Induced Cultural Globalization by an External Vector Field in an Enhanced Axelrod Model

Arezky H. Rodríguez[1], M. del Castillo-Mussot[2], and G.J. Vázquez[3]

[1] Academia de Matemáticas, Universidad Autónoma de la Ciudad de México, Mexico City, Mexico
arezky@gmail.com
[2] Departamento de Estado Sólido, Instituto de Física, Universidad Nacional Autónoma de México (UNAM), Apdo. Postal 20-364, San Ángel 01000, México D.F., Mexico
lldlcstll@yahoo.com
[3] Departamento de Estado Sólido, Instituto de Física, Universidad Nacional Autónoma de México (UNAM), Apdo. Postal 20-364, San Ángel 01000, México D.F., Mexico
jorge@fisica.unam.mx

Abstract. A new model is proposed, in the context of Axelrod's model for the study of cultural dissemination, to include and external vector field (VF) which describes the effects of mass media on social systems. The VF acts over the whole system and it is characterized by two parameters: a non-null overlap with each agent in the society and a confidence value of its information. Beyond a threshold value of the confidence there is induced monocultural globalization of the system lined up with the VF. Below this value, the multicultural states are unstable and certain homogenization of the system is obtained in opposite line up according to that we have called *negative publicity* effect. Three regimes of behavior for the spread process of the VF information as a function of time are reported.

1 Introduction

Agent-Based Models (ABMs) [5, 11] are computer simulations of the local interactions of the members of a population which could be plants and animals in ecosystems [15], vehicles in traffic, people in society [1], etc. Locally interactions at lower-level give rise to the spontaneously emergence of higher-level organizations whose properties are not possessed by the individuals neither directly determined by them. Complex and non-linear phenomena have attracted the attention of the scientific community to study the interplay between the lower and higher levels of organizations [20]. These models typically consist of an environment or framework in which the interactions occur among some number of individuals defined in terms of their behaviors (procedural rules) allowing the tracking of the characteristics of each individual through time.

There are lots of applications to model different aspects of dynamics in society [7, 16, 26]. Specifically, the Axelrod model [1, 2, 8] is an ABM designed to investigate the dissemination of culture among interacting agents in a society. In this model, society is represented by a lattice composed by a 2-dimensional array of vectors (agents) with a number of entries called "features". The definition of its culture is given by the set of traits an agent has in its features. The Axelrod's model has been exhaustively implemented to study a great variety of problems: the nonequilibrium phase transition

Y. Demazeau et al. (Eds.): 7th International Conference on PAAMS'09, AISC 55, pp. 310–318.
springerlink.com

between monocultural and multicultural states [18], the cultural drift driven by noise [19, 22], nominal and metric features [10, 17], propaganda [6], time evolution dynamics [25], the resistance of a society to the spread of a foreign cultural traits [4], finite size effects [24], the impact of the evolution of the network structure with cultural interaction [9] among others.

Some works have been done including in the Axelrod model an extra agent acting as a vector field (VF) over the whole society with the purpose of simulating a mass media effects [12, 13, 14, 23]. In all of them, the interaction between the external VF and agents is similar to that between an agent and its neighbors: they interact only if they have at least one common trait in their corresponding features. In this formalism, the inclusion of an external field, which does not change its values on time, introduces an asymmetry on the lattice, which can now be described as composed by two groups of agents: group A where agents have trait(s) in common with the VF and group B whose do not. All the interactions can now be classified as follows: agents from group A with VF (VF-A), agents from group B with VF (VF-B), between agents from group A (A-A), between agents from group B (B-B) and finally between agents from group A and B (A-B). The VF-B interaction is a null interaction because agents from group B do not share traits with the VF. In this way, the only opportunity of agent B to acquire one VF trait is through a diffusion mechanism with the combined interactions VF-A plus A-B. In those models it is also included the strength of the VF as a probability P of interactions VF-A and VF-B while the probability of interactions A-A, B-B and A-B are given then by $1-P$. Therefore the diffusion mechanism has very low probability as P increases and agents from group B are set apart from the VF information. Furthermore, the mechanism VF-B can be very active (time consuming) but with null effects, and the internal relaxing mechanism B-B is not able to drive agents to the final state in an efficient way. Thus, the final absorbing states obtained in those previous models consisting in multicultural states when the VF strength is increased is then not surprising [12, 13, 14, 23].

Our current interest is to develop a new model for the inclusion of an extra agent acting as a vector field (VF) over the *whole* society to overcome the difficulties achieved by the previous models described above. As mentioned in Ref. [23], the media information is socially processed through personal networks. Then, models will be more realistic is they allow a strong interaction with the VF without loss of interchanges between agents on the lattice. It is also important to say that mass media designs its publicity in a clever way. As mentioned by the anthropologist Gregory Bateson: to produce a change it is necessary to be different but, at the same time, it is necessary to be "close enough" to be taken into account [3]. When acting over the society, mass media always try to have something in common with the people chosen as target of publicity or propaganda. It is designed to offers attractive materials for the whole society: news, sports, soap operas, movies, cartoons, music, arts, etc.

Our goal here is to develop a model to include this effect considering an additional non-zero probability of all agents to copy a trait from the VF, even if they do not share any trait of their features. Section 2 is devoted to that purpose. In section 3 it is exposed some numerical calculations and finally some conclusions are outlined in section 4.

2 The Model

The system consists of L^2 agents as the sites of a square lattice. The state of an agent i is defined as a vector of F *nominal* components called features given by $\sigma_i = (\sigma_{i_1}, ..., \sigma_{i_f}, ..., \sigma_{i_F})$ which characterize the nominal F-dimensional culture of the corresponding agent. This way, each agent has four nearest neighbors but as the fifth it is introduced the VF $\mathscr{M}$ with nominal features $\sigma_{\mathscr{M}} = (\sigma_{\mathscr{M}_1}, ..., \sigma_{\mathscr{M}_f}, ..., \sigma_{\mathscr{M}_F})$. The VF intents to simulate and external mass media or publicity which acts over the whole society. Then, each agent can interact with five agents: its four nearest neighbors and the VF $\mathscr{M}$, all with equal probability $1/5$. Additionally, each feature σ_{i_f} and $\sigma_{\mathscr{M}_f}$ can take any of the values in the set $\{0, 1, ..., q-1\}$ which are the corresponding cultural traits of an agent i or the VF $\mathscr{M}$. Initially, the values of the vectors σ_i and $\sigma_{\mathscr{M}}$ are randomly and independently set with one of the q^F state vectors with uniform probability.

The interaction between different agents is possible only when the two vector have an overlap $0 < l < 1$ where the overlap between two agent i and j is the number of shared traits and it is given by $l(i,j) = \sum_{f=1}^{F} \delta_{\sigma_{i_f}, \sigma_{j_f}}$. Here δ is the Kronecker symbol. The probability, which we call here *nominal* probability, of the interaction between two agents is given by $p(i,j) = l(i,j)/F$. In general, the situation $p(i,j) = 0$ is possible when the overlap between two agents is zero, but the case where the probability between an agent and the VF is zero is not an acceptable situation for a publicity (or mass media) which intentionally designs its interaction in such a way that always there are features which have traits in common with the agent subject of the influence to guarantee that the connection is active.

In order to include this important effect in our model, we included some *effective* features that the VF always shares with each agent when interacting, besides nominal features, with the purpose to simulate phenomenologically in a simple way the almost omnipresent force of today's publicity or propaganda in mass media that offers something for all tastes and ages (magazines, radio programs, TV series, etc). Note that the effective features are related with the dynamics between the agents and the VF while the nominal features are related with the dynamics between agents inside the lattice. The specific nature in real society of the effective features is not of importance here. It will be different for different agents, but the intention is to take into account the specific design of the publicity that mass media does to attract everyone. For $\varepsilon/F < 1$ where ε is defined as the "effective feature", the VF and the agent share more nominal than effective features and the VF can be considered as a "perturbation" to the internal interaction between different agents in the society. In our model, the parameter ε not only takes natural values, but also fractional values, as it will be seen later.

Therefore, the probability of interaction between the external vector and an agent, which we call here *extended* probability, is written as

$$p(i, \mathscr{M}) = \frac{l(i, \mathscr{M}) + \varepsilon}{F + \varepsilon} = \frac{l(i, \mathscr{M})/F + \varepsilon/F}{1 + \varepsilon/F} \tag{1}$$

where $l(i, \mathscr{M})$ is the overlap of the nominal features between agent i and the VF. The probability is zero only when there are not effective features ($\varepsilon = 0$) and the overlap between the agent i and the VF is zero. In contrast, in all the other cases the probability

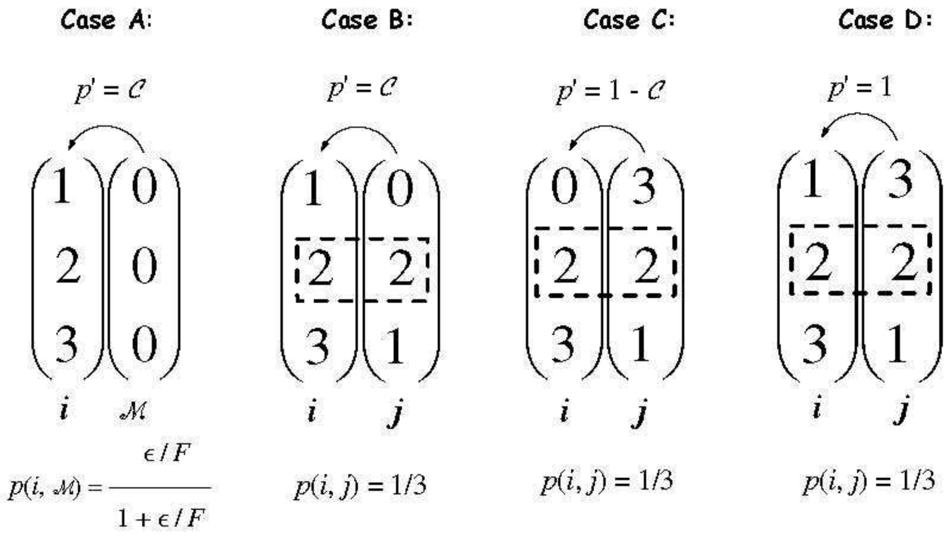

Fig. 1. Four possible cases of interaction for a system with $F = 3$ features. Shared features are indicated inside a dashed rectangle. The probability of interaction $p(i,j)$ (or $p(i,\mathscr{M})$ in Case A) is indicating below. In each case the trait in σ_{i_1} will be deleted by coping trait σ_{j_1} (or by trait $\sigma_{\mathscr{M}_1}$ in Case A). The probability of coping/deleting trait 1 is given by a) $p' = \mathscr{C}$, b) $p' = \mathscr{C}$, c) $p' = 1 - \mathscr{C}$ and c) $p' = 1$.

$p(i,\mathscr{M})$ is always different from zero. For $\varepsilon = 0$ the values for the case with no effective features are recovered. As expected, the probability is lager for larger values of the effective features ε for a given value of $l(i,\mathscr{M})$ and also increases for larger values of the overlap $l(i,\mathscr{M})$ at a given number of ε. Finally, when the agent i and the VF share all the nominal features ($l(i,\mathscr{M}) = 1$) the probability is always one for any value of ε.

In Ref. [21] the authors have used an expression similar to Eq. (1), but with $\varepsilon = 1$ and, as in Ref. [13], they have modelled the strength of the VF as a probability of the interaction between agents and the VF. Increasing values of this probability implies a decreasing value of the probability for agents interacting between each other and then the corresponding diffusion of traits values between agents can be stopped which is not a realistic or desirable effect. They have found that only monocultural states are obtained and only by introducing a noise rate it is possible to drive the system to a multicultural final state.

In our case, we are not interested to include the effects of random perturbation effects, but we instead introduce another parameter related with the confidence of the information belonging to the VF. As "confidence" we understand here the credibility granted by agents to the information possessed by the VF. It is included as an extra probability $\mathscr{C}$ for agent i to copy an entry directly from the VF or an entry from another agent j that belongs to the VF either. It is also included as an extra probability $1 - \mathscr{C}$ when agent i, when coping a trait, deletes and information the VF possesses in the same feature.

To clarify this important concept we show in Fig. 1 four situations of interaction which resume all the possible cases. In Case A it is described an interaction between agent i and the VF which has been set to (0,0,0) without lost of generality. None of the nominal features are shared and then the probability of interaction $p(i,\mathscr{M})$ depends only from the value of the effective features ε according to the expression in the figure and is always larger than zero. In the practical case when agent i copies, for example the first entry, then the VF will be copied with probability $p' = \mathscr{C}$ which characterize the confidence of the information possesses by the VF. In the next cases, the interaction occurs between agents i and j which only share one trait of three posibles. The probability of interaction is then given by $p(i,j) = 1/3$ in all these cases. In Case B the nominal feature that agent i selects to copy from agent j coincides with the value the VF has in the same feature. Then, as in Case A, the corresponding trait is copied with probability $p' = \mathscr{C}$. In Case C, when coping, agent i will delete its trait which is equal to that possessed by the VF. Then, it is deleted with probability $p' = 1 - \mathscr{C}$. Finally, in Case D the traits copied and deleted are not related with the VF and then they are copied with probability $p' = 1$. Note that according with these rules of interaction, when the confidence of traits possessed by the VF are $\mathscr{C} = 1$, these traits are always copied with probability $p' = 1$ and never deleted. Otherwise, if the confidence of the traits possessed by the VF is $\mathscr{C} = 0$, this traits are never copied ($p' = 0$) and are always deleted with probability $p' = 1$. Then, starting from the initial condition described above, the system evolves by iterating the following steps:

(1) Select at random an agent i on the lattice, which is the active element.
(2) Select at random, with equal probability, an agent of interaction. It could be one of the four nearest neighbors or the VF.
(3) Calculate the overlap $l(i,s)$ where $s = j$ for the neighbor or $s = \mathscr{M}$ for the VF. If $s = \mathscr{M}$, the agent i and the VF interact with the extended probability $p(i,\mathscr{M})$. If $s = j$ and $0 < l(i,j) < F$, agents i and j interact with the nominal probability $p(i,j)$.
(4) In case of interaction between agent i and agent s, choose a position trait h at random such that $\sigma_{i_h} \neq \sigma_{j_h}$ (or $\sigma_{i_h} \neq \sigma_{\mathscr{M}_h}$) and then set $\sigma_{i_h} = \sigma_{j_h}$ (or $\sigma_{i_h} = \sigma_{\mathscr{M}_h}$) according to:
  (4.1) if $s = \mathscr{M}$ then set $\sigma_{i_h} = \sigma_{\mathscr{M}_h}$ with probability $p' = \mathscr{C}$,
  (4.2) if $s = j$ and $\sigma_{j_h} = \sigma_{\mathscr{M}_h}$ then set $\sigma_{i_h} = \sigma_{j_h}$ with probability $p' = \mathscr{C}$,
  (4.3) if $s = j$ and $\sigma_{i_h} = \sigma_{\mathscr{M}_h}$ then set $\sigma_{i_h} = \sigma_{j_h}$ with probability $p' = 1 - \mathscr{C}$.
  (4.4) if $s = j$ and both $\sigma_{j_h} \neq \sigma_{\mathscr{M}_h}$ and $\sigma_{i_h} \neq \sigma_{\mathscr{M}_h}$, then set $\sigma_{i_h} = \sigma_{j_h}$ with probability $p' = 1$.

Before studying the effects of the VF in our model let us review the original Axelrod's model. The computational dynamics of this model ends when the system reaches an absorbing state characterized by either $l(i,j) = 0$ or $l(i,j) = F$ for all pairs of closed neighbors (i,j). A class of absorbing state, given by q^F different configurations is called the "monocultural" state which corresponds to the case where $l(i,j) = F$ for all pairs of closed neighbors. In this case, all agents in the network share the same trait at each feature ($\sigma_{i_h} = \sigma_{j_h}$ for all (i,j)) and the dynamics ends. Another class of absorbing state is called "multicultural" state and consist of at least two (or more) homogeneous domains which agents have cultural traits completely different. This way, two agents belonging to two different domains have zero overlap. The multicultural state is reached when each agent in the lattice has full of null overlap with all its neighbors. In these cases, a domain is given by a set of contiguous sites with identical state vector.

Previous works have shown that the system reaches monocultural or multicultural states in dependence of lower ($q < q_c$) or higher values ($q > q_c$) of the cultural diversity

q [8]. To characterize the transition it has been considered two different order parameters: the average fraction of different cultural domains or the average number of agents in the biggest domain $< S_{max} >$ normalized to the number of lattice elements. In our case, it will be shown how our model produces a complex pattern of social behavior with affinities and repulsions to the influence of the VF in dependence of the effective features ε and the confidence $\mathscr{C}$ of the information.

3 Numerical Results

Numerical simulations have been carried out in lattices with $L^2 = 30 \times 30$ agents and $F = 4$ features each. Different absorbing states have been found and we report the average realization number of agent in the largest domain over 50 different initial conditions.

Figure 2 shows the calculations of $< S_{max} >$ as a function of q at the absorbing state. Each panel shows the result for a certain value of the ratio ε/F and different values of the confidence $\mathscr{C}$ in increasing order. The result using the Axelrod model without VF is included for comparison with full square dots. In this case, the system reaches a monocultural state at $q < q_C \approx 18$ and a multicultural state at $q > q_C$. In panel a) we set $\varepsilon = 1.0$. It can be seen that when the value of the confidence $\mathscr{C}$ is small ($\mathscr{C} = 0.05$), the results are very close to those of Axelrod model. The monocultural states remain unchanged at $q < q_C$ but the multicultural state is less “robust” and higher values of $< S_{max} >$ are obtained. For increasing values of $\mathscr{C}$, higher values of $< S_{max} >$ for $q > q_C$ are obtained, as seen with $\mathscr{C} = 0.10$ (hence the number of different domains in the multicultural state are smaller) and finally, at $\mathscr{C} = 0.15$, the multicultural states vanish and the system remains in monocultural states for all values of q. Then, it can be concluded that the increasing value of the confidence induces an homogenization of the cultural information that the system has, even at those values of q where the system

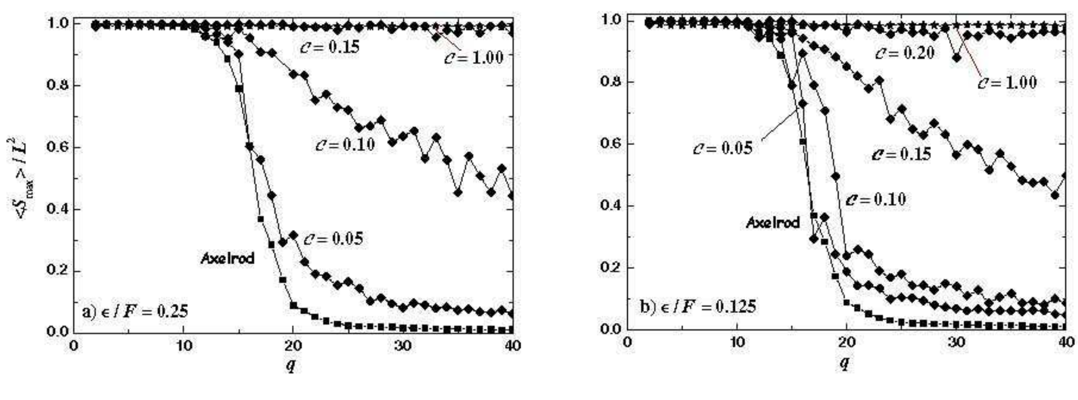

Fig. 2. Calculation of the normalized average number of agent in the greatest domain at an absorbing state as a function of q averaged over 50 realizations. The values of the ratio ε/F are a) 0.25 and b) 0.125. At each panels, different value of the confidence $\mathscr{C}$ is taken into account in increasing order. The case of the Axelrod's model without VF is included in each panel with full square dots. Full rhombus indicate that the corresponding absorbing states do not share the information possessed by the external field, while full stars indicate full coincidence of the absorbing state with the VF.

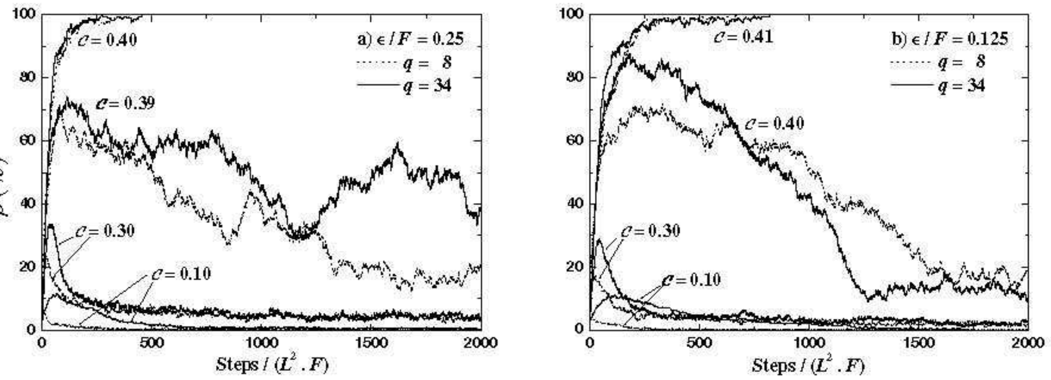

Fig. 3. Percent of the VF information in the lattice as a function of time for different values of the ratio ε/F and the confidence $\mathscr{C}$. In straight lines is used $q = 34$ while in dotted lines it is used $q = 8$.

reaches multicultural states when there is no VF. Then, multicultural states are unstable for increasing values of the confidence $\mathscr{C}$ at this value of the effective trait ε. In panel b) we set $\varepsilon = 0.5$. It can be seen that multicultural states at $q > q_c$ are again obtained for low values of $\mathscr{C}$ but now higher values are needed for the confidence to produce a cultural homogenization ($< S_{max} \approx 1 >$). This can be seen when comparing the results for $\mathscr{C} = 0.15$ in both panels.

Nevertheless, care has to be taken when analyzing the information of the induced monoculture at $q > q_c$ when the VF is present (as the case $\mathscr{C} = 0.15$ in Fig. 2 a) and $\mathscr{C} = 0.20$ in Fig. 2 b)). It is interesting to know whether the greatest domain in the absorbing state characterized by $< S_{max} >$ has, or not, the information possesses by the VF in the corresponding nominal features. This information is indicated in Fig. 2, where all the full dots show absorbing states where the corresponding greatest domain does not possess the information of the VF in any of its features. That is, the overlap between VF and the cultural state of the largest domain is zero. Only at those absorbing states indicated by full stars the corresponding biggest domain fully shares the information at nominal features of the VF. Then, it is obtained the interesting result that at low confidence values $\mathscr{C}$, the culture homogenization induced by the VF results in a negation or cancellation by the agents of the lattice of the information possesses by an external media. Here we call this phenomenon *negative publicity* effect, and it represents the process occurring in society when a group or different groups of people gather together, physically or intellectually, against an external action they consider misconceived.

In order to study in more detail how the information of the VF spreads (or not) into the society, we have defined the parameter ρ which gives the percent of the total amount of the information that agents share in their nominal features with the VF as a function of time. It is given by

$$\rho = 100 \times \frac{1}{L^2 F} \sum_{i=1}^{L^2} \sum_{f=1}^{F} \delta_{\sigma_{i_f}, \sigma_{\mathscr{M}_f}} \tag{2}$$

and it is shown in Fig. 3. The calculation has been done for different values of the ratio ε/F and different values of the confidence $\mathscr{C}$ taking in consideration only one initial state. In straight lines it is considered $q = 34$ while in dotted lines $q = 8$. When the dynamics starts from random initial conditions, the information possessed by the VF is already present in the lattice and are shared by some agents. This anisotropy gives rise to the strong increase of its percent at the beginning. Nevertheless, if the confidence value is low, the information is avoided by agents when traits are copied and it percent decreases with time after some maximum is achieved. This can be seen for $\mathscr{C}$ equal or below 0.39 and 0.40 in Fig. 3 a) and b) respectively. It is necessary a higher value of $\mathscr{C}$ for drive the system to a monocultural state with all agents aligned with the VF information. On the other hand, if the confidence value is high enough, the percentage of the VF information increases continuously until the absorbing state is reached. This can be seen for $\mathscr{C} = 0.40$ and 0.41 in Fig. 3 a) and b). At the same time, there is a sharp discontinuity between the regions of confidence values where the system remains monoculture and multiculture, as can be seen between $\mathscr{C} = 0.39$ and 0.40 in Fig. 3 a) and between $\mathscr{C} = 0.40$ and 0.41 in Fig. 3 b). For higher values of $\mathscr{C}$ the system reaches the monocultural state in only few time steps, while for intermediate values of $\mathscr{C}$ the value of ρ changes slowly, at least until $2000L^2F$ time steps where the simulation was artificially aborted. At those values of $\mathscr{C}$ the system seems to be in a quasi-stationary state and no absorbing state was found in these regions. For enough low values of $\mathscr{C}$, as those shown in Fig. 2, the system reaches the multicultural state and there is almost no VF information on the lattice, as seen for $\mathscr{C} = 0.10$ in Fig. 3 in all panels.

4 Conclusions

It is developed here a new model for the inclusion of an external vector field in the Axelrod model at zero temperature to describe the effects of the mass media on a social system. The clever design of publicity which allows the mass media to have influence over the whole society was included as a non-zero *extended* probability of traits being copied. This important effect is related with a parameter ε which can be interpreted as an extra effective feature or features the mass media could have with all agents in society, beyond the nominal features. This effective feature(s) is(are) used by the VF to reinforce the frequency of certain information already present on the society or to introduce a new one. It is also included in the model a *confidence* value of the information possessed by the VF. It is modelled as a probability of copying/deleting VF information which represents the criteria a person (or a group of persons) has (have) about what the mass media is proposing to the society. Three different regimes were found. First, for very low values of this confidence, the dynamics recovers the Axelrod model with no external field, but an increasing value produces an homogenization on the society which would be multicultural without the external influence. This cultural homogenization is lined up against the acting influence of the VF with a zero overlap with the VF information. We have called here *negative publicity* to this effect. It simulates the behavior of people in society who gathers together against the external information they estimate wrong or incorrect. Second, for large values of the confidence, the system reaches a quasi-stationary state with only slow changes of the amount of VF information in the society and no absorbing state was found for the amount of time steps tested. Finally, in the third case, higher enough values of the confidence produce a completed homogenized lattice lined up with the VF information, a situation that could be the purpose of mass media, politic party, etc. These results are qualitatively in agreement with the intuitive idea that with enough bombardment of the mass media information, that is accepted as valid and trusted by people, there will be a strong induced culture homogenization in the society. In other words, when people assume this information as personal the cultural differences tend to disappear. It is important to mention that the confidence parameter $\mathscr{C}$ can be experimental measured and, therefore, it could advance the results expected on a society when designing publicity.

Acknowledgements

The authors thank UNAM and CONACyT for partial financial support through Grants IN-114208 and 45835-F respectively. AHR also thanks psychologist Gezabel Guzmán for helpful suggestions and enlightening discussions.

References

1. Axelrod, R.: J. Conflict Resolut. 41, 203 (1997)
2. Axelrod, R.: The Complexity of Cooperation. Princeton, New Jersey (1997)
3. Bateson, G.: Mind and Nature: A Necessary Unity. Bantam Books, Toronto (2002)
4. Boccara, N.: arXiv:nlin/0611035v1 (2006)
5. Bonabeau, E.: PNAS 99, 7280 (2002)
6. Carletti, T., Fanelli, D., Grolli, S., Guarino, A.: Europh. Lett. 74, 222–228 (2006)
7. Castellano, C., Fortunato, S., Loreto, V.: arXiv:physics/0710.3256
8. Castellano, C., Marsili, M., Vespignani, A.: Phys. Rev. Lett. 85, 3536 (2000)
9. Centola, D., González-Avella, J.C., Eguíluz, V.M., Miguel, M.S.: J. Conflict Resolut. 51, 2007 (2007)
10. Flache, A., Macy, M.W.: arXiv:physics/0604201v1 (2006)
11. Goldstone, R.L., Janssen, M.A.: TRENDS in Cognitive Sciences 9, 424 (2005)
12. González-Avella, J.C., Cosenza, M.G., Klemm, K., Eguíluz, V.M., Miguel, M.S.: J. Artif. Soc. Soc. Simul. 10, 9 (2007)
13. González-Avella, J.C., Cosenza, M.G., Tucci, K.: Phys. Rev. E 72, 065,102(R) (2005)
14. González-Avella, J.C., Eguíluz, V.M., Cosenza, M.G., Klemm, K., Herrera, J.L., Miguel, M.S.: Phys. Rev. E 73, 046, 119 (2006)
15. Grimm, V., Railsback, S.F.: Individual-based Modeling and Ecology. University Press, Princeton (2005)
16. Helbing, D.: Quantitative Sociodynamics. Kluwer Academic Publishers, Dordrecht (1995)
17. Jacobmeier, D.: Int. Journ. Modern Phys. C 4, 633 (2005)
18. Klemm, K., Eguíluz, V.M., Toral, R., Miguel, M.S.: Phys. Rev. E 67, 026, 120 (2003)
19. Klemm, K., Eguíluz, V.M., Toral, R., Miguel, M.S.: Phys. Rev. E 67, 045, 101(R) (2003)
20. Laughlin, R.: A Different Universe: Reinventing Physics from the Bottom Down. Basic Books, New York (2006)
21. Mazzitello, K.I., Candia, J., Dossetti, V.: Int. J. Mod. Phys. C 18, 1475 (2007)
22. Sanctis, L.D., Galla, T.: arXiv:0707.3428v1 (2007)
23. Shibanai, Y., Yasuno, S., Ishiguro, I.: J. Conflict Resolution 45, 80 (2001)
24. Toral, R., Tessone, C.J.: arXiv:physics/0607252v1 (2006)
25. Vázquez, F., Redner, S.: EPL 78, 18, 002 (2007)
26. Weidlich, W.: Sociodynamics. Dover Publications, New York (2000)