Chapter 1

DYADIC AND SOCIAL INFLUENCE ON THE AXELROD MODEL WITH CLEVER MASS MEDIA

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Abstract

The Axelrod model has been proven to be a fruitful model to study different social phenomena related to the dissemination of cultures. In recent years, it has been widely studied and several settings have been implemented to understand different social situations. Particularly, attention has been dedicated to the case where an external field is present, in order to characterize the competition between agent-agent interactions and the agents' interaction with the external field influencing all of them. Here, we review some fundamental aspects of the Axelrod model. To situate the reader in the context of this review, we first discuss several modifications of the original model. Afterwards, a new way to include an external vector field is studied. The vector field acts over the whole system and remains fixed on time. It has a non null overlap with each agent in the society. We explore the influence of this external agent under different model formulations and analyze the system's behavior when dyadic interaction between agents is changed to *social influence*, as has been recently suggested. Furthermore, we discuss in depth how the results obtained depend on different parameters such as the initial social diversity, the size of the network, the strength of the external agent (here associated to Mass Media), different levels of noise, etc. Our conclusions both summarize what we discuss and points to future challenges.

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1. Introduction.

Economical and social systems have become target systems of nowadays interest. Nevertheless, the elementary components of these systems are much more complex than atoms and molecules. Social systems are composed by agents whose interactions at lower level yield to the spontaneous emergence of higher-level organizations whose properties are not due to the behavior of the single entities but rather to nontrivial collective effects resulting from the interactions of a large number of them.

Some social science research use simplified verbal representations of the social phenomena. In such cases it is difficult to precisely determine the implications of the ideas being put forward [1]. The other approach adopted is the well-known representation in terms of statistical or mathematical equations because it is generally accepted that the understanding of social problems involves model-building. Although this approach is currently much more formal and allows assessing consistency and other desirable properties much easier than verbal representations [2, 3], still there are some disadvantages. Sometimes the equations which one would like to use to represent real social phenomena are rather complicated to be analytically tractable. Of course it is possible to make simplifying assumptions that make the equations solvable, but this should be done with much care as these assumptions are often implausible from a social point of view, leading to a theory that might be seriously misleading.

Nowadays, there is a third approach available for scientists studying social systems: computer simulations (or computer modelling). They involve representations of the model as computer codes which can simulate either quantitative or qualitative theories. The use of computer simulations is equivalent to the use of mathematical equations [1]. In the latter case the target phenomenon to be understood is modeled through a process of abstraction which produces mathematical or statistical equations. The equations are solved and the results are compared with observations in order to validate the proposed model. Besides, this approach allows to evaluate the effects of different input parameters on the behavior of the system.

The breakthrough in computational modeling in the social sciences comes with the development of agent-based models (ABM). The bibliography related with this technique is increasing [4, 5, 6, 7, 8, 9]. The interest in the application of such techniques has grown rapidly mainly as a result of the increasing availability of computational capabilities. Today, computer simulations have become an excellent way to model and understand social processes. The usefulness of social simulation modeling results as much from the process (problem specification, model development, and model evaluation) as the product (the final model and simulations of social system dynamics). Nevertheless, social simulations constitute a theory-guided enterprize. Results will often include the development of explanations, rather than the prediction of specific outcomes [6]. ABM are computer simulations of the local interactions of the members of a population which could be plants and animals in ecosystems [9], vehicles in traffic, people in society [10], etc. Bottom-up models can be formulated and modeled in such a way that local interactions at lower-level give rise to the spontaneously emergence of higher-level organizations whose properties are not possessed by the individuals nor directly determined by them .

On the other hand, ABM are useful to study and characterize social systems composed by heterogeneous agents which are autonomous, with differentiated learning capabilities, where agents can interact each other in differentiated manners under the influence of internal and external factors producing different internal social networks. A typical ABM consists of an environment or framework in which the interactions occur among some number of individuals defined in terms of their behaviors (procedural rules) allowing the tracking of the characteristics of each individual through time. These models are also useful in systems where the geographic landscape could be important and where it is desirable to understand the causal relations between traits and behaviors of agents (micro-scale) with the global properties of the system (macro-scale) [6]. Finally, ABM have a great potential to assist us in the discovery of simple social effects by introducing simple models that focus on some small aspect of the social world in the "artificial society" the models built. As a result, one can uncover how simple principles of agent interaction produce highly nontrivial global complex behaviors.

1.1. The Axelrod model.

There are lots of applications to model different aspects of dynamics in society. In this chapter we are particularly interested in studying the Axelrod Model [10, 11] which is an agent-based model designed to investigate the dissemination of culture among interacting agents on a society (see the recent review in Ref. [12] for other models of social dynamics). Axelrod argued that culture "is something people learn from each other", and hence something that evolves through social influence. At the same time he asks "If people tend to become more alike in their beliefs, attitudes and behavior when they interact, why do not all differences eventually disappear?" [10]. To study the process of cultural propagation Axelrod built a model based on two simple assumptions which are observed empirically:

1. people are more likely to interact with others who share many of their cultural attributes, and
2. these interactions tend to increase the number of cultural attributes they share (thus making them more likely to interact again).

The first assumption is also called the principle of *homophily*, which is the principle of "likes attract".

The Axelrod model consists of a population of agents, each one occupying a single node of a square network of size L and area L^2. The culture of an agent is described by a vector of F integer variables $\{\sigma_f\}$ called *features* (f = 1, ..., F). Each feature can assume q different values between 0 and $q-1$. These are the possible *traits* allowed per feature. See Figure 1. In the original Axelrod model the interaction topology is regular bounded (non-toroidal). Each agent can interact only with its four neighbors (a von Neumann neighborhood) which are the most closer (only one step distant from the target agent of influence) without crossing the borders. Initially, individuals are assigned a random culture and the parameter q, which defines the possible traits in each cultural dimension, can be seen as a measure of the initial disorder or cultural variety in the system. In the temporal dynamics of the model at each time step t, the cultural profile of a randomly selected agent i may be updated through the interaction with a randomly chosen neighbor j. According to the first assumption described above, the probability of this interaction is proportional to the corresponding overlap of their

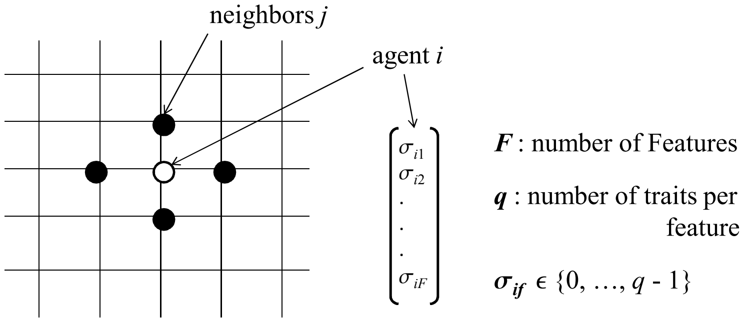

Figure 1. Schematic representation of agents' features in a 2D Axelrod model.

cultural profiles (the amount of features with identical traits) and is normalized with respect to the amount of features F. According to the second assumption above, when interacting, agent j influences agent i causing the last to adopt j's trait on a feature randomly chosen from those that they do not share. Formally, the discrete-time dynamics of the system is defined by iterating the following steps:

1. Select at random an element i in the lattice
2. Select at random a neighbor j of the agent i (from the von Neumann neighborhood)
3. Calculate the cultural overlap $O(i, j)$ (the number of features with the same trait value)
4. If $0 < O(i, j) < F$, agents i and j interact with probability $O(i, j)/F$. In case of interaction choose h randomly such that $\sigma_{i_h} \neq \sigma_{j_h}$ and set $\sigma_{i_h} = \sigma_{j_h}$.

Then the interaction probability between any two agents changes in time because it depends of the number of common traits they share. Particularly, when any two agents are completely different the interaction is stopped and if they are completely equal there is nothing new to copy one from the other and the interaction is also stopped. The process outlined above continues until no cultural change can occur and then. The dynamics then reaches an absorbing state which is one of the two possible final states. This happens when every pair of neighboring agents have cultures that are either identical or completely different.

At this final states it is explored the aggregate behavior by studying the spatial distribution of the emergent cultural regions: sets of spatially contiguous agents who share an identical vector of culture. Two final possible states are possible: only one cultural region is obtained or multiple cultures are obtained separated by a boundary. These states are called monocultural and multicultural, respectively. Studies of this system used two parameters to characterize the final states. One of them is the number of agents in the biggest cultural domain (usually designed by S) and the other is the number of different cultures that exist

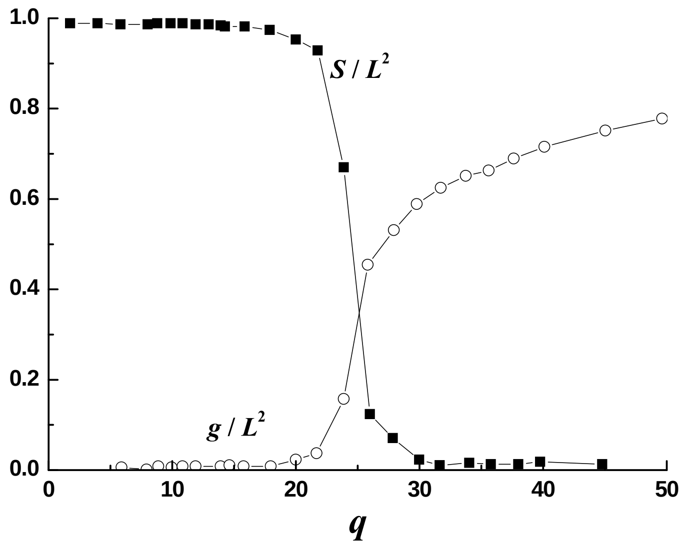

Figure 2. Dependence of S and g with respect to the initial diversity q. We have used for the calculation F = 5 and L = 40. The transition is obtained at a critical value $q_c \approx 25$.

at the final state (usually designed by g). Clearly if the system ends up in a monocultural state we have $S = L^2$ and g = 1, while lower (higher) values of $S\,(g)$ mean that a final multicultural state has been reached.

From the statistical physics point of view, it was shown in Ref. [13] that the system undergoes a first order phase transition between the monocultural to a multicultural state for increasing values of the initial diversity q and $F > 2$. The transition is continuous for F equal or less than 2. The usual dependence of S and g is shown in Fig. 2, where we have set F = 5 and L = 40. The critical value where the transition occurs is $q_c \approx 25$. This value was found to increase as F grows [14].

The Axelrod model incorporated the principle of “likes attract”, or homophily [15] which, combined with social influence generates a self-reinforcing dynamics in which growing similarities strengthen attraction. In its turn, this attraction increases the influence giving rise to greater similarities. Even though this circular dynamics might appear to merely strengthen the tendency towards global convergence, Axelrod’s computational studies showed how local convergence can preserve global diversity by cultural speciation. In the Axelrod model the social interaction becomes impossible between actors who have nothing in common and once influence between regions becomes impossible, their cultures evolve along divergent paths. Then, the Axelrod model has shown how tendencies towards local convergence in cultural influence can help to preserve cultural diversity when the influence between agents is combined with *homophily*.

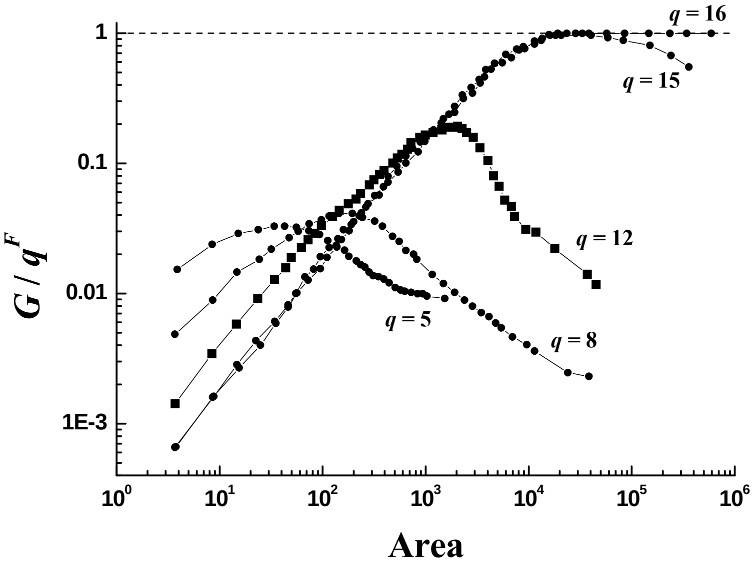

Figure 3. Culture-area relation for F = 3 and five different values of q. The number of cultures G has been normalized to the maximum amount of possible cultures for each value of q used. In the limit of infinite area there are two distinct regimes: monoculture for $q <$ 16 or full multiculturality for $q >$ 16. Data obtained from Ref. [18].

1.2. Two important results of the Axelrod Model which contradict the common sense

Even though the Axelrod model has become a breakthrough inspiring a range of follow-up studies, there are two key problems with Axelrod's explanation of diversity:

1. The first problem is the inability of Axelrod's model to explain diversity in large populations.

2. The second problem is the lack of robustness to noise [16, 17].

Axelrod himself noted the counterintuitive result of his model that generates diversity only for small populations. See Ref. [10], p. 22. Indeed it should be desirable to expect monoculture in small isolated groups (communities or tribal villages) and diversity in large societies.

An extensive examination of the culture-area relation on the Axelrod model was done in Ref. [18]. The authors have counted all the different culture configurations G obtained in the final absorbing state without paying attention to simple connected regions. As there are q^F different possible cultures, the parameter G/q^F is normalized and its maximum value (G/q^F = 1) means a completely multicultural state while low values are related with monocultural final states. The authors obtained a non-monotonic behavior for the culture-area relation for q values below the critical value where the transition occurs ($q < q_c$) while for $q > q_c$ the number of cultures G first increases in a power-law dependence $G \sim A^x$

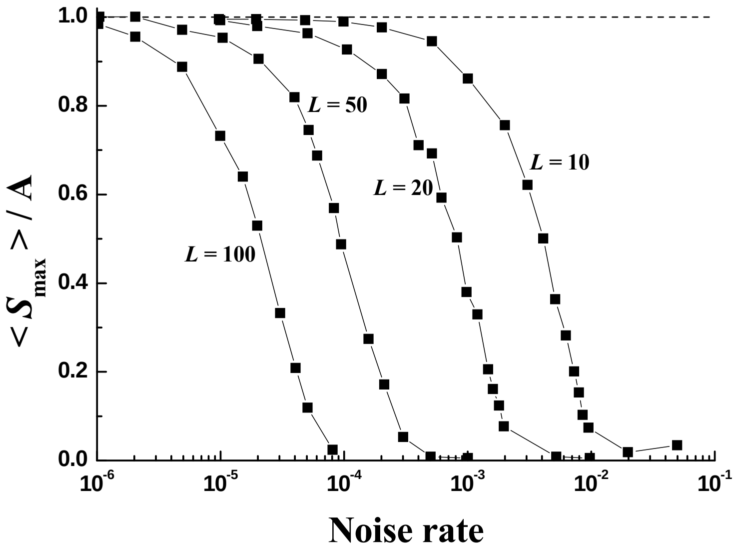

Figure 4. Biggest cultural size $< S_{max} >$ as a function of noise for four different values of L. Data obtained from Ref. [16] where it was used F = 10 and q = 100.

with x = 1 and then gradually flattens when the area becomes of the order of the maximum number of cultures q^F (see Fig. 3). In the limit $L \to \infty$ there are only two possible outcomes: for $q < q_c$ a single culture dominates in an ordered regime, while $G \to q^F$ and all the cultures are represented in the network in a full disordered regime for $q > q_c$. The authors proved the transition between these two regimes to be discontinuous because G jumps from 1 to q^F at $q = q_c$.

Then, multicultural states were shown to be unstable for increasing area and they remain only for sufficiently high values of the initial diversity q. Klemm *et al* in Ref. [16, 17] attempted to find a mechanism present in real life which allow the presence of multicultural state for big societies and introduced noise. These authors relaxed the assumption that cultural traits are entirely determined by the influence from neighbors and allowed for a small probability of random "perturbation" of cultural traits, showing that a small population that exhibits stable diversity under Axelrod's assumptions "drifts" towards monoculture in the presence of very small amounts of random cultural perturbations. This is because random cultural perturbations can disturb the equilibrium in which influence is no longer possible since all neighbors are either identical or totally different, generating a cultural overlap between otherwise perfectly dissimilar neighbors. Besides, perturbations allow for social influence across cultural boundaries to occur. Hence, formerly dissimilar neighbors become increasingly similar until no differences remain and a new cultural boundary forms around a larger region. Eventually this boundary could disappear also by new perturbations. Then, multicultural states keep on remaining unstable, now with respect to very small amount of noise perturbation on the society.

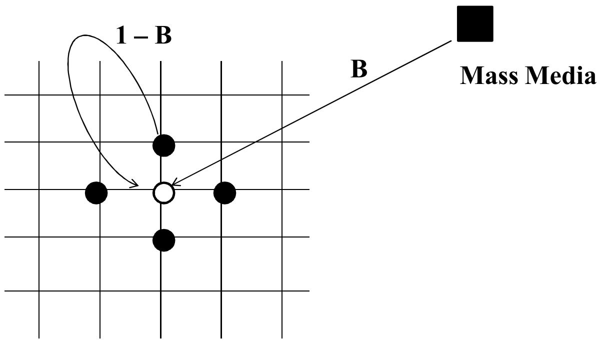

Figure 5. Axelrod model with Mass Media effects included as an external vector. The probability to interact with the Mass Media is B, while the probability of interaction between two agents is $(1 - B)$.

Nevertheless, perturbations can also increase diversity if the rate of perturbations is sufficiently high and the heterogeneity is introduced faster enough as to inhibit social influence from taking advantage of the bridges created by perturbations when dissolving the boundaries between regions. Thus, perturbations are able either to reduce diversity or to increase it. In Ref. [16, 17] it was shown that for increasing population size the introduction of heterogeneities by noise is the predominant mechanism compared to the homogenizing effect. This was an important result which explained both the heterogeneity in large societies and the monoculture in small ones. Still, with this noise mechanism, it was proven that cultural diversity with local convergence is highly fragile with respect to very small changes of noise rate. This is shown in Fig. 4, where it can be seen that cultural diversity with local convergence is obtained only in a narrow window of perturbation rates below which diversity collapses and above which local convergence is destabilized. Moreover, the size of the window closes down as the population size increases and for large populations a strong multicultural state where each agent expresses a different culture is predicted.

Then, it remains necessary to find new mechanisms of interaction that would allow for higher stability against variations of the noise rate.

1.3. Previous Axelrod Models with Mass Media

The Axelrod model just described constitutes an autonomous system where all the complex dynamics originates from the internal rules. An interesting extension of the model consists of studying the system when an external influence is included making the system now non autonomous or forced by an external “force”. This work has been done interpreting the external influence as cultural information broadcasting to the society by different mediums of massive communications (Mass Media), global propaganda, political parties, etc.

An initial work done by Shibanai *et al* in Ref. [19] simulates Mass Media effects by introducing an homogeneous influence applied to all agents with the aim of influencing

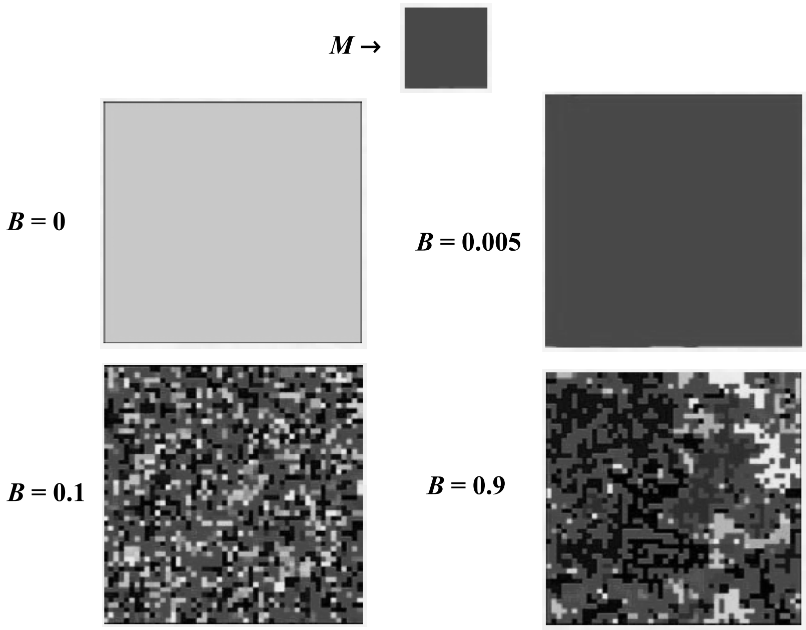

Figure 6. Spatial patterns for different values of the intensity B, for F = 10, $q = 35 < q_c$, and L = 50. The color for the Mass Media (M) vector is indicated for comparison.

globally in all the agents of the network. Surprisingly, even though the globally polarized state has been found to be very fragile and easily disrupted by any perturbation (extensions of the original Axelrod model), it was reported that a factor that aims at homogenizing the system actually favors polarization.

More recently, seminal works have studied more deeply the situation [20, 21, 22]. To do that it has been defined a vector $M = (\sigma_{M_1}, ..., \sigma_{M_F})$ that can interact with all agents on the society as an extra neighbor. Each one of its entries has a value $\sigma_{M_i} \in \{0, ..., q-1\}$, and a parameter $B \in (0, 1)$ quantifies the relative intensity of the Mass Media message with respect to local interactions. It is a probability that the message in M attracts the attention of the agents in the system. This parameter B is uniform, i.e., Mass Media reaches all agents with the same intensity as a uniform field. Thus, each agent in the network possesses a probability B of interacting with the vector M and a probability $(1 - B)$ to interact with one of its neighbors. See Figure 5. As in the original Axelrod model, one initially chooses a target agent at random. Since the vector media M is defined as a virtual agent the interaction follows exactly the same rules as before:

1. Select at random an element i in the lattice.

2. With probability B the element i interacts with the vector M or with a neighbor with probability $(1 - B)$. For each case

3. Calculate the cultural overlap O between the active agent i and the other interacting agent

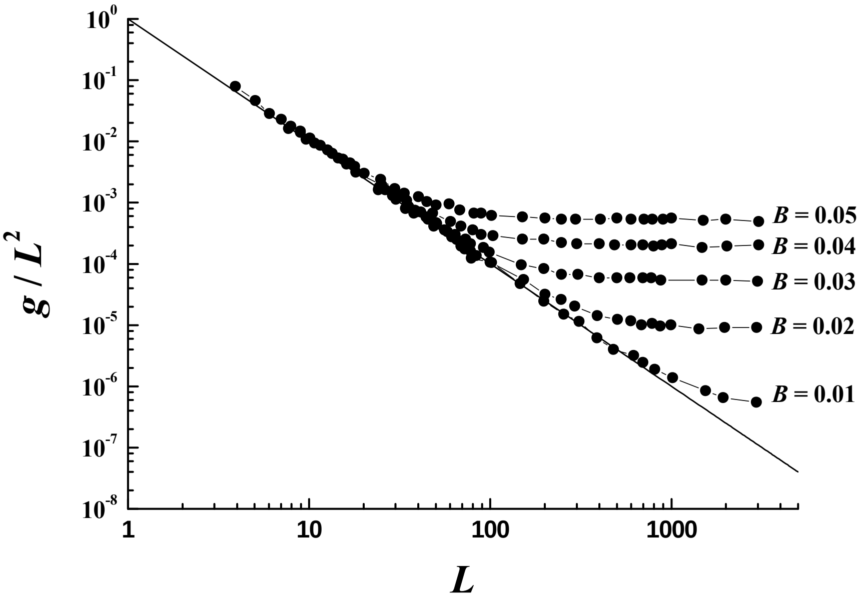

Figure 7. Logarithmic plot for g as a function of the network size L for different values of B. The solid line is $1/L^2$ (the value of g in a monocultural state). F = 5 and q = 5 were used. Data obtained from Ref. [23].

4. If $0 < O < F$ the element i interacts with probability O/F. In case of interaction choose h randomly such that $\sigma_{i_h} \neq \sigma_{M_h}$ ($\sigma_{i_h} \neq \sigma_{j_h}$) for the case of the media (a neighbor) and set $\sigma_{i_h} = \sigma_{M_h}$ ($\sigma_{i_h} = \sigma_{j_h}$).

Note that the original Axelrod model is recovered for B = 0. When the Mass Media cultural influence is applied to the system the order-disorder phase transition shown on Fig. 2 persists, but the critical value q_c for which the transition takes place decreases as the intensity B of the message is increased. In Fig. 6 the spatial configurations of the final absorbing states of the system when the Mass Media is present are shown. We have used F = 10 and $q = 35$ which is a value below the critical one, q_c. Then, when B = 0 the dynamics of the system ends up in a monocultural state as shown in the left-upper case (different gray colors represent different cultural configurations of the agents). As one can see for the case B = 0.005, the final absorbing state corresponds to a monocultural state equal to the culture of the Mass Media. However, its seems there is a critical value of B beyond which the system no longer converges to the state of the message M but reaches a multicultural state with increasing number of cultural domains for increasing B, as can be seen for the cases of B = 0.1 and B = 0.9. This has been a surprising and counterintuitive result. Above some threshold value for the intensity B Mass Media actually promotes cultural diversity on the system and only for sufficiently small values of B the system is driven to a uniform regime with the culture of the Mass Media.

A more careful analysis done in Ref. [23] revealed that the threshold value obtained for

B is just an effect of the finite size of the network used. In Figure 7 we have depicted the dependence of the parameter g (already defined in section 1.1.) with respect to the network size L for different values of B. The parameter g is the number of cultural domains obtained in the final absorbing state. Two or more equal cultural domains are counted separately. Then g is bounded by L^2 and in the uniform regime we have $g/L^2 = 1/L^2$. For each value of B the monocultural state is obtained for low values of the network size L when g/L^2 roughly coincides with the value expected in uniform regime $1/L^2$. Still when sufficiently high values of L are achieved, g/L^2 saturates and the possibility of a vanishing value for g/L^2 as $L \to \infty$ disappears. Then, the presence of a global element influencing the agent's opinion as an external homogenizing factor not only promotes polarization, but this effect is such a powerful factor that even a vanishing small influence (vanishing value of B) is sufficient to destabilize the culturally homogeneous state for very large lattice sizes.

To round off this section, let's mention that the Axelrod model has been exhaustively studied either analytically [24, 25, 26, 27] or numerically [28]. It has also been extended to study the cultural drift driven by noise [16, 14], the effects of combining nominal and metric features [15, 14], propaganda [29], the resistance of a society to the spread of foreign cultural traits [30], finite size effects [31], the impact of the evolution of the network structure with cultural interaction [32], the mobility of social agents [33], the *temperature* as an order parameter [34], and others. Besides, the Axelrod model has also been implemented on non-regular networks [35, 36, 37]. Some of these works were discussed and analyzed early in this section.

We have discussed how the multicultural absorbing state is unstable against noise and when increasing the area of the society. Even the expansion of communication by including higher connectivity on the lattice [26] or by placing agents in small-world and scale-free networks also resulted in cultural homogenization [17]. Additional parameters, as individual nonconformity, has also been considered as a way to explore new mechanisms that make the polarized state stable [38]. Surprisingly, the multicultural state has been found to be stable when a homogenizing external vector is present.

In what follows, we revise some new implementations of the Axelrod model with the presence of Mass Media effects introduced to overcome the drawbacks commented on previously. The rest of the Chapter is organized as follows: in the next section we explore a new approach to study Mass Media effects on the Axelrod model which makes it possible to obtain strong multicultural states when increasing the Mass Media strength. In section 3., a new approach for modeling the interaction between agents is studied, including Mass Media effects, which has been demonstrated to produce robust multicultural final states. Finally, some conclusions are outlined.

2. Axelrod model with clever Mass Media

To explain the counterintuitive results obtained in the previous models where the Mass Media is introduced as an external field, we have included Fig. 8 to analyze the dynamical mechanisms of the system when agents interact. The agent M is represented as a full square external to the lattice. When present, it introduces an asymmetry on the society,

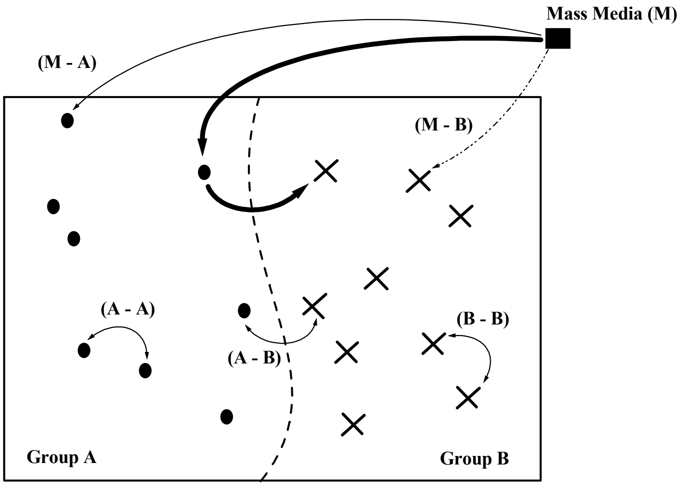

Figure 8. Former models' representation of agents and interaction rules in the lattice when the Mass Media is included (represented by a big square). Agents from group A are indicated with dots while agents in group B are indicated with crosses. Different interactions are represented with different lines. The diffusion mechanism from the agent M to agents in group B is represented by thick lines. Null direct interaction between agent M and agents from group B is represented by a dashed-dotted line.

which can now be described as composed by two groups of agents: group A where agents have trait(s) in common with the agent M and group B whose elements do not share traits with the Mass Media. Agents of group A are represented as dots while agents from group B as crosses. The different lines in Fig. 8 represent all possible interaction between the system's elements, including the agent M. The (M-B) interaction, showed as dash-dot-dot line in the figure, is a null interaction because agents from group B do not share traits with the agent M. In this way, the only opportunity for agent B to acquire one trait from agent M is through a diffusion mechanism with the combined interactions (M-A) plus (A-B), as pointed out in Fig. 8 with a thick line. We model the strength of the agent M using a probability P for the interactions M-A and M-B, while the rest of interactions occurs with probability $1 - P$. Therefore the diffusion mechanism has very low probability as P increases and agents from group B are set apart from the Mass Media information. Furthermore, the mechanism M-B can be very active (time consuming) but with null effects, and the internal relaxing mechanism B-B is not able to drive agents to the final state in an efficient way. Thus, for high values of P the M-A and M-B interactions dominate and the group B will contain high cultural diversity (because of the absence of relaxing mechanism A-B and B-B). Finally a multicultural state is obtained, being it stronger for higher values of the Mass Media strength P. This is, perhaps, a limitation of the model.

Nevertheless, the Mass Media, when acting over the agents of the society, designs its actions in a clever way to always have something in common with the people chosen as targets of publicity or propaganda. Mass media uses language and symbols shared by all the individuals in society to introduce its information with the purpose to homogenize people dressing, way of thinking, etc. At least one way to simulate this common information Mass Media and agents on the society share is the introduction of an extra common feature such that the traits overlap between any agent i and the agent M. Then, the probability of interaction is described by

$$p(i, M) = \frac{O(i, M) + 1}{F + 1} \geq \frac{1}{F + 1} > 0 \tag{1}$$

which has a minimum value min(p_{is}) $= 1/(F+1)$ always greater than zero. The minimum value is obtained when there are not common traits between the agent i and the agent M and the overlap $O(i, M)$ is zero. This procedure was already used in Ref. [39] and Ref. [40] in regular and complex networks with community structure, respectively. As in Ref. [20], they have modelled the strength of the Mass Media as a probability of interaction between this agent and agents on the network. Increasing values of this probability imply a decreasing value of the probability that agents interact with each other. Then, the corresponding socialization process is stopped. The authors have found that only monocultural states are obtained in this case. When introducing noise rates on the system, different multicultural final states are attained.

In this section we are interested in further study this new approach. Nevertheless, as mentioned in Ref. [19], the media information is socially processed through personal networks. Thus, we think it is important to model the Mass Media strength in a way such that its increment does not destroy the interactions between agents in the society.

2.1. The model.

2.1.1. Nominal features

Our system consists of L^2 agents as the sites of a square lattice. The state of an agent i is defined as a vector of F *nominal* components called features given by $\sigma_i = (\sigma_{i_1}, ..., \sigma_{i_f}, ..., \sigma_{i_F})$ which characterize the nominal F-dimensional culture of the corresponding agent. In this way, each agent has four nearest neighbors. The fifth (the Mass Media) is introduced as a vector field M with nominal features $\sigma_M = (\sigma_{M_1}, ..., \sigma_{M_f}, ..., \sigma_{M_F})$. Then, each agent can interact with five agents: its four nearest neighbors and the agent M. In this approach all the interactions has equal probability (1/5 in this case). Additionally, each feature σ_{i_f} and σ_{M_f} can take any of the values in the set $\{0, 1, ..., q-1\}$ which are the corresponding cultural traits of an agent i or the super-agent. As usual, at the beginning, the values of the vectors σ_i and σ_M are randomly and independently set to one of the q^F available state vectors with uniform probability.

The interaction between different agents is possible only when the two cultures have an overlap $0 < O < 1$ where the overlap between two agents i and j is the number of shared traits and is given by $O(i, j) = \sum_{f=1}^{F} \delta_{\sigma_{i_f}, \sigma_{j_f}}$. Here δ is the Kronecker symbol. The probability, which we call here *nominal* probability, of the interaction between two agents is given by $p(i, j) = O(i, j)/F$.

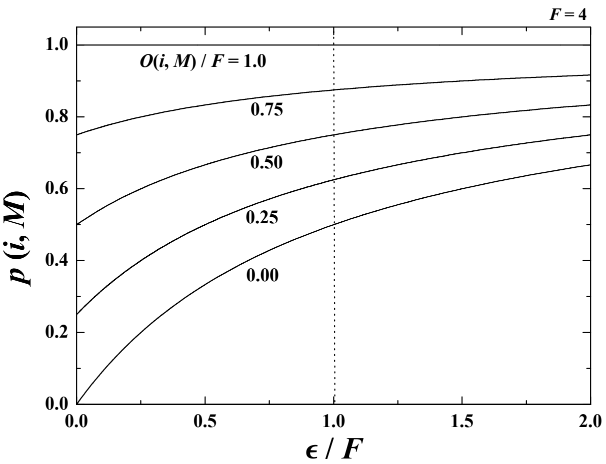

Figure 9. Probability of interaction between the agent i and the vector field M as a function of ϵ/F for fourth nominal traits (F = 4) and different values of the overlap $O(i, M)$.

2.1.2. Effective features

In general, the situation $p(i, j) = 0$ is possible when the overlap between two agents is zero but, as mentioned before, the case where the probability between an agent and the Mass Media is zero is not an acceptable situation for a clever publicity. In this case the connection is always active. In order to include this important effect in our model, we have included some *effective* features ϵ, besides nominal features F, that the agent M always shares with each agent. The specific nature in real society of the effective features is not of importance here. It could be the use of the same language, symbols with common interpretation in every-day life, etc. In general, it could be different for different agents, but the intention is to take into account the clever design of the publicity that Mass Media does to influence everyone. Each individual "understands" the message of the publicity, even if they accept it or not.

Then, in our model any agent has F nominal features and ϵ effective features which always shares with the Mass Media. Therefore, the probability of interaction between the external vector and agents, which we call here *extended* probability, is written as

$$p(i, M) = \frac{O(i, M) + \epsilon}{F + \epsilon} = \frac{O(i, M)/F + \epsilon/F}{1 + \epsilon/F}, \tag{2}$$

where $O(i, M)$ is the overlap of the nominal features between agent i and the agent M. This way, the overlap O counts the number of nominal features shared between agent i and agent j or an agent i and the agent M. This parameter is related mainly to the dynamics between agents because is the only mechanism for them to interact, while the parameter ϵ counts the effective features an agent i and the agent M share. It constitutes the measure of its dynamics. For $\epsilon/F < 1$ the Mass Media and agents share more nominal than effective features and the Mass Media constitutes a "perturbation" to the internal interaction between

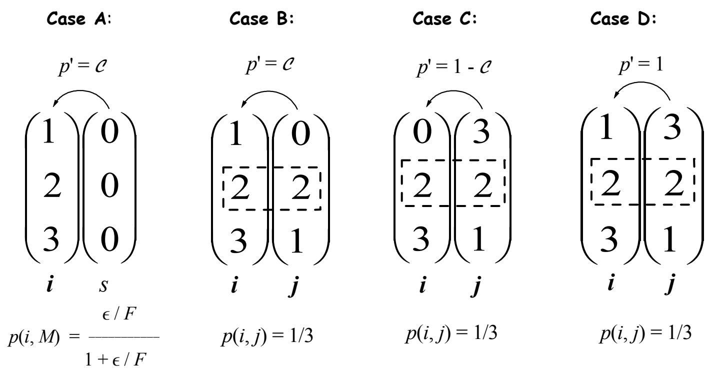

Figure 10. Four possible cases of interaction for a system with $F = 3$ features. Shared features are indicated inside a dashed rectangle. The probability of interaction $p(i, j)$ (or $p(i, M)$ in Case A) is indicating below. In each case the trait in σ_{i_1} will be deleted by copying trait σ_{j_1} (or by trait σ_{M_1} in Case A). The probability of copying/deleting trait 1 is given by a) $p' = \mathcal{C}$, b) $p' = \mathcal{C}$, c) $p' = 1 - \mathcal{C}$ and d) $p' = 1$.

different agents in the society. The case $\epsilon/F > 1$ means that there are more effective features that certainly share each agent with the agent M than the number of nominal features each agent has. Then it is now the society which can be considered as a "perturbation" with respect to the more robust influence of the agent M over the agents. The former expression for the extended probability is similar to Eq. (1), but now, as a generalization, the parameter ϵ can take not only natural values but also fractional ones. It is worth saying that in this case the original Axelrod model is not recovered when $\epsilon \to 0$. If the parameter ϵ is set to zero it only clears the effective features between the Mass Media and the agents of the network, but the external influence is still present.

In Fig. 9 the values of the probabilities $p(i, M)$ as a function of ϵ/F for fourth nominal features ($F = 4$) and different values of $O(i, M)$ are shown. The values of $O/F = 0.00$, 0.25, 0.50, 0.75 and 1.00 are obtained when the agent shares with the vector field 0, 1, 2, 3 and 4 nominal features. As it can be seen, the probability is zero only when there are not effective features ($\epsilon = 0$) and the overlap between the agent i and the agent M is zero. In contrast, in all the other cases the probability $p(i, M)$ is always greater than zero. For $\epsilon = 0$ the values for the case with no effective features are recovered. As expected, the probability is lager for larger values of the effective features ϵ at a given value of $O(i, M)$ and also increases for larger values of the overlap $O(i, M)$ at a given number of ϵ. Finally, when the agent i and the Mass Media share all the nominal features ($O(i, M)$ = 1) the probability is always one for any value of ϵ.

2.1.3. Confidence value for the Mass Media traits

If the dynamics of the system only involves the extra parameter ϵ and it follows the usual rules described on section 1., but now including as a fifth neighbor the agent M and the extended probability of interaction given by Eq. (2), only monocultural final states are obtained. The Mass Media always homogenizes the society to its own values.

A non trivial case appears if we instead include a value to describe the credibility of the information the Mass Media has. This value is clearly tested in massive surveys done by different social organizations in society, for example, when elections are closer in order to know the acceptance of a candidate. It is expected that a high value of this credibility increases the influence of the Mass Media over the whole society and, on the contrary, a low value could make the propaganda perhaps almost invisible. To study these effects we introduce a parameter called here the "confidence". It is included as a probability $p' = \mathcal{C}$ for agent i to copy an entry directly from the agent M or an entry from another agent j with a trait value equal to that of the Mass Media. It is also included as an extra probability $p' = 1 - \mathcal{C}$ when agent i, when copying a trait from agent j, deletes an information the agent i possesses which is equal to that the Mass Media possesses in the same feature.

To clarify this important concept we show in Fig. 10 four situations of interaction which summarize all the possible cases. In Case A it is described an interaction between agent i and the agent M which has been set to (0,0,0) without lost of generality. None of the nominal features are shared and the extended probability of interaction $p(i, M)$ depends only on the value of the effective features ϵ according to the expression in the figure. In the practical case when agent i copies, for example the first entry, the Mass Media trait is copied with probability $p' = \mathcal{C}$ which characterizes the confidence of the information it possesses. In the next cases, the interaction occurs between agents i and j which only share one trait of three possibles. The probability of interaction is then given by the nominal probability $p(i, j) = 1/3$ in all these cases. In Case B the nominal feature that agent i selects to copy from agent j coincides with the value the Mass Media has in the same feature. Then, as in Case A, the corresponding trait is copied with probability $p' = \mathcal{C}$. In Case C, when copying, agent i will delete its trait which is equal to that possessed by the agent M. Then, it is deleted with probability $p' = 1 - \mathcal{C}$. Finally, in Case D the traits copied and deleted are not related with the Mass Media and then they are copied/deleted with probability p' = 1. Note that according to these rules of interaction, when the confidence of traits possessed by the agent M is the highest possible ($\mathcal{C}$ = 1), these traits are always copied with probability p' = 1 and never deleted. Otherwise, if the confidence of the traits possessed by the Mass Media is the lowest possible ($\mathcal{C}$ = 0), these traits are never copied (p' = 0) and are always deleted with probability p' = 1. Then, starting from the initial condition described above, the system evolves by iterating the following steps:

(1) Select at random an agent i on the lattice, which is the active element.

(2) Select at random, with equal probability, an agent for interaction. It could be one of the four nearest neighbors or the agent M.

(3) Calculate the overlap $O(i, x)$ where $x = j$ for the neighbor or $x = M$ for the Mass Media. If $x = M$, the agent i and the agent M interact with the extended probability

$p(i, M)$. If $x = j$ and $0 < O(i, j) < F$, agents i and j interact with the nominal probability $p(i, j)$.

(4) In case of interaction between agent i and agent x, choose a position trait h at random such that $\sigma_{i_h} \neq \sigma_{j_h}$ (or $\sigma_{i_h} \neq \sigma_{M_h}$) and then set $\sigma_{i_h} = \sigma_{j_h}$ (or $\sigma_{i_h} = \sigma_{M_h}$) according to:

(4.1) if $x = M$ then set $\sigma_{i_h} = \sigma_{M_h}$ with probability $p' = \mathcal{C}$,

(4.2) if $x = j$ and $\sigma_{j_h} = \sigma_{M_h}$ then set $\sigma_{i_h} = \sigma_{j_h}$ with probability $p' = \mathcal{C}$,

(4.3) if $x = j$ and $\sigma_{i_h} = \sigma_{M_h}$ then set $\sigma_{i_h} = \sigma_{j_h}$ with probability $p' = 1 - \mathcal{C}$.

(4.4) if $x = j$ and both $\sigma_{j_h} \neq \sigma_{M_h}$ and $\sigma_{i_h} \neq \sigma_{M_h}$, then set $\sigma_{i_h} = \sigma_{j_h}$ with probability p' = 1.

Finally, the full probability that agent i copies a trait from agent $x = \{j, M\}$ is given by

$$\mathcal{P}(i, x) = \begin{cases} \frac{1}{5}\, p(i, j)\, p' & ; x = j \\ \\ \frac{1}{5}\, p(i, M)\, p' & ; x = M \end{cases} \tag{3}$$

2.2. Numerical results

We have performed numerical simulations in lattices with $L^2 = 30 \times 30$ agents and $F = 4$ features each. Different absorbing states have been found and we report the average number of agents in the largest domain over 50 different initial conditions. The absorbing states are obtained when all agents in the network have full or null overlap with each one of its neighbors. The Mass Media is not included to check this condition.

Figure 11 shows the calculations of $< S_{max} >$ as a function of q at the absorbing state. Each panel shows the result for a certain value of the ratio ϵ/F and different values of the confidence $\mathcal{C}$. The result using the Axelrod model without Mass Media is included for comparison with full square dots. In this case, the system reaches a monocultural state at $q < q_c \approx 18$ and a multicultural state at $q > q_c$. In panel a) we set ϵ = 1.0. It can be seen that when the value of the confidence $\mathcal{C}$ is small ($\mathcal{C} = 0.05$), the results are very close to those of the original Axelrod model. The monocultural states remain unchanged at $q < q_c$ but the multicultural state is less "robust" and higher values of $< S_{max} >$ are obtained. For increasing values of $\mathcal{C}$ higher values of $< S_{max} >$ for $q > q_c$ are obtained, as seen with $\mathcal{C} = 0.10$, and finally, at $\mathcal{C} = 0.15$, the multicultural states vanish and the system remains in monocultural states for all values of q. Then, the increasing value of the confidence induces an homogenization of the cultural information the system has, even at those values of q where the system reaches multicultural states when there is no Mass Media influence. Multicultural states are unstable for increasing values of the confidence $\mathcal{C}$ at this value of the effective trait ϵ.

In Fig. 11 b), c) and d) we calculated the $< S_{max} >$ for smaller values of ϵ. It can be seen that multicultural states at $q > q_c$ are again obtained for low values of $\mathcal{C}$ but now higher values are needed for the confidence to produce a cultural homogenization ($< S_{max} > \approx 1$). This can be seen when comparing the results for $\mathcal{C} = 0.15$ in Fig. 11 a) with ϵ = 1 and in

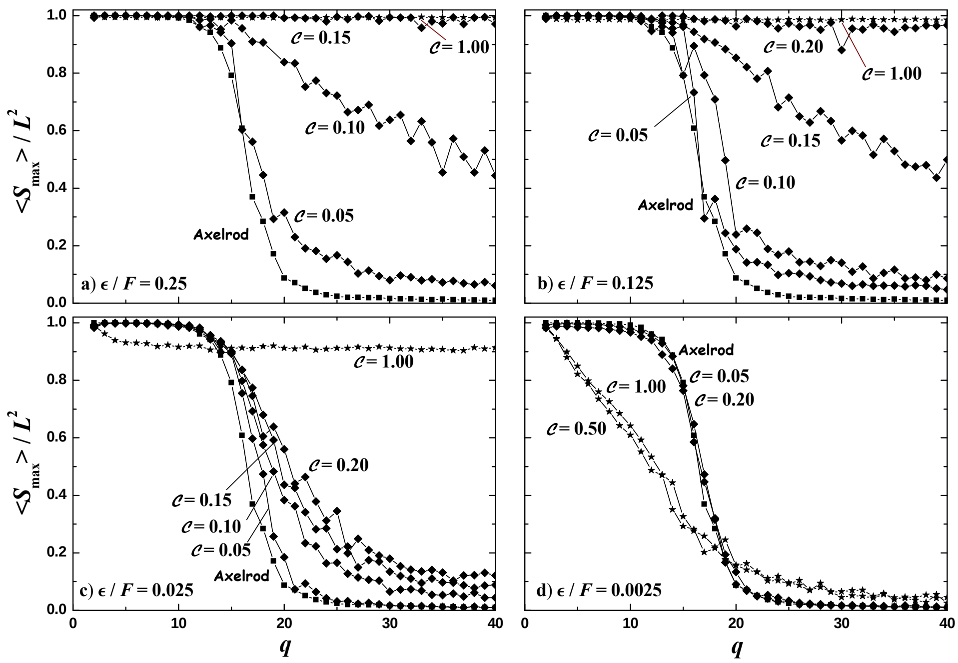

Figure 11. Calculation of the normalized average number of agents in the largest domain at an absorbing state as a function of q averaged over 50 realizations. The values of the ratio ϵ/F are a) 0.25, b) 0.125, c) 0.025 and d) 0.0025. At each panel, different values for the confidence $\mathcal{C}$ are taken into account in increasing order. The case of the original Axelrod Model without Mass Media is included in each panel with full square dots. Full rhombus indicate that the corresponding absorbing states do not share the information possessed by the external field, while full stars indicate full coincidence of the absorbing state with the Mass Media.

Fig. 11 b) with $\epsilon = 0.5$. In Fig. 11 c) it can be seen that at a very small value of ϵ ($\epsilon = 0.1$) the system has a multicultural state for $q > q_c$ with small increments of $< S_{max} >$ for increasing $\mathcal{C}$, and even at $\epsilon = 0.01$ (Fig. 11 d)) the transition from a monoculture to a multiculture at $q \approx 18$ is independent of the confidence for values between 0.0 and 0.2.

Nevertheless, care has to be taken when analyzing the information of the induced monoculture at $q > q_c$ when the agent M is present (as the case $\mathcal{C} = 0.15$ in Fig. 11 a) and $\mathcal{C} = 0.20$ in Fig. 11 b)). It is interesting to know whether the greatest domain in the absorbing state characterized by $< S_{max} >$ has, or not, the information possesses by the Mass Media in the corresponding nominal features. This information is indicated in Fig. 11, where all the full rhombus show absorbing states where the corresponding greatest domain does not possess the information of the agent M in any of its features. That is, the overlap between the Mass Media and the cultural state of the largest domain is zero. Only at those absorbing states indicated by full stars ($\mathcal{C} = 1.00$ in all panels and $\mathcal{C} = 0.50$ in Fig. 11 d)) the corresponding biggest domain fully shares the information at nominal features of the agent M. Then, it is interesting that, at low confidence values $\mathcal{C}$, the culture homogenization induced by the Mass Media results in a negation or cancellation by the agents of the lattice of the information possesses by an external media. Here we call this phenomenon *negative publicity* effect. It represents the process occurring in a society when a group or different groups of people gather together, physically or intellectually, against an external action they consider misconceived.

3. Axelrod Model with *Social Influence*

As already discussed in section 1.2., the Axelrod model has two important limitations when studying the dependence of the final absorbing states with respect to the network size and to the presence of noise which were analyzed in Fig. 3 and Fig. 4: (1) the original Axelrod model predicts cultural diversity in very small societies, but monoculture in larger ones [18] and (2) when cultural perturbation is present, diversity is obtained only in a very narrow window of noise level and this window decreases with increasing population size [16, 17]. This result has been justified as a consequence of the *dyadic interaction* considered between the agents of the society which, at each time step, only comprises the source and the target of influence in a particular interaction [41]. The authors have argued that the influence a person feels from the society is a *social* phenomenon that cannot be reduced to the interactions within a dyad given by a source-target couple of persons because the social pressure on the target to adopt an opinion is proportional to the number of people that the target perceives are supporting this opinion. On the contrary, the Axelrod model assumes that influence is interpersonal (dyadic). We are then in front of a two completely different assumption with respect to the way people interact in society:

1. dyadic interaction where it is supposed that two people in a relationship interact in isolation from others and

2. social influence which is multilateral and involves all network neighbors simultaneously.

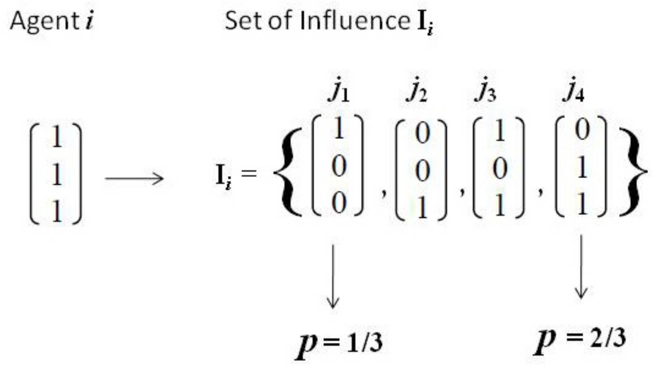

Figure 12. Social interaction between agent i and its neighbors $\{j_1, ..., j_\alpha, ...\}$. Each neighbor is included with probability p_{ij_α} into the set of influence I_i.

In Ref. [41] the authors explained that the central implications of Axelrod's model profoundly change if the *dyadic interaction* is changed to *social influence*. They have shown that the combination of social influence with *homophily* (the principle that "likes attract") solves the two important problems already mentioned. Besides, Ref. [41] proposes an alternative model which uses social influence instead of dyadic interaction. The procedure is defined as follows: When an agent i is randomly selected for possible influence, all of its neighbors $\{j_1, ..., j_\alpha, ...\}$ are stochastically included in a set of influence I_i with probability $p_{ij_\alpha} = O(i, j_\alpha)/F$, where $O(i, j_\alpha)$ is the cultural overlap between agent i and neighbor j_α. The set I_i obtained by this procedure becomes a set of traits' influence over agent i's traits.

Figure 12 illustrates the social influence procedure. In the example shown, the agent i selected has cultural traits (1,1,1) and the neighborhood is formed by the most closed neighbors (von Neumann neighborhood). We have supposed that all the fourth neighbors have entered into the set of influence I_i on the present example (for clarity, we have explicitly assumed that only two neighbors are in I_i, whose probabilities p_{i,j_α} are indicated). Once the current influence set I_i is established, an agent i's feature is selected at random with a trait value different from the corresponding value of at least one agent of the set I_i in the same feature. The decision of which trait value agent i adopts (if any change takes place) is done in such a way that agent i imitates (or copy) the most common trait in the set I_i for the feature selected. If, for example, in the situation depicted in Fig. 12, the feature selected on agent i were the second (that in the middle), this trait value ($\sigma_{i,2} = 1$) will be changed to $\sigma_{i,2} = 0$ due to highest frequency of this last value in the set I_i. If the feature selected were the third (from top to down), the value $\sigma_{i,3} = 1$ will remain due to the highest frequency of this value on the set I_i. The last possible case represented in Fig. 12 occurs if the first feature were selected. In this case, on the set of influence I_i, we have the same frequency of appearance for the values 0 and 1. In this situation, one of the two values is chosen at random and then, the trait value $\sigma_{i,1} = 1$ in agent i could remain with probability $1/2$ or changed to $\sigma_{i,1} = 0$ with the same probability. Then, when social interaction is at work, the trait value of agent i is changed to the value of the corresponding trait with highest

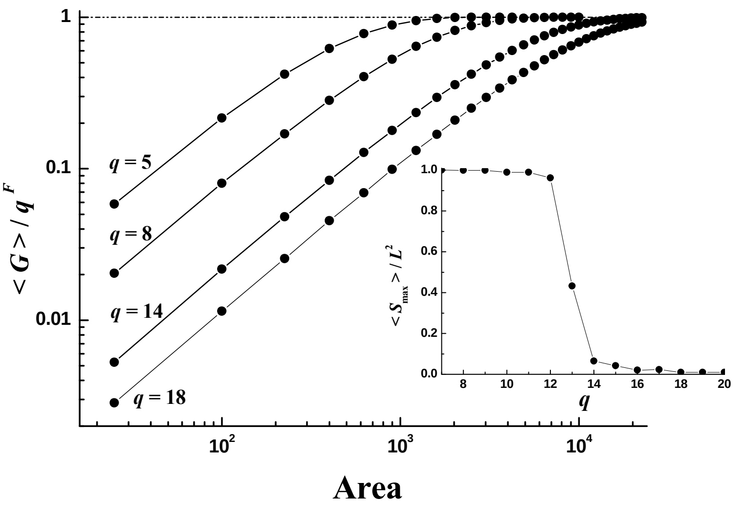

Figure 13. Culture-area relation when social influence is at work for F = 3, L = 50 and different values of q. 200 random initial conditions were used. In the inset, we have included the dependence with q of the maximum culture at the final absorbing state for the original case of dyadic interaction (the same parameters were used, but averages are over 5 initial random conditions.

frequency of appearance on I_i. If there are more than one, it is changed to one of them with equal probability. The trait value of agent i does not change if this is the only one with the highest frequency of appearance on I_i.

The results obtained when social influence is at work are included in Fig. 13 and 14. In the first figure, we have reported the dependence of the amount of different cultures (the same parameter reported in Fig. 3) as a function of the network size. For the parameters used, if dyadic interaction were used a transition from a monocultural to a multicultural state at $q_c \approx 13$ would be obtained, as shown in the inset. The corresponding behavior of the number of cultures in the final state as a function of network size will correspond qualitatively with that shown on Fig. 3. Contrarily, when the model is changed including instead of dyadic interaction social influence the problematic result of the Axelrod model which produces a multicultural final state only for small societies [18] is overcome and only multicultural final states are obtained for values of q below and above the critical value q_c. Moreover, the authors of Ref. [41] have reported more robust behavior of the final state with respect to noise, as can be seen when comparing Fig. 14 with Fig. 4. When social influence is present in the system there is no variation of the parameter S for a wide range of noise rates, contrary to the behavior observed in Fig. 4.

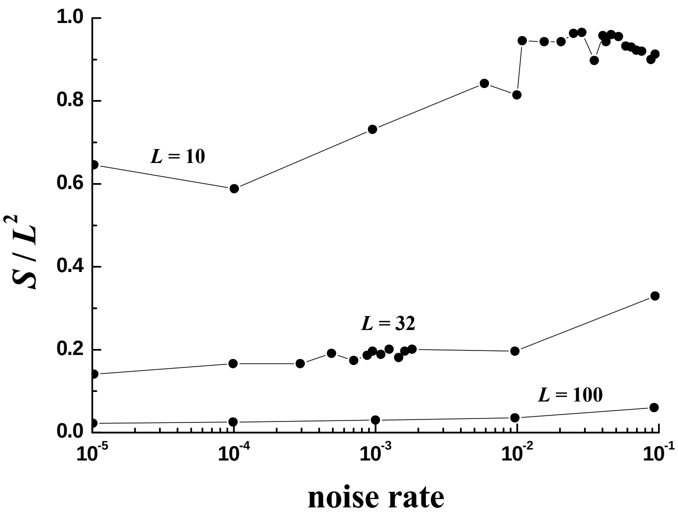

Figure 14. Effects of the noise level on the final absorbing state when social influence is present. F = 5 and q = 15. Data obtained from Ref. [41].

3.1. The model for self-included social influence with Mass Media.

3.1.1. Description of the model

In this section we discuss an extension of the formalism of social interaction in the Axelrod model to include Mass Media effects. Let's characterize the neighborhood of any agent by a parameter p which controls how many agents can be visited (in unitary steps) in each direction starting from that agent until the perimeter of the neighborhood is reached. One unit-step allows to move only in north-south or east-west direction. When p = 1 the neighborhood obtained is therefore the well-known von Newmann neighborhood N_1, which only includes the fourth closer neighbors of an agent ($N_1 = \{j_1, j_2, j_3, j_4\}$). The parameter p can take higher values to define bigger neighborhoods. In general, for a given value of p it is obtained $N_p = \{j_1, ..., j_\alpha\}$ where $\alpha = \alpha(p) = 2p(p+1)$. This give fourth neighbors ($\alpha = 4$) for one step neighborhood ($p = 1$), twelve neighbors ($\alpha = 12$) for two step neighborhood ($p = 2$) and so forth. In our model we have decided to include also agent i into the set I_i with probability $p_{ii} = 1$. This allows a self-reflection of agent i about its own traits when comparing with the traits of its neighbors. Then, each agent is aware of its own traits when being influenced by its neighbors and its own traits count when deciding to change, or not, its corresponding value for a different one.

We have also included the Mass Media effects as an extra neighbor that each agent of the society has. It is an extra agent to be analyzed for inclusion on the set of influence of any agent i. The probability $p(i, M)$ to be included is given by Eq. (2). Then, the agent M has always a non-zero probability to be included in the set of influence I_i. The higher the value of the strength ϵ, the higher the minimal inclusion probability, see Fig. 9. Moreover, the model is implemented using periodic boundary conditions (toroidal society) to avoid boundary effects.

In the case of dyadic interaction and in the absence of noise, when analyzing the possible final absorbing states, it is simpler to establish it by checking when each agent has full or null overlap with each of its neighbors. In the case of social influence the problem is more involved. However, it is also possible to establish some technical conditions to be checked to see if an agent is active (i.e., can interact with its neighbors according to the dynamical rules established in the model) or not taking in consideration the set of influence I_i. When all the agents of the society are inactive, then an absorbing state is obtained.

Here we have also implemented the procedure developed in Ref. [18] where a list of active agents is built. Instead of randomly selecting agents of the society in each time step, the agents are randomly chosen directly from this list. This procedure strongly increases the efficiency of the dynamical evolution of the system and allows to save computational time. Therefore, when the system is initialized by randomly assigning different cultures to each agent of the society, the first list of active agents is built. Next, at each time step, when the influence is established and an agent of the society changes its cultural value, the list of active agents is updated analyzing the agent itself and all its neighbors to check which of them are now active. The dynamical iteration keeps on until the length of the active agents list is reduced to zero. It is worth noticing that in our case (where social influence is established), some runs did not settle into a final, well-defined absorbing state. In these cases the list of active agents reduces to one element and each time this agent changes its cultural values and becomes inactive, one of its neighbors becomes active. This propagation seems to go on indefinitely. We have neglected these cases from our calculations and we have only considered runs which finish in a precise well-defined final absorbing state.

We have implemented different computational experiments to study the Mass Media effects in the Axelrod model when the social influence is the mechanism at work for agents interaction. As there is no qualitative difference on the dynamical behavior of the system for $F \geq 3$, we have set $F = 3$ in the present study, a network size with $L = 50$ (2500 agents unless otherwise stated) and averages have been taken over 200 different initial random configurations.

3.1.2. Culture-area dependence

We have studied the influence of the Mass Media in the culture-area relation when the interaction on the society is social. We have calculated the amount of different cultures present in the final absorbing state as indicated by the previous definition of the parameter G on section 1.2..

In Fig. 15 it is included with black circles the final cultural diversity G for a fixed value of the Mass Media strength ϵ. It can be seen that at $\epsilon = 0$ the amount of cultures obtained in the final absorbing state increases for increasing value of the area and reaches the maximum multicultural state for sufficiently high values of A. The calculation done with any other value of q gives the same dependence of the area which includes the same slope and only a parallel shift. The other curves were then not included for simplicity. We have then qualitatively reproduced the results obtained in Ref. [41]. In our case, a calculation of the slope yields $x = 0.42 \pm 0.02$, which is different to that reported in Ref. [18]. We think that the deviation is due to the difference that arises when dyadic interaction or social influence

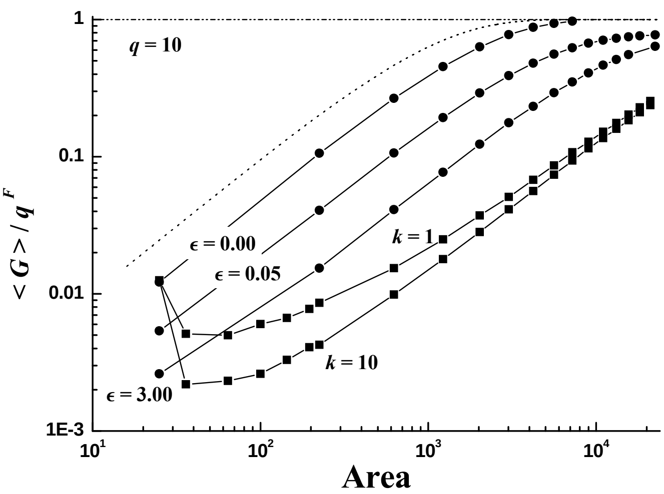

Figure 15. Culture-area relationship for q = 10 and different values of ϵ. For the dotted line the calculation is done for the corresponding value of ϵ indicated in labels. Squares represent the cases for which calculations were done according to the relation given by Eq. 4, with the value of k indicated with labels. The area is obtained according to $A = L^2$.

is included.

When effective features are included setting $\epsilon > 0$ the values of G are lower for higher strength ϵ at the same value of the area, but the slope of the curve is the same as the case ϵ = 0. The agent M also prevents the system to reach the full multicultural state ($< G > /q^F =$ 1) for higher values of the area for ϵ ¿ 0. This can be seen in the curve for ϵ = 0.05, which saturates at values of $A > 10^4$. The same seems to occur with ϵ = 3.00. Calculations taking in consideration two-step neighbors were also carried out with the same qualitative results (including slope and saturation). Only a shift to decreasing values of G was obtained for the cases of ϵ = 0.00 and 0.05, but the shift finally disappears for ϵ = 3.00 meaning that for strong enough Mass Media the relative importance of the amount of neighbors including in the set of influence is weak (at least for the value of q = 10).

Furthermore, on the dynamics of the system there are two parameters which compete to produce opposite effects for increasing values of each one. For increasing area of the network the system tends to reach a multicultural state while an increasing value of the Mass Media strength ϵ pushes the system to a more cultural global absorbing state. In order to study the relative weight these parameters have over the system, we have also calculated the final absorbing state when both the area A and the strength ϵ increase. To accomplish this purpose we have established a dependence of the strength ϵ to the area given by the following relation:

$$\epsilon = k(A - 5) \tag{4}$$

In Fig. 15 it is included, with black squares the results for k = 1 and 10. The values of the diversity G obtained are higher for k = 1, than for k = 10, due to the lower rate of increase of the strength ϵ and consequently a more multicultural final state is induced. Nevertheless, both curves increase for increasing area, which means that the area of the network has more weight than the strength ϵ on the dynamics of the system, as expected if we examine Eq. (2). In our case, the maximum overlap between the agent M and any agent of the network is max$(O(i, M)) = F = 3$. Then, for $\epsilon/F \gg 1$ (which occurs rapidly for increasing values of A in Eq. (4)) it is obtained than $p_{is} \longrightarrow 1$ and the relative increase of the probability p_{is} is cancelled out with the effects of the increasing area. This explains also why the curves for k = 1 and 10 tend to the same values as the area is increased.

Then, when social interaction is present according to the present model, increasing network size always drives the system to a multicultural state, while increasing Mass Media strength prevents the system to reach the maximum possible of cultural configurations. Additionally, the saturation value of G seems independent of the value of the strength of the Mass Media, as far as our calculations have shown.

We have also included in dotted line in Fig. 15 the analytical expression reported in Ref. [18]

$$< G >= q^F \left(1 - \mathrm{e}^{-A/q^F}\right) \tag{5}$$

which is the average number of cultures in the totally disordered configuration where agents are randomly assigned with one of the q^F available cultures. The expression is valid for A and q^F very large. As can be seen, the prediction from Eq. (5) overestimates the values for the cultural diversity as a function of the area. The results are more different the higher the values of the Mass Media strength ϵ are, since it is a factor that decreases the cultural differences between neighboring agents.

3.1.3. Culture-noise dependence

We next consider the same model but now including noise. In this case it is not possible in general to define a final absorbing state and the dynamical evolution of the system is stopped by defining some criterion related with a definition for the stationary state that the system reaches. In our case, we have included noise *only* in those agents which are active on each time step, and do not in the rest of the society. This has allowed us to reach final absorbing states even when noise is present.

Following Ref. [41] we have included *interaction* errors as well as *copying* errors. The interaction error relaxes the previous deterministic procedure used in former works of the Axelrod model when deciding the possible interaction between two agents (dyadic interaction). Both copying and interaction errors can randomly alter the outcome of an interaction event. The interaction error acts over the selection procedure as follow: if the normal procedure of selecting an agent j for being included in the set of influence I_i results in its inclusion, then with probability r' the neighbor j is removed from I_i. If the neighbor has not been selected into the influence set, then with probability r' the agent j will be included into I_i. This error creates the possibility that cultural influence occurs across the boundary of two disconnected cultural regions if a neighbor with zero overlap is included in the influence set. This error can also reduce the social pressure against adopting different trait values when a cultural identical neighbor is excluded from the set of influence and increases the possibility for the target agent i to adopt trait values from a completely different culture.

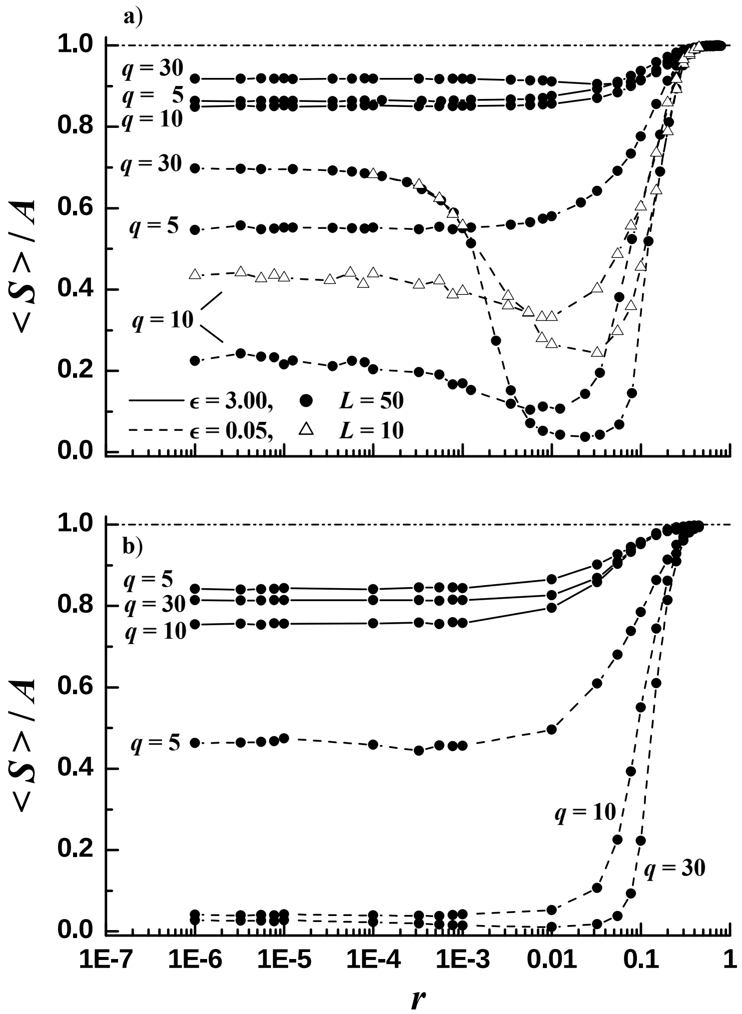

Figure 16. Dependence of the final absorbing state with respect to different noise levels and different values of q. In solid line it is included the case with ϵ = 3.00 while dashed lines are the cases with ϵ = 0.05. In black circles the final absorbing states calculated with a network size of L = 50 are reported, while the white triangles correspond to the case with L = 10. a) Calculation done including one-step neighbors (von Neumann neighborhood). b) Calculation done including two-step neighborhood.

On the other hand, the copying error acts after agent i has adopted (or not) a different trait value and the new trait value has been already set up. In this case, the corresponding trait value adopted by agent i is changed to a new randomly selected value with probability r. Notice that the new value randomly generated could be that one the agent i already has or just deleted.

In Fig. 16 it is reported the dependence of the parameter $< S > /A$ with respect to different noise levels for several values of q, ϵ and L. In general, both interaction and copying errors are conceptually different, but for simplicity we have used here the same value $r = r'$. In panels a) and b) we have considered one- and two-steps neighborhoods, respectively. The noise included ranges from 0 to 0.45. As it can be seen when comparing with Fig. 4, social influence makes absorbing states more stable to a bigger range of noise level than dyadic interaction, as already reported in Ref. [41]. In general there is no qualitative change for at least three orders of magnitude for all the values of q, ϵ and L used. For higher strength of Mass Media (ϵ = 3.00, min(p_{is}) = 0.5) the final absorbing state remains almost monocultural for fourth orders of noise level and finally for noise values higher than 0.1 the system is driven to a full monocultural state reached at r = 0.45 approximately. The situation is different for low values of Mass Media strength (ϵ = 0.05). In this case the final absorbing state is also stable to different noise levels, but only for three orders of magnitude. In panel a) of the Figure it can be seen that for q = 5 and noise level higher than r = 0.01, the system is driven to a monoculture. For increasing noise level at q = 10 and 30 (for ϵ = 0.05) there is first a reinforcement of the polarized state, given by decreasing values of $< S > /A$ (stronger induced for q = 30), and later for higher values of r it is induced a monoculture. All the calculations shown in black circles have been done with a network size of L = 50. In order to study whether the stability of the final absorbing state to noise is robust with respect to the network size, we have also explored the case with network size L = 10. Quantitative differences arise only for q = 10 and 30, for ϵ = 0.05 (white triangles in Fig. 16 a)). In general it is observed that the final absorbing state is stable to the same range of noise independent of the network size. The difference as given by the higher values of $< S > /A$ for q = 10 and the higher minimum for q = 30 at r = 0.05 (both for ϵ = 0.05) are due to the lower size of the network (L = 10). There are no quantitative differences for the other parameters due to the rather trivial case of q = 5 (for any value of $\epsilon > 0$) and for the high value of ϵ used.

In Fig. 16 b) we have included the same results of panel a) calculated with L = 50, but now considering two-step neighbors. The results are qualitatively the same for ϵ = 3.00 (no matter the values of q) and for q = 5 and ϵ = 0.05. For the case of ϵ = 0.05 and q = 10 and 30 the system ends up in a multicultural state for almost all the range of noise levels. Nevertheless, for sufficiently high noise level the system is abruptly driven to a monocultural state, as in all the others cases.

A possible explanation of the results obtained with this experiment is a s follows. There are three parameters involved: the initial cultural diversity q, the strength ϵ of the agent M and the noise level r, and two different error mechanisms: the copying and including errors. For low values of ϵ the agent M will appear on the set of influence I_i with low frequency. This frequency increases for increasing ϵ. For low values of q the majority of the neighbors of the target agents will be included on I_i because of the low cultural diversity, while the opposite occurs for large enough values of q. Nevertheless, as can be seen from the figure, this complex interplay does not has an important impact on the dynamics of the system along a wide range of noise level and the value of $< S >$ remains almost constant. When $r \geq 0.1$ the copying error becomes the dominant mechanism because independently of the cultural trait an agent has copied from its neighbors, the copying error changes it to any one randomly selected and as it was already said, it deletes the boundaries between different cultural regions. It is also important that the error mechanism stop when the agent becomes inactive. Then, the social influence is the mechanism which allows an agent to be active/inactive and it is also the mechanism which switch on/off the copying error. Hence, as the copying error connects two completely different cultures the homogenization is favored and agents become inactive (and noise stops) when their culture is completely equal to that of its neighbors. Thus, the final monoculture obtained at r = 0.45.

At lower values of noise a non monotonic behavior is obtained for q = 10 and 30 when the strength ϵ has low values (ϵ = 0.05 in this case). The value of $< S >$ first decreases for increasing noise reinforcing the multicultural state. This effect is strongly pronounced at q = 30. In these cases, the initial diversity makes it possible that the set I_i be formed only by the target agent and some of its neighbors. More neighbors will be present on the set of influence for q = 10 than for q = 30. For increasing noise the including error makes the set I_i populated by both agents from cultures which share some traits with the target agent and also with agent neighbors which do not. The agent M will also be included. This interplay seems to produce strong local convergence and drives the system to a multicultural state for a range of noise between 0.01 and 0.1. Finally the copying error drives the system to a monoculture state for even higher values of r.

3.1.4. Dependence on the diversity of the initial culture

In Fig. 17 we represent the normalized value of S as a function of the initial diversity q with no noise (r = 0). Panel a) corresponds to calculations with one-step neighborhood (p = 1) while in panel b) the calculation was done for p = 2. This means that the amount of neighbors of each agent in panel a) are only four plus the agent M while in case b) each agent has now twelve neighbors besides the Mass Media. Different values of the Mass Media strength ϵ are included in both cases. All the calculations shown with full circles correspond to F = 3 while F = 5 was used for the calculations shown with empty ones. Averages over 200 initial random conditions were done. It can be seen that for low values of q the system reaches a close-monocultural state for any value of the strength ϵ. These are rather trivial cases because these extremely low values of q mean a very low initial diversity of the system and a final close-monoculture state is then expected.

A close monocultural states are also induced for high enough values of q but the system is now more sensitive to the values of ϵ than in the region of low q values. In this case, higher values of ϵ induce stronger global final states given by higher values of S. To explain this result, we note that higher values of q mean that initially there is a higher degree of cultural diversity on the society and this is reflected in sets of influence I with low number of neighbors, i.e., each set of influence I_i will be frequently composed by the own agent i and by the agent M, and with low probability by the neighbors since they probably do not share any of their trait values with agent i. The probability of the agent M to be included in the set of influence I_i increases when the value of ϵ is increased. For higher q values, in

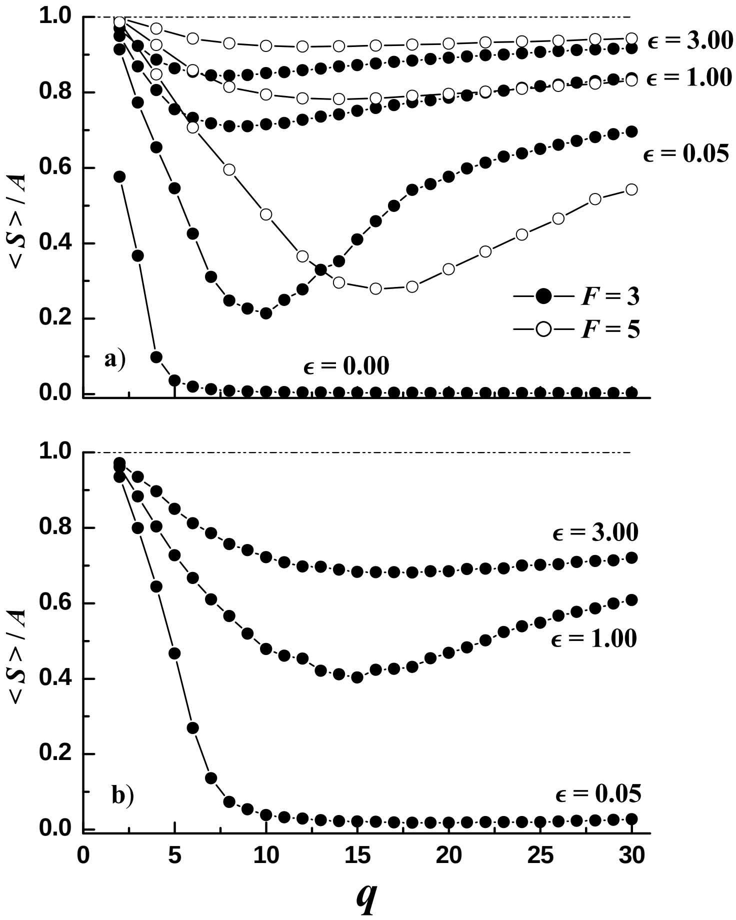

Figure 17. Final absorbing states in the Axelrod model with social influence as a function of q for different values of ϵ. Calculations with F = 3 are included in full circles, while the case F = 5 is shown with empty ones (only in panel a)). a) Calculation done including one-step neighbors (von Neumann neighborhood). b) Calculation done including two-step neighborhood.

cases where I_i is formed by agent i and agent M, the last one will be able to introduce its own value on agent i with probability 0.5. The iteration of this process in time drives the system to a close monocultural state.

A more interesting situation occurs for middle values of q where it is observed a minimum of the $< S > /A$ values as a function of q. The minimum of $< S >$ is very pronounced for ϵ = 0.05 in Fig. 17 a) at q = 10 and 17 for F = 3 and 5 respectively, while in Fig. 17 b) is strongly observed at q = 15 for ϵ = 1.00. For values of q close to the corresponding minimum, the initial diversity is such that besides agent i and M, some of the agent's neighbors are also included in the set of influence I. Then the interacting dynamics involves higher cultural diversity on the set of influence I_i and the agent M fails to induce a strong monocultural final state.

When comparing Fig. 17 a) and b) for calculations with F = 3 it is interesting to note that the increment of the number of neighbors included in the social interaction decreases the size of the biggest culture in the absorbing state and then a more pronounced multicultural state is reached for the same values of q and ϵ. The value of q where the minimum of S is attained also increases. This is a consequence of a direct competition between the higher diversity on the set I_i and the homogenized influence of the Mass Media. In Fig. 17 b), the amount of neighbors to be analyzed for inclusion on the set I_i is twelve. Three times the case in Fig. 17 a), which is only fourth. Then with p = 2, at any value of q there will be, with higher probability, bigger diversity of trait values than in the case of p = 1 and, therefore, a weaker homogenizing effect of the Mass Media is expected. This decreases the possibility of the agent M to drive the system to a monocultural absorbing state because the probability for a trait value of the agent M to appear with the highest frequency on the set of influence is lower and consequently lower values of S are obtained. Then, when considering more neighbors in the social interaction the higher local diversity reinforces the final multicultural state, even with the presence of a Mass Media. The opposite effect is obtained when comparing in Fig. 17 a) the cases with F = 3 and F = 5 for ϵ= 1.00 and ϵ = 3.00. In both cases, the increasing number of features of the agents favors the monocultural states, being $< S >$ for F = 5 slightly above the corresponding value for F = 3. The case of ϵ = 0.05 with F = 5 is qualitatively the same of that with F = 3, but the minimum occurs at a higher value of q.

3.1.5. Culture-Mass Media strength dependence

Figure 18 shows the values of the same parameter $< S > /A$ now as a function of the Mass Media strength ϵ for three different values of q. In this case, noise has also been neglected. One and two-steps neighborhoods are included in panels a) and b) respectively. Calculations are done with F = 3 and F = 5, and also with L = 50 and L = 10, for comparison. For any combination of parameter, a full global state is strongly induced as ϵ increases due to the constant presence of the agent M on the set of influence. As ϵ increases the agent M becomes a factor of "normalization" making its own values constant in time along the dynamical evolution of the system. Even for q = 10 and F = 3 (with L = 50), when the set of influence includes with high probability some of the neighbors of the target agent and the agent M trait values are not on the majority frequency of appearance (see Fig. 17), the latter succeeds in inducing a monoculture as the strength ϵ increases even when for low

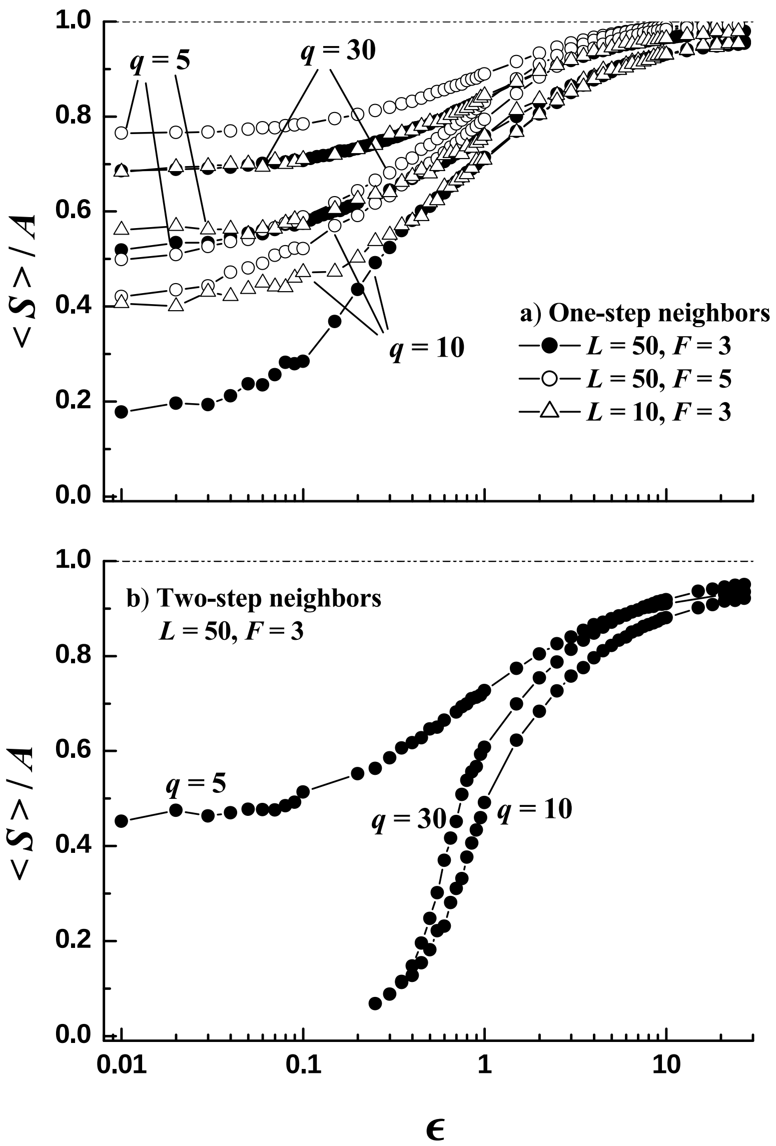

Figure 18. Final absorbing states in the Axelrod model with social influence as a function of the Mass Media strength ϵ for different values of q, F and L. a) Calculation done including one-step neighbors (von Neumann neighborhood). b) Calculation done including two-step neighbors.

values of ϵ the system is strongly polarized. In particular, for $\epsilon \geq 1$, an almost global state with $< S > /A \geq 0.7$ is already induced, i.e., at least 70% of the society belongs to the biggest culture for all values of q included. For $\epsilon = 1$, we have min($p(i, M)$) = 0.25 but the agent M can also induce most of the society to belong to the bigger culture for any initial diversity q. The maximum value of ϵ included on Fig. 18 is $\epsilon = 27$, which gives min(p_{is}) = 0.90.

Moreover, we have also explored other dependencies. Panel a) include results for three groups of parameters. In black circles we show calculations with $F = 3$ and $L = 50$. To study the influence of the network size, calculations were also made with $L = 10$ (shown with white triangles). We found no differences for $q = 40$, while a very small deviation is obtained for $q = 5$ and $\epsilon \leq 0.03$. The greatest differences are for $q = 10$ at values of ϵ below 0.21. In this region of parameters, the values of $< S >$ are higher for lower network size L. Yet, for increasing ϵ calculations with $L = 10$ and $L = 50$ (with $F = 3$) fully coincide.

Panel a) also includes calculations for $L = 50$ and $F = 5$ (white circles). When comparing with $L = 50$ and $F = 3$, one can see that for both $q = 5$ and $q = 10$, the values of $< S >$ are higher and then, as seen in Fig. 17 the increments in the number of features reinforces the monocultural final state of the system. Only for $q = 30$ the values of $< S >$ are lower, but this result is in agreement with that obtained in Fig. 17, where the effect of the same parameter was investigated. Furthermore, when comparing in Fig. 18 panels a) and b) to study the influence of increasing neighborhood, the same results are obtained as when comparing Fig. 17 a) and b) for $L = 50$ and $F = 3$. In this case, the inclusion of two-steps neighbors in the social influence dynamics of the system strongly decreases the value of S/A for the cases $q = 10$ and 30.

4. Conclusion.

We have explored the Axelrod model for the study of culture dissemination. Its achievements and drawbacks have been discussed. In particular, we have examined the extensions of the model that include an external vector field, fixed on time, which simulates Mass Media effects. Different works have been carried out to study this problem. Counterintuitively, it results that the Mass Media induces polarized regimes instead of full global final culture aligned to the external message.

We have also described in section 2. our work that explores a new way to include the external field with dyadic interactions and also in combination with a confidence parameter for the Mass Media. In our case, the Mass Media has the ability to influence all agents of the network with a probability of interaction that is proportional to an extra parameter ϵ (here interpreted as the Mass Media strength). We have called this a *clever* Mass Media. The confidence parameter can also be experimentally estimated on real phenomena through official surveys and therefore it could be correlated with expected results when designing publicity campaigns. The results obtained indicate that for low values of the confidence the system closely reproduces the Axelrod's original results. Increasing confidence values induces the globalization of the final absorbing state, first orthogonal to the external vector and later, for sufficient high values of the confidence, aligned to the Mass Media information. These three phases have been recently obtained in other similar systems [42]. We consider important to perform further studies where the confidence value be an internal parameter with values defined by the internal status and dynamics of the system, as well as studying the dependence of the results obtained with respect to the network size and different noise rates.

Additionally, on section 3. we explored a new mechanism recently proposed to avoid the results of the original Axelrod model that are considered as limitations of the model: the social influence between agents. This mechanism was implemented in combination with clever Mass Media. Similar to the results discussed on Ref. [41] when this mechanism is implemented without the presence of the external field, the system is driven to a polarized final state for all initial diversities. Nevertheless, as far as our calculations have shown, the full polarized state is not attained when the Mass Media is present and a maximum value for cultural diversity is obtained. This maximum value seems to be independently of the Mass Media strength. Furthermore, we obtained that the number of final cultures also follows a power-law dependence when the social influence is at work, but the exponent value found here is lower than that reported for the original Axelrod model with dyadic interaction [23].

The model at non-zero temperature, representing errors when copying traits and in the formation of the social influence set for interaction was also addressed. Contrary to previous works where the noise is present on the whole society, we have included here noise effects only in those agents which are active according to the rules of the social interaction. This allowed us to reach well-defined final absorbing states and correspondingly a higher precision in the description of the noise effects. The results show that social influence makes the system dynamics more stable against the presence of noise and that the latter only has a marginal influence on the general qualitative picture obtained without any errors. It is however worth stressing that noise has in general a positive effect in the formation of monocultural final states, giving rise to global societies for large enough values of it, independently of the other parameters ruling the size of the social influence and the strength of the mass media effect.

Moreover, the dynamics of the system is such that at low and high values of initial social diversity q, a global state is attained with a stronger dependency on the mass media strength at large q values. In the first case ($q \sim 1$) the Mass Media is providing information that is already on the largest frequency of the society and then the results are rather trivial. For the other case ($q \gg 1$) the Mass Media successes in homogenizing the system through the mechanism represented by the additional probability to interact with any agent. The strong diversity isolates agents of the society from its neighbors and each of them becomes subject of influence from the external vector field. An interesting behavior was obtained for intermediate values of q, where the system dynamics attains a minimum in the size of the biggest monocultural cluster. At those values of q the initial diversity is such that the mass media has to compete with a larger number of neighbors of the agent and thus its information is not necessary on the majority. Then, in this case the mass media fails to drive the system to an homogeneous cultural state. These situations resemble the sentence of anthropologist Gregory Bateson: "to produce a change it is necessary to be *different* but, at the same time, it is necessary to be *close enough* to be taken into account" [43]. We think that we have shown that when the local diversity fulfills this condition, the multicultural state is robust enough and the Mass Media fails to homogenize the system. However, we should add that increasing the mass media strength for a fixed network size always reinforces the monocultural state.

Finally, we have shown different calculations for higher values of agents' size (F). The increment of this parameter reinforces the global final state because it makes more probably the interaction of two agents through higher values of the cultural overlap between them. Consequently, the monocultural state becomes more robust for greater values of the initial diversity. For this reason, the critical value q_c where the phase transition occurs when dyadic interaction is at work is higher. Additionally, the polarized state was found to be favored when the agents' neighborhood is increased. This result is a direct consequence of the high local diversity obtained when more neighbors are considered in the set of influence.

As for future research, we think that it would be interesting to conduct further studies to better explore the inter-relations between the dyadic interaction and social influence on a social system. It could be important to elucidate the robustness of the results discussed in this review.

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