sny die y-as by G. Die x-afsnit van lyn BC is F(p ; 0) en die inklinasiehoek van BC met die positiewe x-as is α. Die hoeklyne van die reghoek sny by M.



3.1 Bereken die koördinate van M. (2)

3.2 Skryf die gradiënt van BC in terme van p neer. (1)

3.3 Bereken vervolgens die waarde van p. (3)

3.4 Bereken die lengte van DB. (3)

3.5 Bereken die grootte van α. (2)

3.6 Bereken die grootte van $O\hat{G}B$. (3)

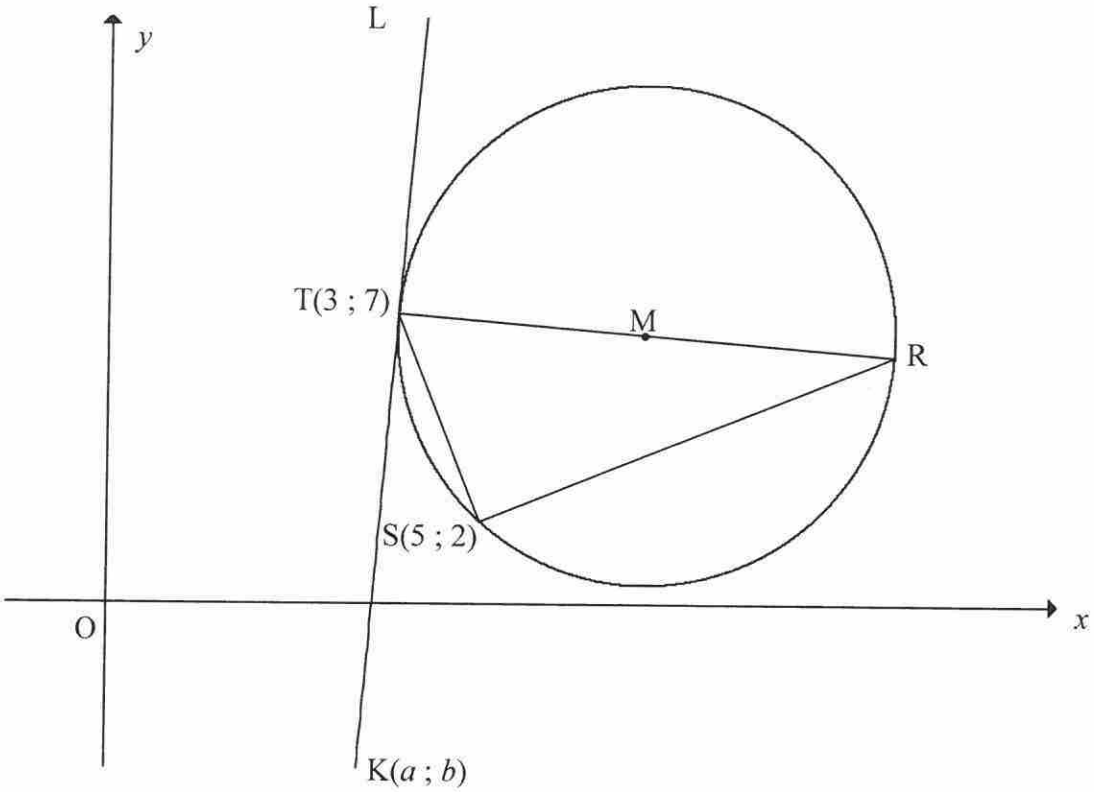
3.7 Bepaal die vergelyking van die sirkel wat deur punt D, B en C gaan in die vorm $(x-a)^2 + (y-b)^2 = r^2$. (3)

3.8 Indien AD so geskuif word dat ABCD 'n vierkant word, sal BC 'n raaklyn wees aan die sirkel wat deur punt A, M en B gaan, waar M nou die snypunt van die hoeklyne van die vierkant ABCD is? Motiveer jou antwoord. (2)

[19]

VRAAG 4

In die diagram gaan die sirkel, met middelpunt M, deur T(3 ; 7), R en S(5 ; 2). RT is 'n middellyn van die sirkel. K(a ; b) is 'n punt in die 4de kwadrant sodat KTL 'n raaklyn aan die sirkel by T is.



4.1 Gee 'n rede waarom $T\hat{S}R = 90°$. (1)

4.2 Bereken die gradiënt van TS. (2)

4.3 Bepaal die vergelyking van die lyn SR in die vorm $y = mx + c$. (3)

4.4 Die vergelyking van die sirkel hierbo is $(x-9)^2 + \left(y - 6\frac{1}{2}\right)^2 = 36\frac{1}{4}$.

4.4.1 Bereken die lengte van TR in wortelvorm. (2)

4.4.2 Bereken die koördinate van R. (3)

4.4.3 Bereken $\sin R$. (3)

4.4.4 Toon dat $b = 12a - 29$. (3)

4.4.5 As TK = TR, bereken die koördinate van K. (6)

[23]

VRAAG 5

5.1 Gegee: $\sin 16° = p$

Bepaal, **sonder om 'n sakrekenaar te gebruik**, die volgende in terme van p.

5.1.1 $\sin 196°$ (2)

5.1.2 $\cos 16°$ (2)

5.2 Gegee: $\cos(A - B) = \cos A \cos B + \sin A \sin B$

Gebruik die formule vir $\cos(A - B)$ en lei 'n formule af vir $\sin(A + B)$ (3)

5.3 Vereenvoudig $\dfrac{\sqrt{1 - \cos^2 2A}}{\cos(-A).\cos(90° + A)}$ volledig, gegee dat $0° < A < 90°$. (5)

5.4 Gegee: $\cos 2B = \dfrac{3}{5}$ en $0° \leq B \leq 90°$

Bepaal, **sonder om 'n sakrekenaar te gebruik**, die waarde van ELK van die volgende in die eenvoudigste vorm:

5.4.1 $\cos B$ (3)

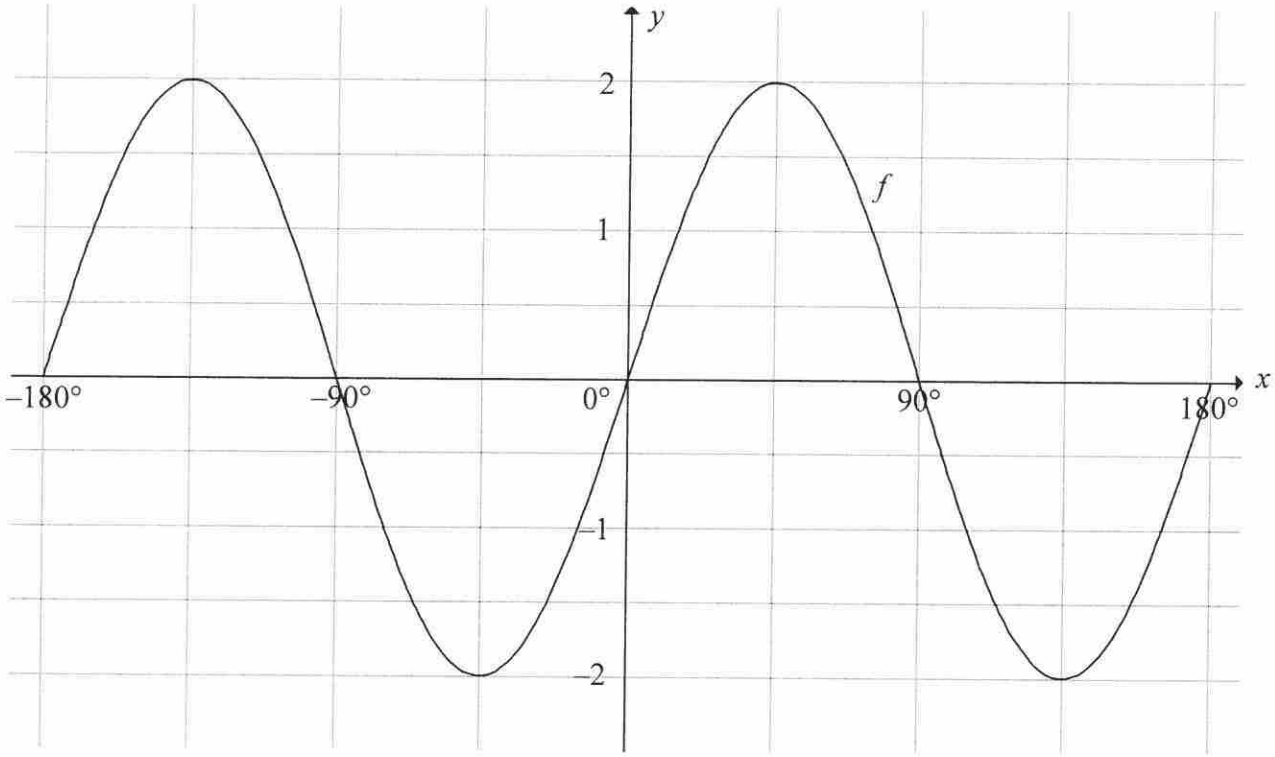
5.4.2 $\sin B$ (2)

5.4.3 $\cos (B + 45°)$ (4)

[21]

VRAAG 6

In die diagram is die grafiek van $f(x) = 2\sin 2x$ geskets vir die interval $x \in [-180° ; 180°]$.



6.1 Skets, op die assestelsel waarop f in die ANTWOORDEBOEK geskets is, die grafiek van $g(x) = -\cos 2x$ vir $x \in [-180° ; 180°]$. Toon duidelik alle afsnitte met die asse, die koördinate van die draaipunte en die eindpunte van die grafiek. (3)

6.2 Skryf die maksimum waarde van $f(x) - 3$ neer. (2)

6.3 Bepaal die algemene oplossing van $f(x) = g(x)$. (4)

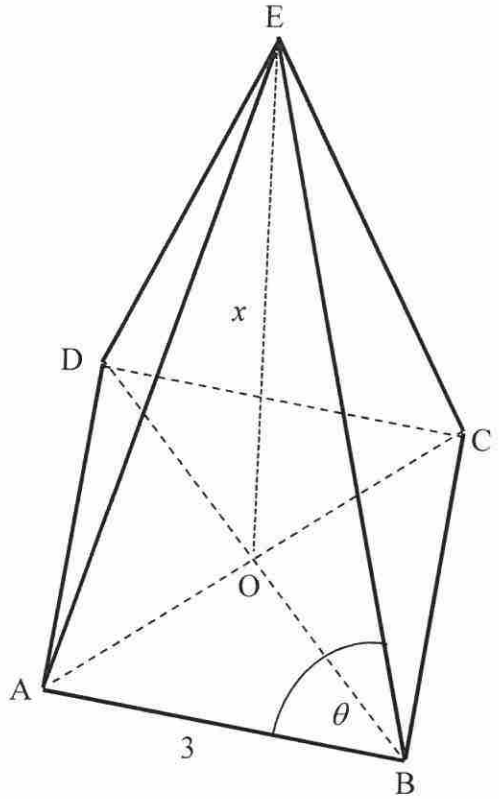
6.4 Bepaal vervolgens die waardes van x waarvoor $f(x) < g(x)$ in die interval $x \in [-180° ; 0°]$. (3)

[12]

VRAAG 7

E is die toppunt van 'n piramide met 'n vierkantige basis ABCD. O is die middelpunt van die basis. $E\hat{B}A = \theta$, AB = 3 m en EO, die loodregte hoogte van die piramide, is x.

$$\text{Volume van piramide} = \frac{1}{3}(\text{oppervlakte van basis}) \times (\perp \text{ hoogte})$$



7.1 Bereken die lengte van OB. (3)

7.2 Toon dat $\cos\theta = \dfrac{3}{2\sqrt{x^2 + \frac{9}{2}}}$ (5)

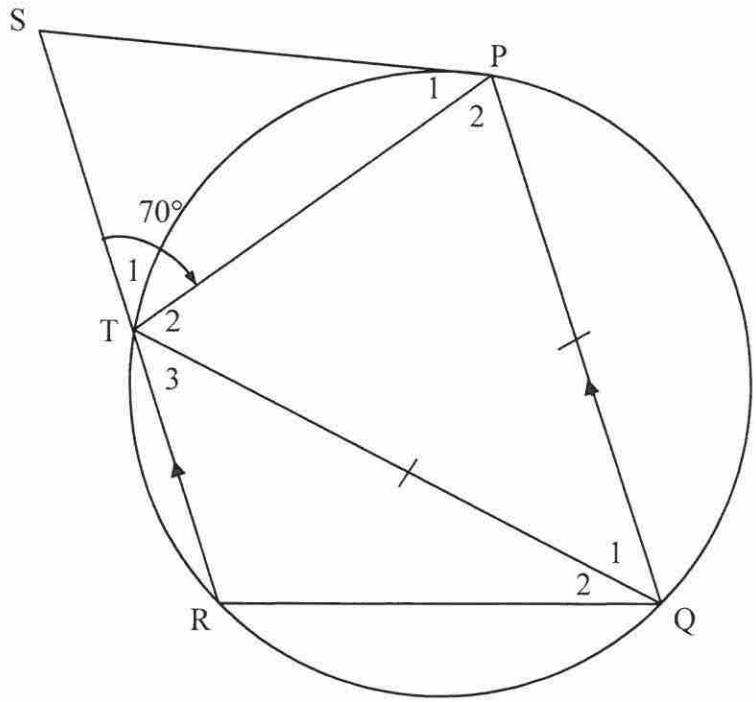
7.3 As die volume van die piramide $15\ m^3$ is, bereken die waarde van θ. (4)

[12]

Gee redes vir ALLE bewerings en berekeninge in VRAAG 8, 9 en 10.

VRAAG 8

8.1 In die diagram hieronder is PQRT 'n koordevierhoek met RT || QP. Die raaklyn by P ontmoet RT verleng by S. QP = QT en $P\hat{T}S = 70°$.



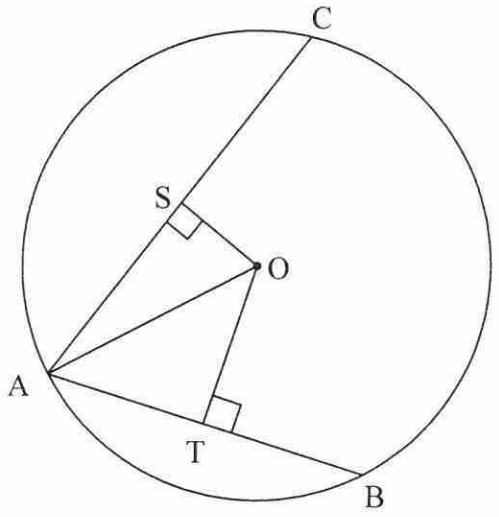
8.1.1 Gee 'n rede waarom $\hat{P}_2 = 70°$. (1)

8.1.2 Bereken, met redes, die grootte van:

(a) $\hat{Q}_1$ (3)

(b) $\hat{P}_1$ (2)

8.2 A, B en C is punte op die sirkel met middelpunt O. S en T is punte op AC en AB onderskeidelik sodat $OS \perp AC$ en $OT \perp AB$. $AB = 40$ en $AC = 48$.



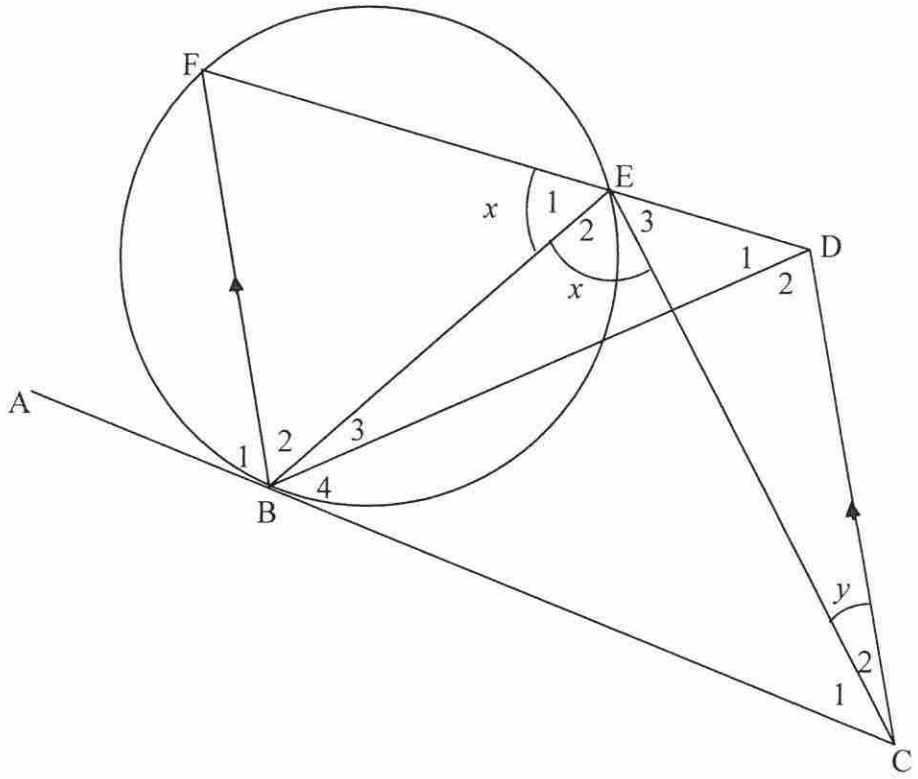
8.2.1 Bereken AT. (1)

8.2.2 As $OS = \frac{7}{15}OT$, bereken die radius OA van die sirkel. (5)

[12]

VRAAG 9

ABC is 'n raaklyn aan die sirkel BFE by B. Vanaf C word 'n reguitlyn ewewydig aan BF getrek om FE verleng by D te ontmoet. EC en BD word getrek. $\hat{E}_1 = \hat{E}_2 = x$ en $\hat{C}_2 = y$.



9.1 Gee 'n rede waarom ELK van die volgende WAAR is:

9.1.1 $\hat{B}_1 = x$ (1)

9.1.2 $B\hat{C}D = \hat{B}_1$ (1)

9.2 Bewys dat BCDE 'n koordevierhoek is. (2)

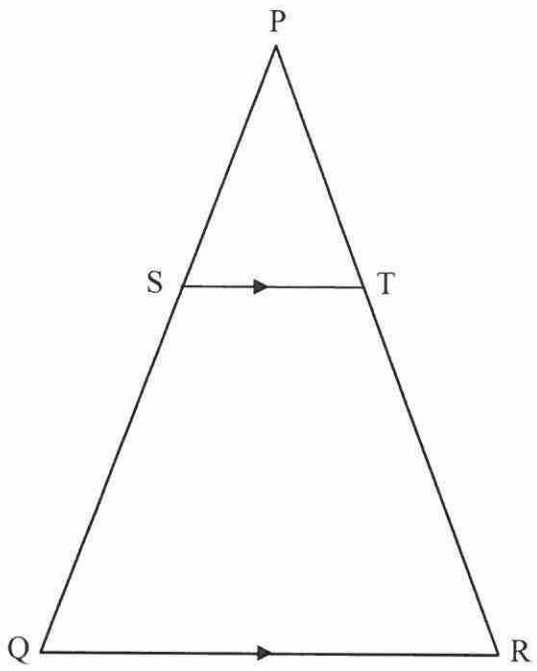
9.3 Watter ander TWEE hoeke is elk gelyk aan x? (2)

9.4 Bewys dat $\hat{B}_2 = \hat{C}_1$. (3)

[9]

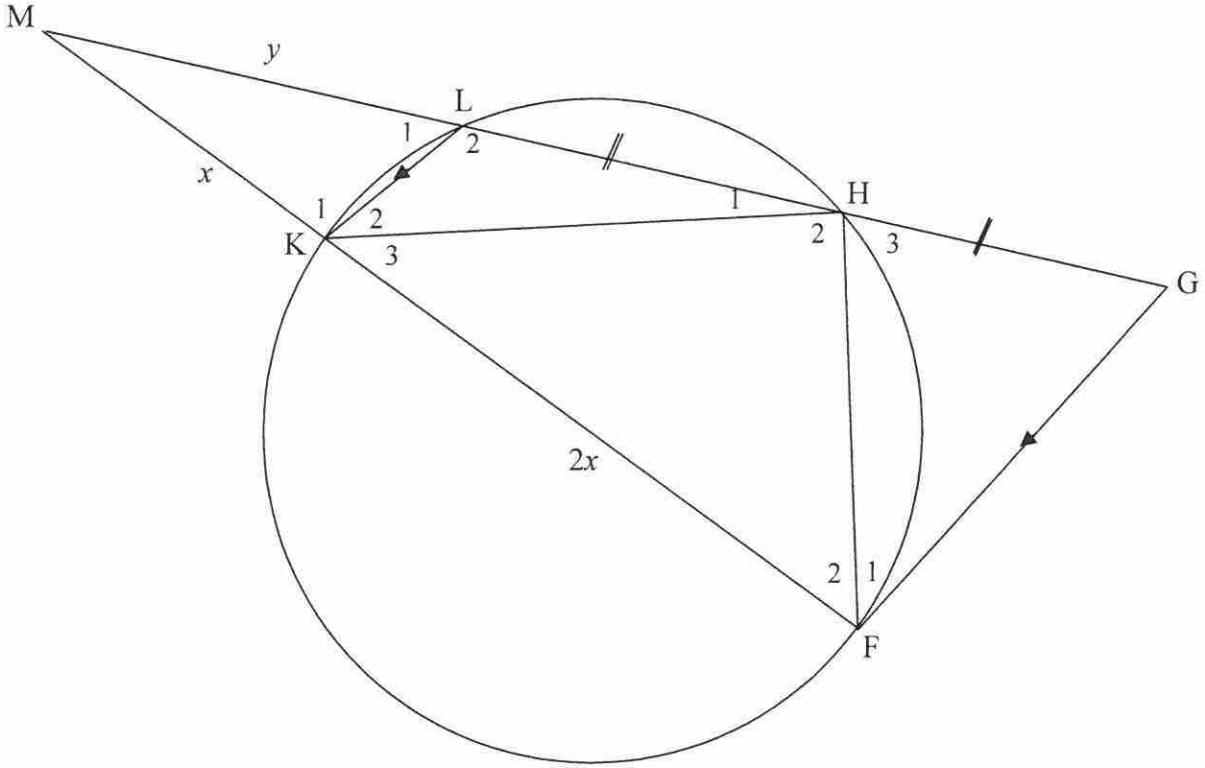
VRAAG 10

10.1 In die diagram is ΔPQR geskets. S en T is punte op sy PQ en PR onderskeidelik sodat $ST \parallel QR$.



Bewys die stelling wat beweer dat $\frac{PS}{SQ} = \frac{PT}{TR}$. (6)

10.2 In die diagram is HLKF 'n koordevierhoek. Die koorde HL en FK is verleng en ontmoet by M. Die lyn deur F ewewydig aan KL ontmoet MH verleng in G. $MK = x$, $KF = 2x$, $ML = y$ en $LH = HG$.



10.2.1 Gee 'n rede waarom $G\hat{F}M = L\hat{K}M$. (1)

10.2.2 Bewys dat:

(a) $GH = y$ (3)

(b) $\Delta MFH \,|||\, \Delta MGF$ (5)

(c) $\dfrac{GF}{FH} = \dfrac{3x}{2y}$ (2)

10.2.3 Toon dat $\dfrac{y}{x} = \sqrt{\dfrac{3}{2}}$ (3)

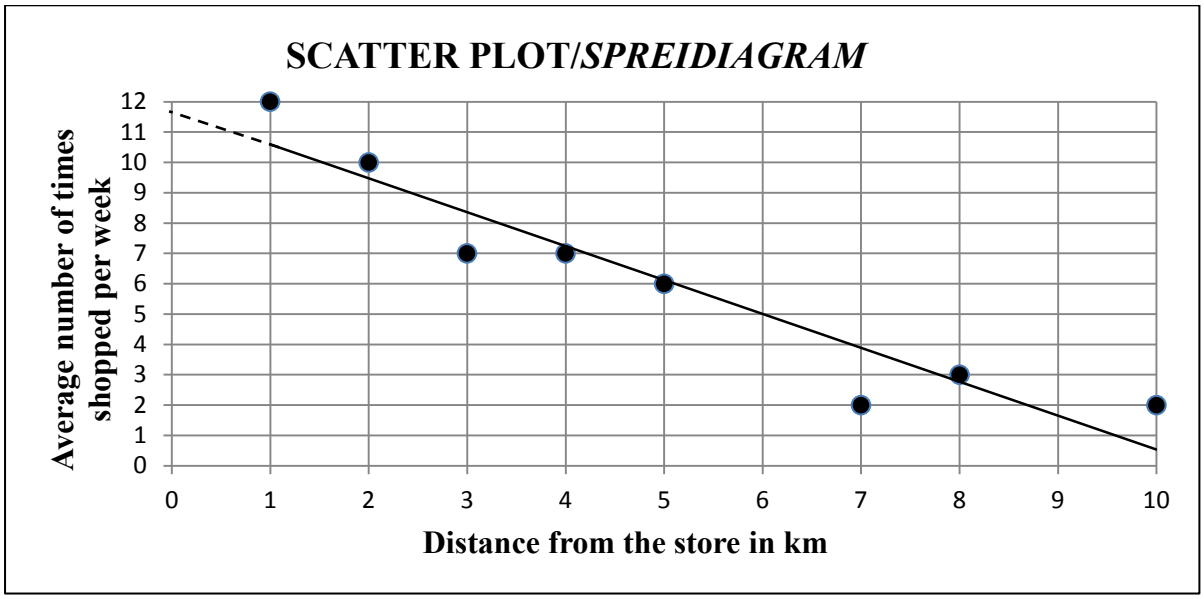
[20]

TOTAAL: 150

Memo

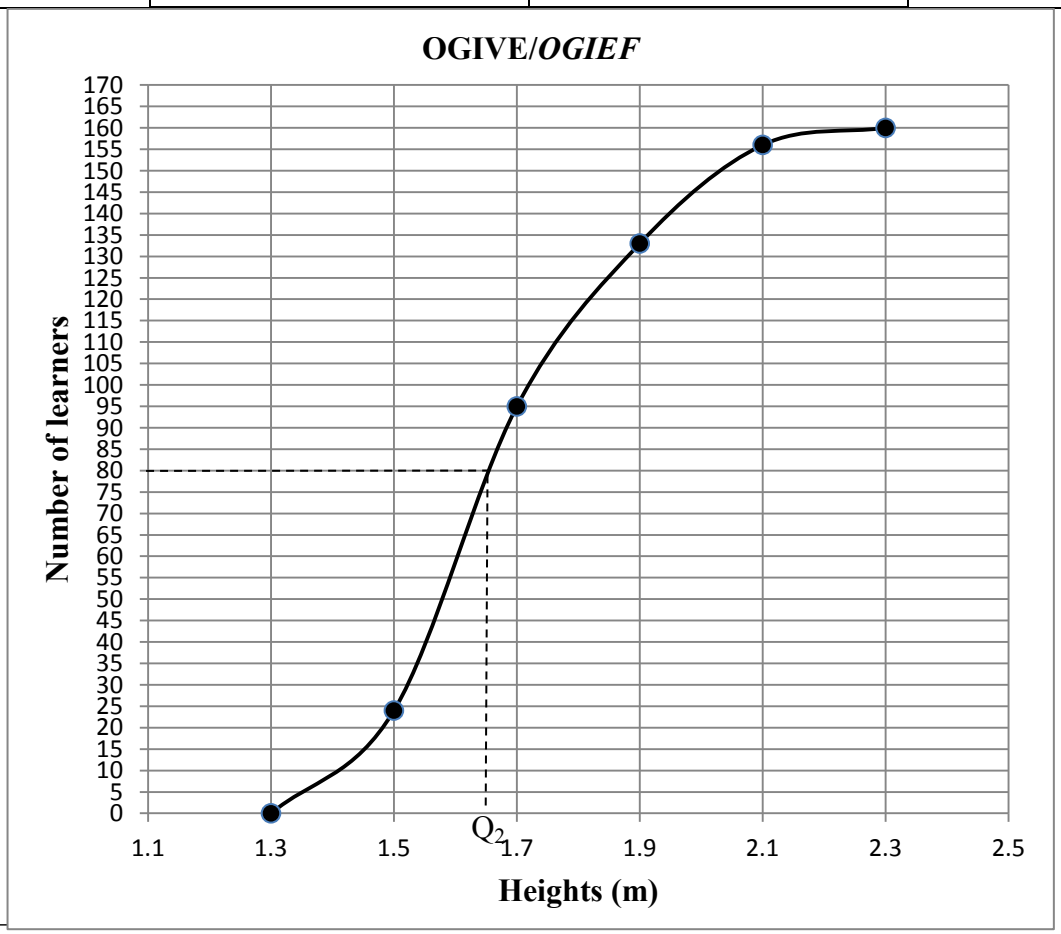
QUESTION/*VRAAG* 1

Distance from the store in km ***Afstand vanaf die winkel in* km**	1	2	3	4	5	7	8	10
Average number of times shopped per week ***Gemiddelde aantal keer wat kopers die winkel per week besoek***	12	10	7	7	6	2	3	2

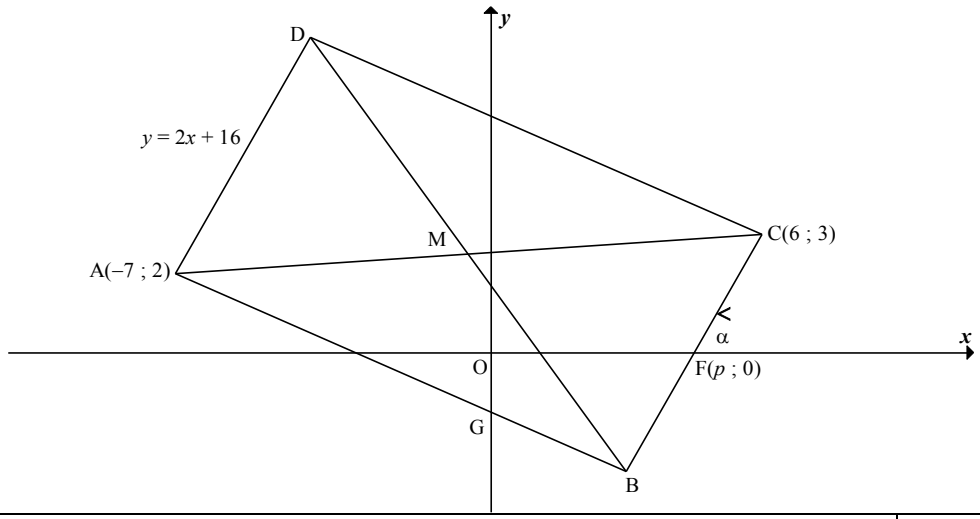


1.1	Strong/*Sterk*	✓ (1)
1.2	–0,95 (–0,9462..)	✓ (1)
1.3	$a = 11{,}71$ (11,7132…) $b = -1{,}12$ (–1,1176…) $\hat{y} = -1{,}12x + 11{,}71$	✓ value of a ✓ value of b ✓ equation/*vgl* (3)
1.4	$\hat{y} = -1{,}12(6) + 11{,}71$ = 5 times	✓ substitition ✓ answer (2)
1.5	On scatter plot/*Op spreidiagram*	✓✓ A line close to any 2 of the following points: (5 ; 6) or (10 ; $\frac{1}{2}$) or (6 ; 5) or (0 ; 11,7) (2) **[9]**

QUESTION/*VRAAG* 2

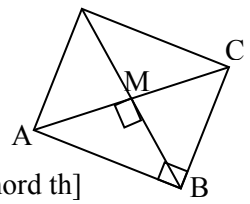
<table>
<tr><td>2.1</td><td>Positively skewed OR skewed to the right/positief skeef OF skeef na regs</td><td>✓ answer
(1)</td></tr>
<tr><td>2.2</td><td>Range/Omvang = 2,21 – 1,39 = 0,82 m</td><td>✓ subtract values
✓answer
(2)</td></tr>
<tr><td>2.3</td><td>
<table>
<tr><th>Intervals
Klasse</th><th>Cumulative frequency
Kumulatiewe frekwensie</th></tr>
<tr><td>$1{,}3 \le x < 1{,}5$</td><td>24</td></tr>
<tr><td>$1{,}5 \le x < 1{,}7$</td><td>95</td></tr>
<tr><td>$1{,}7 \le x < 1{,}9$</td><td>133</td></tr>
<tr><td>$1{,}9 \le x < 2{,}1$</td><td>156</td></tr>
<tr><td>$2{,}1 \le x < 2{,}3$</td><td>160</td></tr>
</table>
</td><td>✓95 , 133, 156
✓160
(2)</td></tr>
<tr><td>2.4</td><td> 
</td><td>✓ upper limits / boonste limiete
✓ cum f/ kum f
✓ shape/ vorm
✓ grounded geanker
(4)</td></tr>
<tr><td>2.5</td><td>method (using 80 to determine the height)
1,65 (accept any value between 1,6 and 1,69)</td><td>✓ method
✓ answer
(2)</td></tr>
<tr><td>2.6.1</td><td>The mean would change by 0,1 m
Die gemiddelde sal met 0,1 m verander</td><td>✓ answer
(1)</td></tr>
<tr><td>2.6.2</td><td>No influence/change as there is no difference in variation of data./Geen invloed /verandering aangesien daar geen verskil in die variasie van die data is nie.</td><td>✓ answer
(1)
[13]</td></tr>
</table>

QUESTION/*VRAAG 3*

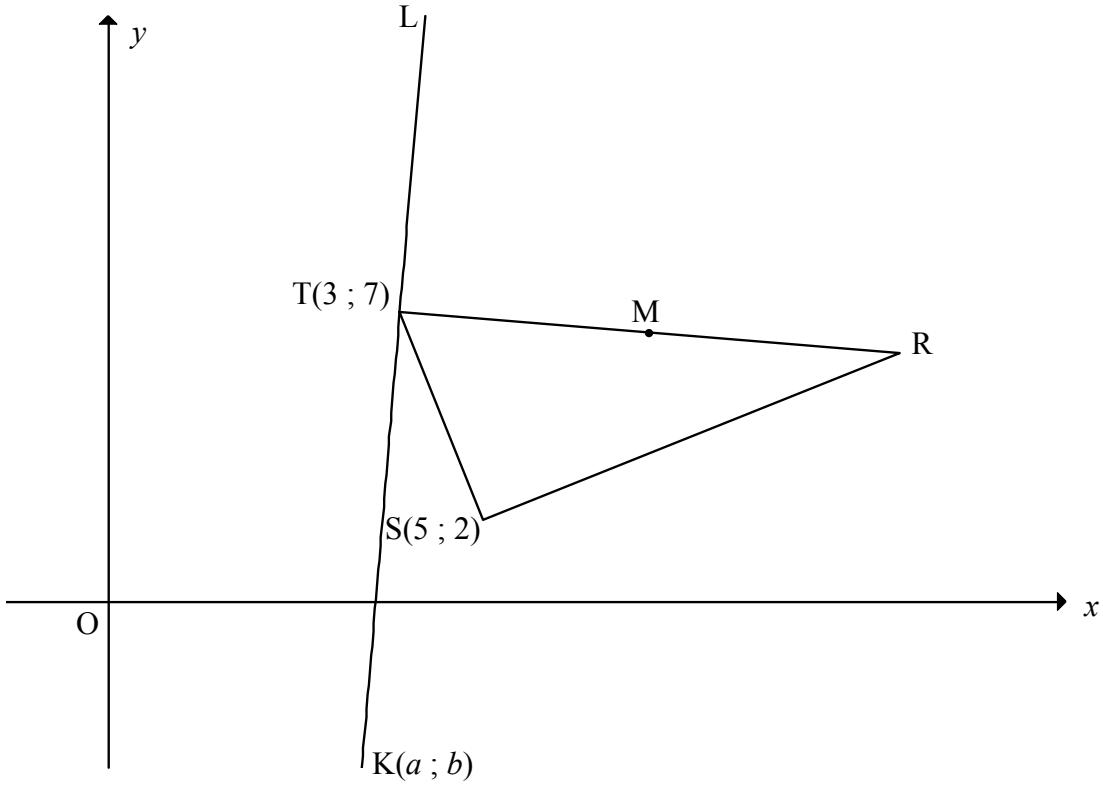


3.1	M = Midpt of AC [diags of rectangle bisect/ *hoekl v reghoek halveer*] $= \mathrm{M}\left(\frac{-7+6}{2}\ ;\ \frac{2+3}{2}\right)$ $= \mathrm{M}\left(-\frac{1}{2}\ ;\ \frac{5}{2}\right)$	✓ x-value of M ✓ y-value of M (2)
3.2	$m_{\mathrm{BC}} = \frac{3-0}{6-p} = \frac{3}{6-p}$ **OR/*OF*** $m_{\mathrm{BC}} = \frac{0-3}{p-6} = \frac{-3}{p-6}$	✓answer (1) ✓answer (1)
3.3	$m_{\mathrm{AD}} = m_{\mathrm{BC}}$ [AD | | BC] $m_{\mathrm{BC}} = 2$ $\frac{3}{6-p} = 2$ $3 = 12 - 2p$ $p = 4\frac{1}{2}$ **OR/OF** $y - y_1 = 2(x - x_1)$ $C(6\,;3)$ $y - 3 = 2(x-6)$ $\therefore y = 2x - 9$ $but\ y = 0$ $\therefore x = 4\frac{1}{2} = p$ **OR/OF**	✓ $m_{\mathrm{BC}} = 2$ ✓ equating ✓answer (3) ✓ $m_{\mathrm{BC}} = 2$ ✓ substituting (6 ; 3) ✓answer (3)

	$y = 2x + c$ $3 = 12 + c$ $-9 = c$ $y = 2x - 9$ $0 = 2x - 9$ $x = \frac{9}{2}$ $\quad \therefore p = \frac{9}{2}$	✓ $m_{BC} = 2$ ✓ substituting ✓answer (3)
3.4	DB =AC [diag of rectangle = / *hoekl v reghoek* =] $AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AC = \sqrt{(6+7)^2 + (3-2)^2}$ $AC = \sqrt{13^2 + 1^2}$ $AC = \sqrt{170}$ $\therefore$ DB = $\sqrt{170}$ or 13,04	✓ substitution ✓ length of AC ✓ AC = BD (3)
3.5	$\tan\alpha = m_{BC} = 2$ $\therefore \alpha = 63{,}43°$	✓ $\tan\alpha = m_{BC}$ ✓ $\alpha = 63{,}43°$ (2)
3.6	In quadrilateral OFBG: $O\hat{F}B = 63{,}43°$ [vert opp ∠s/*regoorst ∠e*] $F\hat{O}G = G\hat{B}F = 90°$ $\therefore O\hat{G}B = 360° - [90° + 90° + 63{,}43°]$ [sum ∠s quad/*som ∠e vierh* = 360°] $\therefore O\hat{G}B = 116{,}57°$ **OR/*OF*** $m_{AB} = -\frac{1}{2}$ $90° + O\hat{G}A = 153{,}43°$ $\therefore O\hat{G}A = 63{,}43°$ $O\hat{G}B = 180° - 63{,}43°$ $= 116{,}57°$ **OR/*OF*** $F\hat{O}G = G\hat{B}F = 90°$ $\therefore$GOFB is cyc quad $O\hat{G}B = 180° - 63{,}43°$ [∠s of cyc quad = 180°] $= 116{,}57°$ **OR/OF** $O\hat{F}B = 63{,}43°$ $X\hat{O}G = F\hat{B}G = 90°$ $\therefore$ OGBF is a cyclic quad $\therefore O\hat{G}B = 180° - 63{,}43°$ $O\hat{G}B = 116{,}57°$	✓ size of $O\hat{F}B$ ✓ S ✓ answer (3) ✓ $m_{AB} = -\frac{1}{2}$ ✓ S ✓ answer (3) ✓ S ✓ S ✓ answer (3) ✓ S ✓ S ✓ answer (3)

3.7	$M\left(-\frac{1}{2};\frac{5}{2}\right)$ is the centre/*is die middelpunt* $r=\frac{\sqrt{170}}{2}$ = radius [BD is diameter/*middellyn*] $\left(x+\frac{1}{2}\right)^2+\left(y-\frac{5}{2}\right)^2=\left(\frac{\sqrt{170}}{2}\right)^2=\frac{85}{2}=42{,}5$	✓ M is centre ✓ $r=\frac{\sqrt{170}}{2}$ ✓ equation (3)
3.8	$C\hat{B}M = B\hat{A}M = 45°$ [diag of square bisect ∠s/*hoekl v vierk halv ∠e*] ∴ BC will be a tangent [converse tan chord th/*omgekeerde raakl-koordst*] **OR/*OF*** $A\hat{M}B = 90°$ [diag of square bisect ⊥] ∴ AB is diameter BC ⊥ AB ∴ BC is tangent [line ⊥ radius *or* converse tan-chord th]  	✓S ✓ R (2) ✓S ✓ R (2) **[19]**

QUESTION/*VRAAG 4*



4.1	∠ in semi circle/ ∠ at centre = 2∠ on circle *∠ in halfsirkel* /∠ by middelpt = 2∠ op sirkel	✓ R (1)
4.2	$m_{TS} = \frac{7-2}{3-5}$ $= -\frac{5}{2}$	✓ substitution ✓ m_{TS} (2)
4.3	$m_{TS} \times m_{RS} = -1$ [TS⊥ SR] $\therefore m_{RS} = \frac{2}{5}$ $y = \frac{2}{5}x + c$ $2 = \frac{2}{5}(5) + c$ $c = 0$ $y = \frac{2}{5}x$ **OR/*OF***	✓ m_{RS} ✓substitution *m* and (5 ; 2) ✓ equation (3)

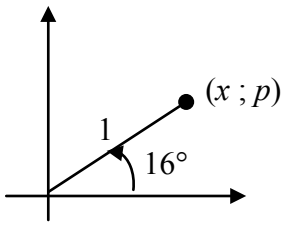
	$m_{TS} \times m_{RS} = -1$ [TS $\perp$ SR] $\therefore m_{RS} = \frac{2}{5}$ $y - y_1 = \frac{2}{5}(x - x_1)$ $y - 2 = \frac{2}{5}(x - 5)$ $y = \frac{2}{5}x$	✓ m_{RS} ✓substitution m and (5 ; 2) ✓ equation (3)
4.4.1	$r = \sqrt{36\frac{1}{4}}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$ **OR/OF** $TM = \sqrt{(3-9)^2 + \left(7 - 6\frac{1}{2}\right)^2} = \frac{\sqrt{145}}{2}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$	✓ r ✓ answer (2) ✓ substitution ✓ answer (2)
4.4.2	$M\left(9 ; 6\frac{1}{2}\right)$ $\therefore \frac{x_R + 3}{2} = 9$ and $\frac{y_R + 7}{2} = 6\frac{1}{2}$ $\therefore$ R(15 ; 6) Answer only: full marks Answer only: only 1 coordinate correct (1 mark) **OR/*OF*** $M\left(9 ; 6\frac{1}{2}\right)$ $\therefore R\left(9 + 6 ; 6\frac{1}{2} - \frac{1}{2}\right) = R(15 ; 6)$ **OR/OF**	✓ M ✓ x coordinate ✓ y coordinate (3) ✓ M ✓ x coordinate ✓ y coordinate (3)

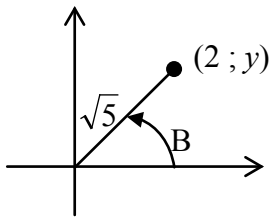
	$m_{TM} = \dfrac{9-3}{6\frac{1}{2}-7} = -\dfrac{1}{12}$ $TM: \quad 7 = -\dfrac{1}{12}(3) + c \quad y = -\dfrac{1}{12}x + \dfrac{29}{4}$ .....(1) $SR: \quad y = \dfrac{2}{5}x$ .....(2) $\dfrac{2}{5}x = -\dfrac{1}{12}x + \dfrac{29}{4}$ $\dfrac{29}{60}x = \dfrac{29}{4}$ $\therefore x = 15$ $\therefore y = \dfrac{2}{5}(15) = 6$	✓ equating ✓ x coordinate ✓ y coordinate (3)
4.4.3	$\text{ST} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\text{ST} = \sqrt{(5-3)^2 + (2-7)^2}$ $\text{ST} = \sqrt{4+25} = \sqrt{29}$ $\sin \text{R} = \dfrac{\text{TS}}{\text{TR}} = \dfrac{\sqrt{29}}{\sqrt{145}}$ *or* $\dfrac{\sqrt{5}}{5}$ *or* $\dfrac{1}{\sqrt{5}}$ *or* 0,45 **OR/OF** $TS = \sqrt{29}$ $SR = 2\sqrt{29}$ area of $\Delta\text{TSR} = \dfrac{1}{2}(\sqrt{29})(2\sqrt{29}) = 29$ $29 = \dfrac{1}{2}(\sqrt{145})(2\sqrt{29})\sin R$ $\sin R = \dfrac{\sqrt{5}}{5}$ *or* $\dfrac{1}{\sqrt{5}}$	✓substitution ✓ answer ✓ ratio (3) ✓area ✓ rule ✓ ratio (3)
4.4.4	$m_{\text{TR}} = \dfrac{7-6\frac{1}{2}}{3-9} = -\dfrac{1}{12}$ **OR/*OF*** $m_{\text{TR}} = \dfrac{7-6}{3-15} = -\dfrac{1}{12}$ $m_{\text{TR}} \times m_{\text{KTL}} = -1$ [$r \perp$ tangent] $m_{\text{KTL}} = 12$ $y - y_1 = 12(x - x_1)$ $y - 7 = 12(x-3)$ $y = 12x - 29$ substitute $\text{K}(a;b)$: $b = 12a - 29$ **OR/*OF***	✓ $m_{\text{TR}} = -\dfrac{1}{12}$ ✓ $m_{\text{KTL}} = 12$ ✓ $y = 12x - 29$ (3)

	$m_{TR} = \dfrac{7-6\frac{1}{2}}{3-9} = -\dfrac{1}{12}$ $m_{TR} \times m_{KTL} = -1$ [$r \perp$ tangent] $\dfrac{b-7}{a-3} = 12$ $b-7 = 12(a-3)$ $b = 12a - 29$ **OR/*OF*** $KR^2 = TR^2 + TK^2$ $(a-15)^2 + (b-6)^2 = (15-3)^2 + (6-7)^2 + (a-3)^2 + (b-7)^2$ $-30a + 225 - 12b + 36 = 144 + 1 - 6a + 9 - 14b + 49$ $2b = 24a - 58$ $b = 12a - 29$	✓ $m_{TR} = -\dfrac{1}{12}$ ✓ $m_{KTL} = 12$ ✓substitution (3 ; 7) & (a ; b) (3) ✓subst into Pyth ✓multiplication ✓simplification (3)
4.4.5	$TK = TR$ $\sqrt{(a-3)^2 + (b-7)^2} = \sqrt{145}$ $(a-3)^2 + (b-7)^2 = 145$ Substitute $b = 12a - 29$ [from 4.4.4] $(a-3)^2 + (12a - 29 - 7)^2 = 145$ $(a-3)^2 + (12a - 36)^2 = 145$ $a^2 - 6a + 9 + 144a^2 - 864a + 1296 - 145 = 0$ $145a^2 - 870a + 1160 = 0$ $a = \dfrac{870 \pm \sqrt{(870)^2 - 4(145)(1160)}}{290}$ $a = 2$ or $a = 4$ $\therefore b = 12(2) - 29$ or $b = 12(4) - 29$ $= -5$ $= 19$ $\therefore$ K(2 ; –5) **OR/*OF***	✓ substitution into distance formula ✓ substitution of $b = 12a - 29$ ✓standard form ✓ subst into formula or factorise ✓ values of a ✓ value of b (6)

	TK = TR $\sqrt{(a-3)^2+(b-7)^2}=\sqrt{145}$ $(a-3)^2+(b-7)^2=145$ Substitute $b = 12a - 29$ [from 4.4.4] $(a-3)^2+(12a-29-7)^2=145$ $(a-3)^2+(12a-36)^2=145$ $(a-3)^2+144(a-3)^2=145$ $(a-3)^2=1$ $a-3=\pm 1$ $a = 2$ or 4 $\therefore b = 12(2) - 29$ or $b = 12(4) - 29$ $= -5$ $= 19$ $\therefore$ K(2 ; –5)	✓ substitution into distance formula ✓ substitution of $b = 12a - 29$ ✓ $(a-3)^2=1$ ✓ ± 1 ✓ values of a ✓ value of b (6)
	OR/*OF* $KR^2=TR^2+TK^2$ $(a-15)^2+(b-6)^2=145+145$ $(a-15)^2+(12a-29-6)^2=290$ $(a-15)^2+(12a-35)^2=290$ $a^2-30a+225+144a^2-840a+1225=290$ $145a^2-870a+1160=0$ $a^2-6a+8=0$ $\therefore (a-2)(a-4)=0$ $a=2$ or $a=4$ $\therefore b=12(2)-29$ or $b=12(4)-29$ $=-5$ $=19$ K(2 ; – 5)	✓ substitution ✓ substitution of $b = 12a - 29$ ✓standard form ✓ factors ✓ values of a ✓ value of b (6)
		[23]

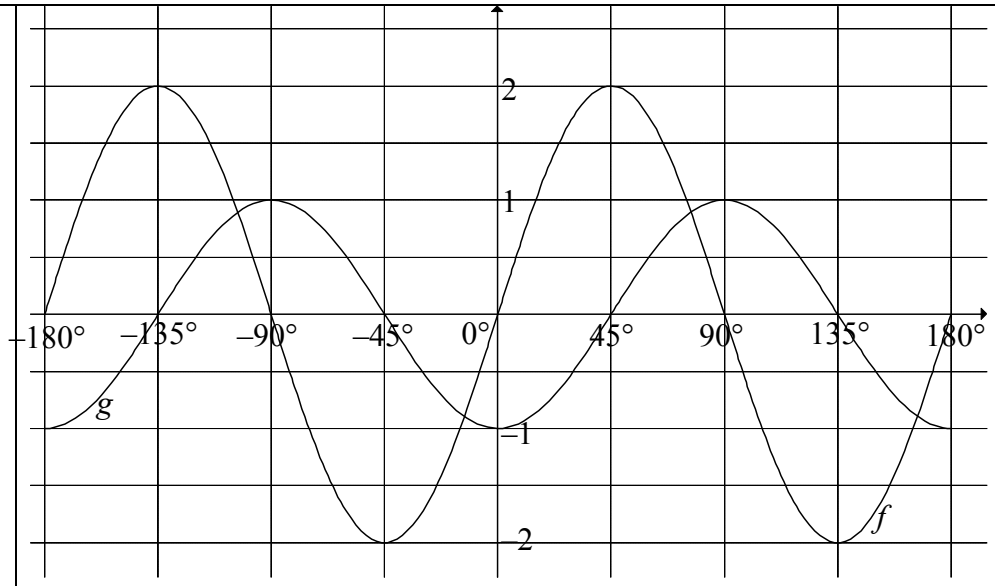
QUESTION/*VRAAG 5*

5.1.1	$\sin 196° = -\sin 16°$ $= -p$	✓reduction ✓answer (2)
5.1.2	$\cos 16° = \sqrt{1-\sin^2 16°}$ $= \sqrt{1-p^2}$ **OR/*OF*** $x^2 + p^2 = 1$ $x = \sqrt{1-p^2}$ $\therefore \cos 16° = \dfrac{\sqrt{1-p^2}}{1} = \sqrt{1-p^2}$ 	✓statement ✓answer (2) ✓x in terms of p ✓answer (2)
5.2	$\sin(A+B) = \cos[90° - (A+B)]$ $= \cos[(90° - A) - B]$ $= \cos(90° - A)\cos B + \sin(90° - A)\sin B$ $= \sin A\cos B + \cos A\sin B$	✓co-ratio ✓correct form ✓expansion (3)
5.3	$\dfrac{\sqrt{1-\cos^2 2A}}{\cos(-A).\cos(90°+A)}$ $= \dfrac{\sqrt{\sin^2 2A}}{\cos A.(-\sin A)}$ $= \dfrac{\sin 2A}{\cos A.(-\sin A)}$ $= \dfrac{2\sin A\cos A}{\cos A.(-\sin A)}$ $= -2$ **OR/OF** $\dfrac{\sqrt{1-\cos^2 2A}}{\cos(-A)\cos(90°+A)} = \dfrac{\sqrt{1-(2\cos^2 A-1)^2}}{\cos A.-\sin A}$ $= \dfrac{\sqrt{1-(4\cos^4 A - 4\cos^2 A + 1)}}{\cos A.-\sin A} = \dfrac{\sqrt{4\cos^2 A - 4\cos^4 A}}{\cos A.-\sin A}$ $= \dfrac{\sqrt{4\cos^2 A(1-\cos^2 A)}}{\cos A.-\sin A} = \dfrac{\sqrt{4\cos^2 A\sin^2 A}}{\cos A.-\sin A}$ $= \dfrac{2\cos A.\sin A}{\cos A.-\sin A}$ $= -2$ **OR/OF**	✓$\sqrt{\sin^2 2A}$ ✓$\cos A$ ✓$-\sin A$ ✓$2\sin A\cos A$ ✓ answer (5) ✓$2\cos^2 A - 1$ ✓$\cos A$ ✓$-\sin A$ ✓identity ✓ answer (5)

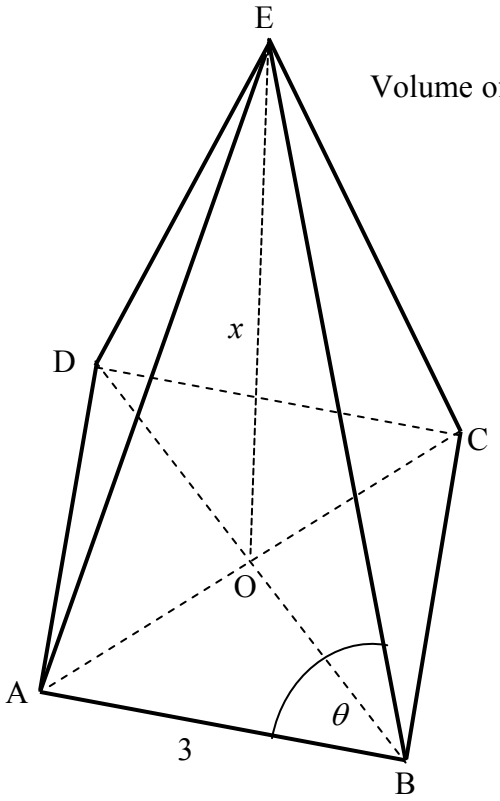
	$\dfrac{\sqrt{1-(1-2\sin^2 A)^2}}{\cos A.-\sin A}$ $=\dfrac{\sqrt{1-(1-4\sin^2 A+4\sin^2 A)}}{\cos A.-\sin A}$ $=\dfrac{\sqrt{4\sin^2 A(1-\sin^2 A)}}{\cos A.-\sin A}$ $=\dfrac{2\sin A\sqrt{\cos^2 A}}{\cos A.-\sin A}$ $=-2$	✓ $1-2\sin^2 A$ ✓ $\cos A$ ✓ $-\sin A$ ✓ identity ✓ answer (5)
5.4.1	$\cos 2B=\dfrac{3}{5}$ $2\cos^2 B-1=\dfrac{3}{5}$ $\cos^2 B=\dfrac{4}{5}$ $\therefore\ \cos B=\sqrt{\dfrac{4}{5}}\ or\ \dfrac{2}{\sqrt{5}}\ or\ \dfrac{2\sqrt{5}}{5}$ $[\,0°\le B\le 90°]$ **OR/*OF*** $\cos B=\dfrac{\sqrt{\cos 2B+1}}{2}$ $=\dfrac{\sqrt{\dfrac{3}{5}+1}}{2}$ $=\dfrac{2\sqrt{5}}{5}$	✓ identity ✓ value of $\cos^2 B$ ✓ answer (3) ✓ $=\dfrac{\sqrt{\cos 2B+1}}{2}$ ✓ value of $\cos^2 B$ ✓ answer (3)
5.4.2	$\sin^2 B=1-\cos^2 B$ $=1-\left(\dfrac{2}{\sqrt{5}}\right)^2$ $=\dfrac{1}{5}$ $\therefore\ \sin B=\dfrac{1}{\sqrt{5}}\ or\ \dfrac{\sqrt{5}}{5}$ **OR/*OF*** $(2)^2+y^2=(\sqrt{5})^2$ $4+y^2=5$ $y^2=1$ $y=1$ $\therefore\ \sin B=\dfrac{1}{\sqrt{5}}\ or\ \dfrac{\sqrt{5}}{5}$  	✓ $\sin^2 B=\dfrac{1}{5}$ ✓ answer (2) ✓ $y=1$ ✓ answer (2)

	OR/*OF* $\cos 2B = \frac{3}{5}$ $1 - 2\sin^2 B = \frac{3}{5}$ $\sin^2 B = \frac{1}{5}$ $\therefore\ \sin B = \frac{1}{\sqrt{5}}\ or\ \frac{\sqrt{5}}{5}$	✓ $\sin^2 B = \frac{1}{5}$ ✓ answer (2)
5.4.3	$\cos(B + 45°) = \cos B.\cos 45° - \sin B.\sin 45°$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{2}{\sqrt{10}} - \frac{1}{\sqrt{10}}$ $= \frac{1}{\sqrt{10}}\ or\ \frac{\sqrt{10}}{10}$ **OR/*OF*** $\cos(B + 45°) = \cos B.\cos 45° - \sin B.\sin 45°$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{2\sqrt{2}}{2\sqrt{5}} - \frac{\sqrt{2}}{2\sqrt{5}}$ $= \frac{\sqrt{2}}{2\sqrt{5}}\ or\ \frac{\sqrt{10}}{10}$	✓ expansion ✓ $\left(\frac{1}{\sqrt{2}}\right)$ ✓ $\left(\frac{2}{\sqrt{5}}\right)$ & $\left(\frac{1}{\sqrt{5}}\right)$ ✓answer (4) ✓ expansion ✓ $\left(\frac{1}{\sqrt{2}}\right)$ ✓ $\left(\frac{2}{\sqrt{5}}\right)$ & $\left(\frac{1}{\sqrt{5}}\right)$ ✓answer (4)
		[21]

QUESTION/*VRAAG* 6

6.1		✓ x- intercepts/ *afsnitte* ✓ y- intercept/ *afsnit* ✓ turning pts/ *draaipte* (3)
6.2	$f(x)-3=2\sin 2x-3$ $\therefore$ maximum value = 2 – 3 = –1	✓ ✓ answer (2)
6.3	$2\sin 2x=-\cos 2x$ $\tan 2x=-\frac{1}{2}$ $ref\angle=26{,}57°$ $2x=153{,}43°+k.180°;k\in Z$ $x=76{,}72°+k.90°;k\in Z$ *or* $x=-13.28°+k.90°,k\in Z$ **OR/*OF*** $2\sin 2x=-\cos 2x$ $\tan 2x=-\frac{1}{2}$ $ref\angle=26{,}57°$ $2x=153{,}43°+k.360°$ *or* $333{,}43°+k.360°,k\in Z$ $x=76{,}72°+k.180°$ *or* $166{,}72°+k.180°\ ;k\in Z$	✓ $\tan 2x=-\frac{1}{2}$ ✓ $2x=153{,}43°$ or – 26,56° ✓ 76,72° or – 13,28° ✓ $k.90°;k\in Z$ (4) ✓ $\tan 2x=-\frac{1}{2}$ ✓ $2x=153{,}43°$ & 333,43° ✓ 76,72° & 166,72° ✓ $k.180°;k\in Z$ (4)
6.4	$x\in(-103{,}28°;-13{,}28°)$ **OR/*OF*** $-103{,}28°<x<-13{,}28°$	✓✓ values ✓notation (3) ✓✓ values ✓notation (3) **[12]**

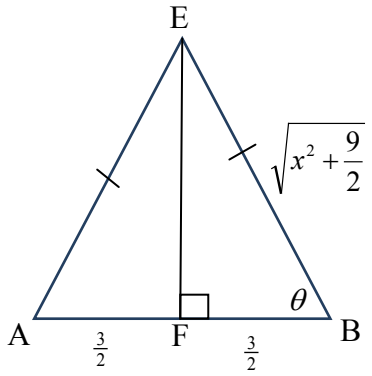
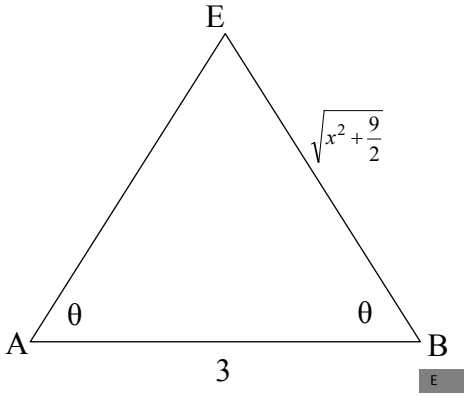
QUESTION/*VRAAG* 7



$$\text{Volume of pyramid} = \frac{1}{3}(\text{area of base}) \times (\perp \text{ height})$$

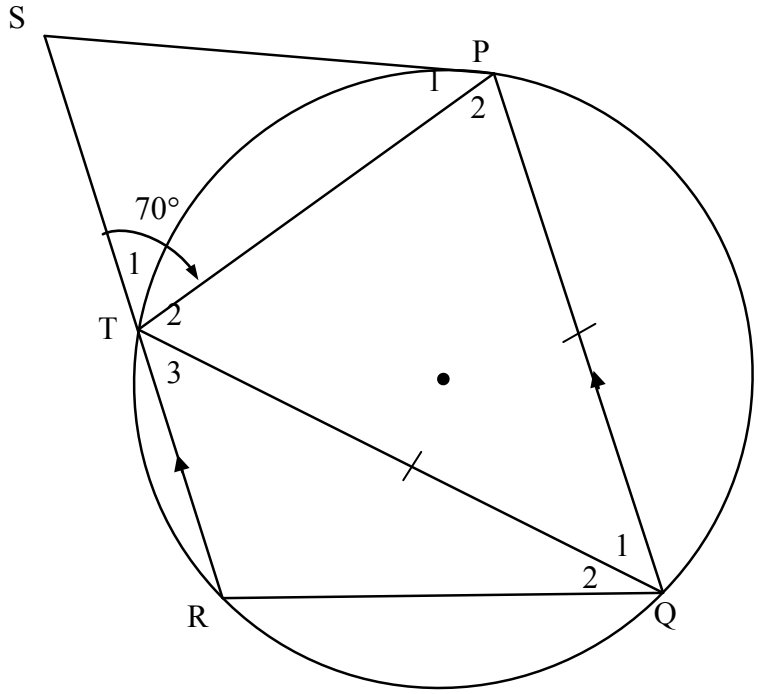
7.1	$DB^2 = 3^2 + 3^2$ [Theorem of Pyth] $= 18$ $DB = \sqrt{18}$ $OB = \frac{1}{2}DB = \frac{\sqrt{18}}{2}$ *or* $\frac{3}{\sqrt{2}}$ *or* $\frac{3\sqrt{2}}{2}$ *or* 2,12 **OR/*OF*** $\sin 45° = \frac{OB}{3}$ $OB = 3\sin 45°$ $OB = \frac{3\sqrt{2}}{2}$ *or* $\frac{3}{\sqrt{2}}$ *or* 2,12 **OF/OR** $\cos 45° = \frac{OB}{3}$ $\frac{1}{\sqrt{2}} = \frac{OB}{3}$ $OB = \frac{3}{\sqrt{2}}$ *or* $\frac{3\sqrt{2}}{2}$ *or* 2,12	✓ substitution into Pyth ✓ value of DB ✓ answer (3) ✓ correct ratio ✓ OB as subject ✓ answer (3) ✓ correct ratio ✓ special angle ✓ answer (3)

	OR/OF $A\hat{O}B = 90°$ (diagonals bisect ⊥) $OB = OA$ $AB^2 = AO^2 + BO^2$ [pyth] $\therefore AB^2 = 2OB^2$ $2OB^2 = 3^2$ $\therefore OB = \frac{3}{\sqrt{2}}$ or $\frac{3\sqrt{2}}{2}$ or 2,12	✓ OB = OA ✓ Pyth ✓ answer (3)
7.2	$BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $AE^2 = AB^2 + EB^2 - 2AB.EB\cos\theta$ $\cos\theta = \frac{AB^2 + EB^2 - AE^2}{2AB.EB} = \frac{AB^2}{2AB.EB}$ [EB = AE] $\cos\theta = \frac{AB}{2EB}$ $\cos\theta = \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$ **OR/*OF*** $BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $AE^2 = AB^2 + EB^2 - 2AB.EB\cos\theta$ $\left(\sqrt{x^2 + \frac{9}{2}}\right)^2 = 9 + \left(\sqrt{x^2 + \frac{9}{2}}\right)^2 - 2(3)\left(\sqrt{x^2 + \frac{9}{2}}\right).\cos\theta$ $\cos\theta = \frac{9}{6\sqrt{x^2 + \frac{9}{2}}}$ $= \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$	✓ substitution into Pyth ✓ length of BE ✓correct cosine rule ✓ cos θ as subject ✓ simplification (5) s ✓ substitution into Pyth ✓ length of BE ✓correct cosine rule ✓ substituting ✓ cos θ as subject (5)

	OR/*OF* $BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $\cos\theta = \frac{\frac{3}{2}}{\sqrt{x^2 + \frac{9}{2}}}$ $= \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$  	✓ substitution into Pyth ✓ length of BE ✓ sketch with values ✓ $\frac{3}{2}$ ✓ substitution (5)
	OR/OF $\hat{E} = 180° - 2\theta$ $\sin E = \sin 2\theta$ $\therefore \frac{3}{\sin 2\theta} = \frac{\sqrt{x^2 + \frac{9}{2}}}{\sin\theta}$ $\therefore \frac{3}{2\sin\theta\cos\theta} = \frac{\sqrt{x^2 + \frac{9}{2}}}{\sin\theta}$ $\therefore \frac{3}{2\cos\theta} = \sqrt{x^2 + \frac{9}{2}}$ $\cos\theta = \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$  	✓ $\hat{E} = 180° - 2\theta$ ✓ $\sin E = \sin 2\theta$ ✓ subst into sine rule ✓ diagram ✓ $2\sin\theta\cos\theta$ (5)
7.3	Volume $= \frac{1}{3}$(area of base) $\times$ ($\perp$ height) $15 = \frac{1}{3}(9) \times x$ $x = 5$ $\cos\theta = \frac{3}{2\sqrt{25 + \frac{9}{2}}}$ $\therefore \theta = 73{,}97°$	✓ substitution ✓ x-value ✓ substitution ✓ answer (4) **[12]**

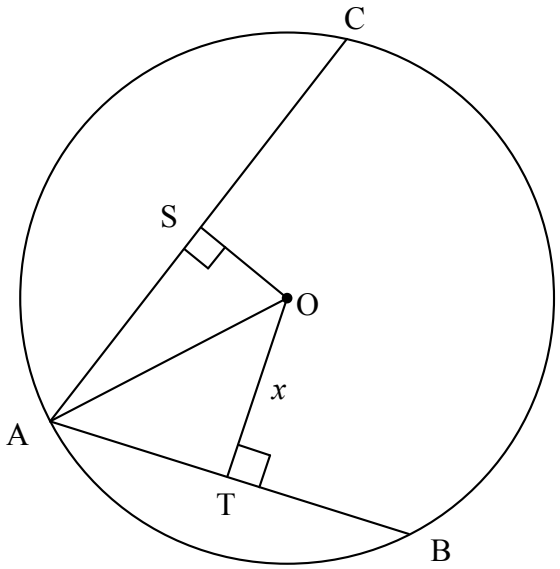
QUESTION/*VRAAG 8*

8.1



8.1.1	Alternate angles / *verwiss hoeke*, PQ ∥ SR	✓ R (1)
8.1.2(a)	$\hat{T}_2 = 70°$ [∠s opp = sides/*∠e teenoor = sye*] $\therefore \hat{Q}_1 = 180° - 2(70°)$ [∠s/*e* Δ = 180°] $= 40°$	✓ S ✓R ✓ answer (3)
8.1.2(b)	$\hat{P}_1 = 40°$ [tangent chord th/*raakl-koordst*]	✓ S ✓R (2)

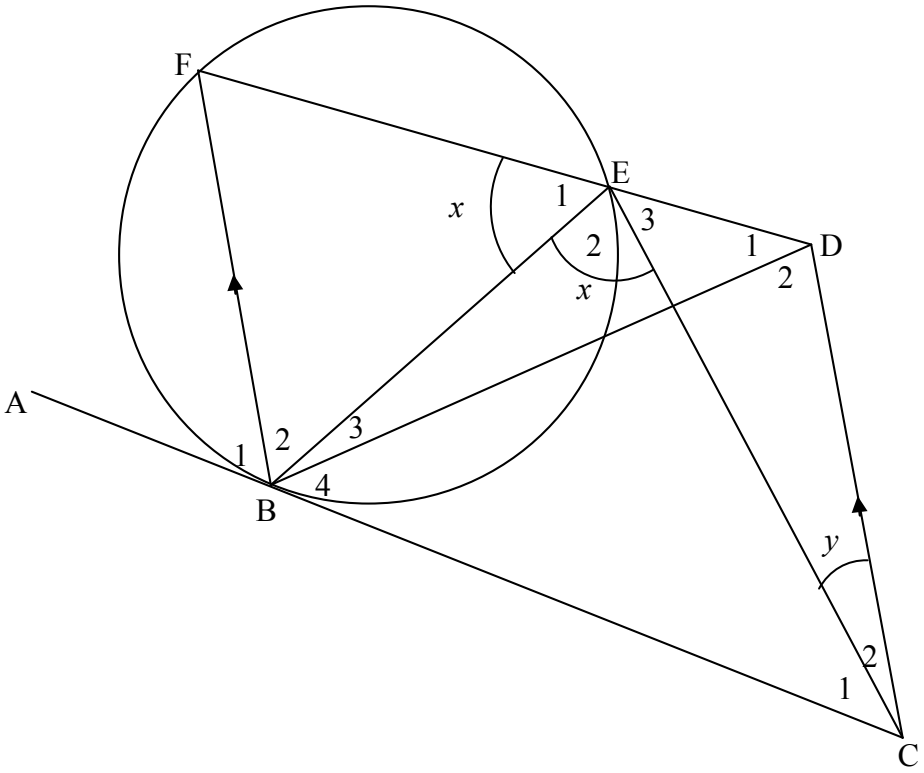
8.2



8.2.1	AT = 20 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*]	✓S (1)
8.2.2	$AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $OT^2 + AT^2 = OS^2 + AS^2$ [Pyth : ΔAOT] But AS = 24 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*] $OT^2 + 400 = \left(\frac{7}{15}OT\right)^2 + 576$ $176 = \frac{176}{225}OT^2$ $OT^2 = 225$ $OT = 15$ $\therefore AO = \sqrt{225 + 400}$ $= 25$ **OR/OF** Let OS = 7, then OT = 15 In ΔAOT: $AO^2 = 20^2 + 15^2$ $= 625$ $AO = 25$ In ΔAOS: $AO^2 = 24^2 + 7^2$ $= 625$ $AO = 25$ ∴ OA = 25 **OR/OF**	✓ equating ✓ AS = 24 ✓ substitution $OS = \frac{7}{15}OT$ ✓ OT ✓ radius (5) ✓✓ testing in ΔAOT ✓✓ testing in ΔAOS ✓conclusion (5)

	$AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $OT^2 + AT^2 = OS^2 + AS^2$ [Pyth : ΔAOT] Let OT = 15x. Then OS = 7x But AS = 24 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*] $(15x)^2 + 400 = (7x)^2 + 576$ $225x^2 + 400 = 49x^2 + 576$ $176x^2 = 176$ $x = 1$ $\therefore AO = \sqrt{225 + 400}$ $= 25$	✓ equating ✓ AS = 24 ✓ substitution ✓ $x = 1$ ✓ radius (5)
	OR/OF AS = 24 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*] $AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $= \left(\frac{7}{15}OT\right)^2 + AS^2$ $AO^2 = \frac{49}{225}(AO^2 - 20^2) + 24^2$ [Pyth : ΔAOT] $\frac{176}{225}AO^2 = \frac{4400}{9}$ $AO^2 = 625$ $AO = 25$	✓ AS = 24 ✓ substitution $OS = \frac{7}{15}OT$ ✓ equating ✓ subst Pyth ✓ radius (5) **[12]**

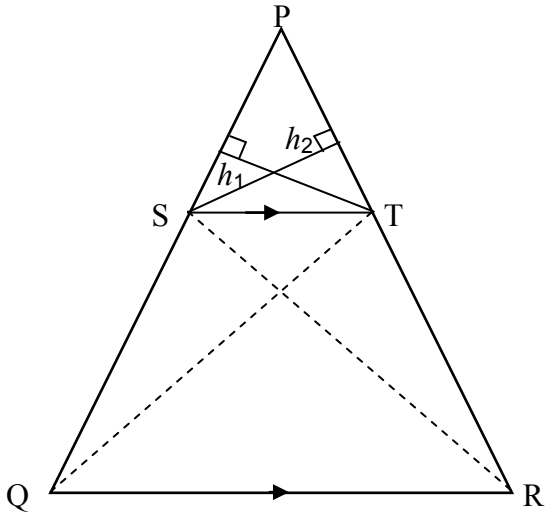
QUESTION/*VRAAG 9*



9.1.1	tangent chord theorem/*raaklyn-koordstelling*	✓ R (1)
9.1.2	corresponding/*ooreenkomstige* ∠s/*e*; FB ∥ DC	✓ R (1)
9.2	$\hat{E}_1 = B\hat{C}D$ $\therefore$ BCDE = cyclic quad [converse ext ∠ cyc quad/*omgek: buite∠kdvh*]	✓ S ✓ R (2)
9.3	$\hat{D}_2 = \hat{E}_2$ [∠*s* in the same segment/∠*e in dies segment*] $\hat{D}_2 = F\hat{B}D$ [alt ∠*s*, BF ∥ CD/*verwiss ∠e,BF*∥*CD*]	✓ S ✓ S (2)
9.4	$\hat{B}_3 = y$ *OR* $\hat{B}_3 = \hat{C}_2$ [∠s in the same segment/∠*e in dies segment*] $\hat{B}_2 = x - y$ *OR* $\hat{B}_3 + \hat{B}_2 = x$ [from 9.3 and 9.4] $\hat{C}_1 = x - y$ [from 9.2 and 9.3] $\therefore \hat{B}_2 = \hat{C}_1$ **OR/*OF*** In ΔBFE and ΔBEC $\hat{E}_1 = \hat{E}_2$ [$= x$] $\hat{F} = \hat{B}_3 + \hat{B}_4$ [tan - chord theorem] $\therefore \Delta BFE /// \Delta CBE$ [∠, ∠, ∠] $\therefore \hat{B}_2 = \hat{C}_1$	✓ S ✓ S ✓ S (3) ✓ identifying Δ's ✓ S ✓ S (3) **[9]**

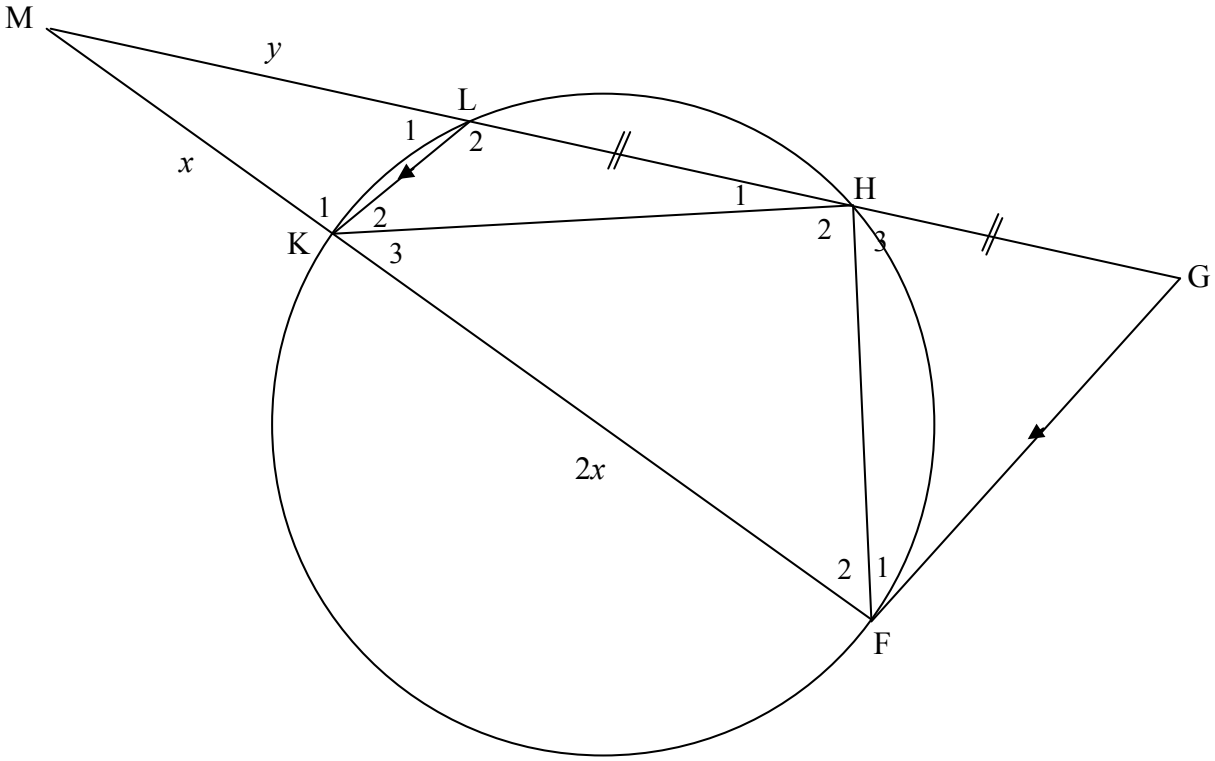
QUESTION/*VRAAG* 10

10.1



10.1	Constr : Join S to R and T to Q and draw h_1 from S $\perp$ PT and h_2 from T $\perp$ PS/ *Verbind SR en TQ en trek h_1 van S $\perp$ PT en h_2 van T $\perp$ PS*]	✓ constr/*konstruksie*
	Proof :	
	$\frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\frac{1}{2}PS \times h_2}{\frac{1}{2}SQ \times h_2} = \frac{PS}{SQ}$ equal altitudes	✓ $\frac{\text{area } \Delta PST}{\text{area } \Delta QST}$ $= \frac{\frac{1}{2}PS \times h_2}{\frac{1}{2}SQ \times h_2}$
	$\frac{\text{area } \Delta PST}{\text{area } \Delta STR} = \frac{\frac{1}{2}PT \times h_1}{\frac{1}{2}TR \times h_1} = \frac{PT}{TR}$ equal altitudes	✓ $\frac{\text{area } \Delta PST}{\text{area } \Delta STR} = \frac{PT}{TR}$
	area ΔPST = area ΔPST [common]	
	But area ΔQST = area ΔSTR [same base, height; ST || QR]	
	$\therefore \frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\text{area } \Delta PST}{\text{area } \Delta STR}$	✓ S ✓ R
	$\therefore \frac{PS}{SQ} = \frac{PT}{TR}$	✓ S
		(6)

10.2



10.2.1	Corresponding/*Ooreenkomstige* ∠s/*e*; GF ∥ LK	✓ R (1)
10.2.2(a)	$\frac{GL}{LM} = \frac{FK}{KM}$ OR $\frac{GL}{y} = \frac{2x}{x}$ [prop theorem/*eweredighst*; GF ∥ LK] $\frac{2GH}{y} = \frac{2x}{x}$ [LH = HG] $\therefore GH = y$	✓S ✓ R ✓ GL = 2GH (3)

10.2.2(b)	$\hat{K}_1 = G\hat{F}M$ [corresponding/*ooreenkomst* ∠ s; GF ∥ LK] $L\hat{K}M$ or $\hat{K}_1 = M\hat{H}F$ [ext ∠ cyclic quad/*buite∠koordevh*] $M\hat{H}F = G\hat{F}M$ In ΔMFH and ΔMGF: $\hat{M} = \hat{M}$ [common/*gemeen*] $M\hat{H}F = G\hat{F}M$ [proven/*bewys*] ∴ ΔMFH ||| ΔMGF [∠∠∠] **OR/*OR*** $\hat{K}_1 = G\hat{F}M$ [corresponding/*ooreenkomst* ∠ s; GF ∥ LK] $L\hat{K}M$ or $\hat{K}_1 = M\hat{H}F$ [ext ∠ cyclic quad/*buite∠koordevh*] $M\hat{H}F = G\hat{F}M$ In ΔMFH and ΔMGF: $\hat{M} = \hat{M}$ [common/*gemeen*] $M\hat{H}F = G\hat{F}M$ [proven/*bewys*] $\hat{F}_2 = \hat{G}$ [∠s of Δ = 180°] ∴ ΔMFH ||| ΔMGF	✓S ✓ R ✓ S ✓ S ✓ R (5) ✓S ✓ R ✓ S ✓ S ✓ S (5)
10.2.2(c)	$\therefore \frac{GF}{FH} = \frac{MF}{MH}$ [||| Δs] $= \frac{3x}{2y}$	✓S ✓R (2)
10.2.3	$\frac{MF}{MH} = \frac{MG}{MF}$ [||| Δs] $\frac{3x}{2y} = \frac{3y}{3x}$ [from 10.2.2(*c*)] $\frac{y^2}{x^2} = \frac{9}{6} = \frac{3}{2}$ $\frac{y}{x} = \sqrt{\frac{3}{2}}$	✓ S ✓substitution ✓ simplification (3) **[20]**
	TOTAL MARKS	**150**